For these problems, use the simulation "Build a Mach-Zehnder Interferometer".

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down five things about the controls and displayed quantities that you have found out.

2) Consider the experimental setup with only one beam splitter present. The mirrors and second beam splitter are not inserted.

(a) Using matrix multiplication, show that the superposition state $\frac{1}{\sqrt{2}}\left(\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\right)$ is generated by having the matrix representation of the beam splitter act on the initial input state of the photon. What happens to this superposition state at the moment the photon is detected?

(b) For this experimental setup, what are the probabilities for the photon to be detected in Detector 1 and Detector 2? Relate these probabilities to the mathematical representation of the photon after it has passed the beam splitter.

(c) After passing the beam splitter, has the photon split into two distinct photons, each with half the energy of the original photon? Justify your answer with the help of the simulation.

3) Now consider the experimental setup with the mirrors and both beam splitters present (the Mach-Zehnder interferometer).

(a) Using matrix multiplication, determine the quantum state of the photon after passing the second beam splitter. Relate this quantum state to the probabilities for detection in Detector 1 and Detector 2.

(b) Compare the behaviour of the photon at the first beam splitter in the Mach-Zehnder interferometer with the case where only a single beam splitter was present. Is the behaviour of the photon at the first beam splitter changed in any way by adding a second beam splitter?

(c) After encountering the first beam splitter, is the single photon taking both paths simultaneously, or is it actually taking only one path or the other (but we can't know which one)? Justify your answer by considering the detection probabilities shown in the simulation.