For these questions, use the simulation "Gaussian wave packet in an infinitely deep square well (wave packet revivals)" and work through the simulation, including the step-by-step exploration (click on the "Step-by-step Exploration" tab).

1) Explain in detail why the initial wave packet $\psi(x,t)$ is exactly reformed after the revival time $T_r = 4ma^2/\pi\hbar$ (i.e., spell out in detail the derivation in the last step of the "step-by-step exploration").

2) Shown in the simulation is the so-called mirror revival time, equal to half the full revival time, after which the initial wave packet is exactly reformed in shape, but is reflected with respect to the centre of the well.

a) Assume that the initial wave packet at time $t = 0$ is given by $\psi(x)$. With the help of a sketch, show that $\psi(a - x)$ reflects the wave packet about a vertical line through the middle of the well.

b) Show that for the infinitely deep square well energy eigenfunctions $u_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$, the eigenfunction reflected about a vertical line through the middle of the well is $u_n(a - x) = -(-1)^n u_n(x)$.

c) Using your results from a) and b), show that $\psi\left(x, \frac{T_r}{2}\right) = -\psi(a - x, 0)$, i.e., that after half the revival time, the wave packet is exactly reformed, but mirrored with respect to the centre of the well (so the sign of ψ changes as well). Hence, show that the probability densities $\left|\psi\left(x, \frac{T_r}{2}\right)\right|^2 = |\psi(a - x, 0)|^2$ and compare with the simulation.