For these questions, use the simulation “Density matrices for a two-level spin system” and work through the simulation, including the step-by-step exploration (click on the “Step-by-step Exploration” tab).

1) Calculate the density matrix ρ and its square ρ^2 explicitly for the following situations shown in the simulation:

a) an equal mixture of particles in eigenstates $|z_+>$ and $|z_->$

b) particles in the eigenstate $|x_+>$

c) particles in the eigenstate $|x_->$

d) an equal mixture of particles in eigenstates $|x_+>$ and $|x_->$.

Which of the states described in a) to d) are pure states, which are mixed states, which are superposition states with respect to the basis $|z_+>$ and $|z_->$? Verify your answers by determining the traces $Tr(\rho)$ and $Tr(\rho^2)$.

2) Imagine the Stern-Gerlach Apparatus shown in the simulation were oriented at an angle θ to the z-axis, in the xz plane. After passing through the SGA, the beam deflected in the positive θ direction consists of particles in the eigenstate
$|\theta_+>= \cos(\theta/2)\,|z_+> + \sin(\theta/2)\,|z_->$.

a) Interpret this formula for $|\theta_+>$ for the special cases $\theta = 0°$ and $\theta = 90°$ shown in the simulation.

b) Determine the density matrix for the state $|\theta_+>$.