Supplementary material: EOM approach

Using a notation as in the main text, the current follows from the retarded GF $G_{dd}^R(t) = -i\Theta(t)\langle\{\Psi_d(t), \Psi_d^\dagger(0)\}\rangle$, which is a 2×2 matrix in Nambu space with spinors $\Psi_d = (d, e^{-i\chi}d^\dagger)^T$. In energy ($\epsilon$) space, it obeys the EOM

$$(\epsilon - E_0 + V_g + i\hat{\Gamma})G_{dd}^R = 1 + 4E_c\Gamma_{Ndd}^R, \qquad (1)$$

where $V_g = 2E_c n_g$, $E_0 = \begin{pmatrix} E_c & 0 \\ 0 & 3E_c \end{pmatrix}$, and $\hat{\Gamma} = \sum_j \Gamma_j(1+s_j\sigma_x)$. Γ_{Ndd}^R in Eq. (1) is the first in a hierarchy of vertex functions ($m = 1, 2, \ldots$),

$$\Gamma_{N^m dd}^R(t) = -i\Theta(t)\langle\{\hat{N}^m(t)\Psi_d(t), \Psi_d^\dagger(0)\}\rangle, \qquad (2)$$

which are generated sequentially through their EOM. In particular,

$$(\epsilon - E_0 + V_g + i\hat{\Gamma})\Gamma_{Ndd}^R = A - i\tilde{\Gamma}G_{dd}^R + 4E_c\Gamma_{N^2dd}^R, \qquad (3)$$

with $\tilde{\Gamma} = \sum_j \Gamma_j s_j \sigma_x$ and

$$A = \langle\{\hat{N}\Psi_d, \Psi_d^\dagger\}\rangle = \begin{pmatrix} \langle\hat{N}\rangle & 0 \\ 0 & \langle\hat{N} - (1-\hat{n}_d)\rangle \end{pmatrix}. \qquad (4)$$

Next we discuss an approximation closing the above set of equations. We see from Eqs. (1) and (3) that the energy dependence of each higher-order vertex function produces a pole in G_{dd}^R, and the scale of the energy spacing between the poles is set by the prefactor $4E_c$ with which the vertex functions appear in the EOMs. For $E_c \gtrsim \Gamma$, the physics is therefore controlled by a small number of poles only, which allows us to close the EOM hierarchy by truncation (effectively keeping just a few poles). An approximation keeping only two poles can be achieved by imposing the variational condition

$$\Gamma_{N^2dd}^R = B\Gamma_{Ndd}^R, \qquad (5)$$

with a Nambu matrix B. From this we obtain the closed equation

$$\Big[(\epsilon - E_0 + i\hat{\Gamma} - 4E_cB + V_g)(\epsilon - E_0 + i\hat{\Gamma} + V_g) + 4iE_c\tilde{\Gamma}\Big]G_{dd}^R = \epsilon - E_0 + i\hat{\Gamma} + 4E_c(A - B) + V_g. \qquad (6)$$

For symmetric contacts, $\Gamma_L = \Gamma_R$, we have $\tilde{\Gamma} = 0$, $\hat{\Gamma} = \Gamma$, all matrices become diagonal, and

$$G_{dd}^R = \frac{B^{-1}A}{\epsilon - E_0 + i\Gamma - 4E_cB + V_g} + \frac{1 - B^{-1}A}{\epsilon - E_0 + i\Gamma + V_g}. \qquad (7)$$

The vertex function correspondingly reads

$$\Gamma_{Ndd}^R = \frac{A}{\epsilon - E_0 + i\Gamma - 4E_cB + V_g}. \qquad (8)$$

This EOM implementation is valid for gate voltages V_g close to resonance for either the $[G_{dd}^R]_{11}$ or $[G_{dd}^R]_{22}$ matrix entries. The on-resonance entry will then be dominated by a single central pole, while the other entry is off-resonant due to the $2E_c$ energy shift between the two entries in the matrix E_0. Through this shift, the Fermi surface lies almost in the center between the two poles of G_{dd}^R, and keeping just these two poles is sufficient for $E_c \gtrsim \Gamma$. For $E_c < \Gamma$, however, higher-order poles become important. These come from higher-order fluctuations of $\hat{N}^m$ in the neglected vertex functions. Hence Eq. (7) can be interpreted as a truncation of fluctuations in the number of Cooper pairs. Therefore, while we obtain quantitatively accurate conductance results for $E_c > \Gamma$, due to the truncation of Cooper pair fluctuations we get only a lower bound for the conductance when $E_c < \Gamma$. Within the restrictions imposed by the truncation, however, we achieve an optimal solution for G_{dd}^R by exact fulfillment of the following sum rules.

From Eqs. (7) and (8), a self-consistent computation of four parameters is required, namely $\langle\hat{N}\rangle$ and $\langle\hat{n}_d\rangle$ appearing in the matrix A [see Eq. (4)], and the two diagonal matrix entries B_{11} and B_{22} of B [see Eq. (5)]. (Alternatively, it can be advantageous to fix $\langle\hat{n}_d\rangle$ but adjust V_g self-consistently.) These values are determined from exact sum rules,

$$-\frac{1}{\pi}\int d\epsilon f_d(\epsilon)\mathrm{Im}[G_{dd}^R(\epsilon)]_{11} = \langle\hat{n}_d\rangle, \qquad (9)$$

$$-\frac{1}{\pi}\int d\epsilon f_d(\epsilon)\mathrm{Im}[G_{dd}^R(\epsilon)]_{22} = 1 - \langle\hat{n}_d\rangle, \qquad (10)$$

$$-\frac{1}{\pi}\int d\epsilon f_d(\epsilon)\mathrm{Tr}\{\mathrm{Im}\Gamma_{Ndd}^R(\epsilon)\} = \langle\hat{N} - (1-\hat{n}_d)\rangle, \qquad (11)$$

where f_d is the distribution function on the TS wire; in equilibrium, $f_d = f$. (Note that through the trace over $\mathrm{Im}\Gamma_{Ndd}^R$, averages of the form $\langle\hat{N}\hat{n}_d\rangle$ cancel out.) However, when using Eq. (7), we have to self-consistently adjust four parameters with three sum rules only. We thus impose $\mathrm{Tr}B = B_{11} + B_{22} = \langle\hat{N}\rangle + (1 - \langle\hat{n}_d\rangle)$, which reproduces in the large-E_c limit the resonances found with the ZBWM and master equation approaches around $\langle\hat{n}_d\rangle = 1/2$. The EOM results shown in Fig. **??** then follow (with $T = 0$ and given ratio E_c/Γ) by self-consistently solving for G_{dd}^R.