For these questions, use the simulation “Tritium decay” and work through the simulation, including the step-by-step exploration (click on the “Step-by-step Exploration” tab).

1) Explain the “sudden approximation” in quantum mechanics. Explain its relevance for the tritium decay described in the simulation.

2) Consider a wave function in spherical coordinates $\psi(r, \theta, \phi)$. As the energy eigenfunctions of the hydrogen atom electron ψ_{nlm} form a complete orthonormal set, one can expand this wave function $\psi(r, \theta, \phi)$ in terms of the ψ_{nlm}:

$$\psi(r, \theta, \phi) = \sum_{n=\cdots}^{\cdots} \sum_{l=\cdots}^{\cdots} \sum_{m=\cdots}^{\cdots} c_{nlm}\, \psi_{nlm}(r, \theta, \phi)$$

What are the ranges for the sums over n, l, and m?

Justifying each step of your derivation briefly, show that the expansion coefficient c_{nlm} can be determined as the overlap integral of ψ and ψ_{nlm}, i.e.,

$$c_{nlm} = \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \psi_{nlm}^{*}(r, \theta, \phi)\, \psi(r, \theta, \phi)\, d\phi\, sin\theta\, d\theta\, r^2 dr$$

3) An electron is in the ground state of tritium as shown in the simulation. The ground state electronic wave function for hydrogen-like atoms (atoms with a single electron) is $\psi_{100}(r, \theta, \phi) = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} e^{-\frac{Zr}{a_0}}$ where Ze is the nuclear charge.

a) Explain why the electron is not anymore in an energy eigenstate after the decay. How does the ground state energy eigenfunction differ before and after the decay?

b) Determine the probability that an electron originally in the ground state of tritium is in the ground state after the nuclear decay.

c) Without detailed calculation, what is the probability for an electron originally in the ground state of tritium to be in a state with angular momentum quantum number $l > 0$ after the decay? Explain your reasoning.