For these questions, use the simulation “Expansion in eigenstates” and work through the simulation, including the step-by-step exploration (click on the “Step-by-step Exploration” tab).

1) For the symmetric triangular wave function shown in the simulation, determine an analytic expression for the wave function $u(x)$ inside the well, including an explicit expression for the normalization constant in terms of $L$, the width of the well.

2) This wave function $u(x)$ can be expanded in terms of the infinite square well energy eigenfunctions $u_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$ with expansion coefficients $c_n$:
$u(x) = \sum_{n=1}^{\infty} c_n u_n(x)$.

a) Write down an integral expression for the expansion coefficient $c_n$.

b) Using symmetry arguments, explain which expansion coefficients will be zero.

c) By integration, derive a general formula for the non-zero coefficients and interpret this formula.

d) Calculate numerical values for the first three non-zero expansion coefficients and compare with the values given in the simulation.

e) Explain the signs of these three coefficients in light of the wave function $u(x)$. What will be the sign of the coefficient $c_7$?

f) If a measurement of energy were made of the particle described by the the wave function $u(x)$, with what probability would one find the energy of the second excited state?