For these questions, use the simulation “Successive energy measurements” and work through the simulation, including the step-by-step exploration.

The energy eigenfunctions and energy eigenvalues for a particle confined to a one-dimensional infinite square well with impenetrable walls at $x = 0$ and $x = a$ are $u_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$ and $E_n = \frac{\pi^2\hbar^2}{2ma^2} n^2$ respectively.

(a) i) Write down an explicit expression for the wavefunction $\psi(x,t)$ and the probability density $|\psi(x,t)|^2$ shown in the simulation prior to any energy measurement.

ii) Show that $\psi(x,t)$ is properly normalized.

iii) Find an expression for the oscillation period of the probability density. Assuming the wave function depicts an electron in a well of length $10^{-10}m$, determine the oscillation period in seconds. (electron mass: $9.11 \times 10^{-31}kg$, $\hbar = 1.05 \times 10^{-34}Js$)

(b) What is the probability of a first energy measurement finding i) the ground state energy, ii) the first-excited state energy, iii) the second-excited state energy? Verify your answers qualitatively by running the simulation multiple times.

(c) Assume that a first energy measurement has found the ground state energy. Write down an explicit expression for $\psi(x,t)$ after this first energy measurement. What is the wave function of the particle if a second measurement of energy is performed? Does the result depend on whether or not the second measurement is done immediately after the first? Explain your answer.