For these questions, use the simulation “Spherical Harmonics” and work through the simulation, including the step-by-step exploration.

1) a) Describe what is shown in the graphs of the simulation. Explain how you can determine the probability of finding the electron along a particular direction from the graphs. Do the graphs tell you anything about the distance from the nucleus you are most likely to find the electron?

b) i) What physical quantities are determined by the two quantum numbers $l$ and $m$?
ii) How many different values of $m$ are there for a given value of $l$?

c) Consider the case $m = 0$. Rank the angular probability densities shown in the simulation in terms of the probability of finding the electron along the $z$-axis, from highest to lowest probability.

d) Which angular probability densities are

i) independent of the azimuthal angle $\phi$;
ii) independent of the polar angle $\theta$?

e) Consider the case $m = l$. Are there any directions in space for which the probability of finding the electron is zero?

f) For a fixed value of $l$, which value or values of $m$ maximize the probability of finding the electron in the $x$-$y$ plane?

g) What would you expect the angular probability density to look like for the case $m = l$ in the limit that $l$ tends to infinity?

2) a) Using the Uncertainty Principle, explain why for a quantum particle the value of the z-component of angular momentum $L_z$ must always be less than the magnitude of angular momentum $L$.

b) Calculate the minimum angle between the z-axis and the angular momentum vector as a function of angular momentum quantum number $l$.

c) From your formula to part b), what is the limit of this minimum angle for $l \to \infty$? Interpret your result physically.