For these questions, use the simulation “Comparison of the half-harmonic and harmonic oscillator” and work through the simulation, including the step-by-step exploration (click on the “Step-by-step Exploration” tab).

Consider a particle of mass m confined to one dimension moving in a half-harmonic oscillator potential given by $V(x) = +\infty$ for $x \leq 0$ and $V(x) = \frac{1}{2}m\omega^2x^2$ for $x > 0$.

a) Make a sketch of the potential.

b) Write down the Schrödinger equation for the region $x > 0$. Explain why the eigenfunctions $u_n(x)$ of the simple harmonic oscillator potential $V_{sho}(x) = \frac{1}{2}m\omega^2x^2$ for $-\infty < x < \infty$ are also solutions of the Schroedinger equation for V.

c) By considering the appropriate boundary conditions, write down the first three energy eigenfunctions of the half-harmonic oscillator in terms of the normalized simple harmonic oscillator eigenfunctions $u_n(x)$. Make qualitative sketches of these eigenfunctions. Using the normalization and symmetry of the simple harmonic oscillator eigenfunctions $u_n(x)$, show that your half-harmonic oscillator eigenfunctions are correctly normalized. Also write down the corresponding eigenenergies.

d) Would the eigenenergies and eigenfunctions change if the potential were $V(x) = \frac{1}{2}m\omega^2x^2$ for $x < 0$ and $V(x) = +\infty$ for $x \geq 0$? If so, how would they change?

e) Describe the motion of a classical particle with total energy $E > 0$ and amplitude A in this potential . Using energy conservation, determine the speed $v(x)$ as a function of position, and sketch $v(x)$.

Defining P_{CL} as the classical probability density, $P_{CL}dx$ is the probability that a measurement of the position of a particle will find it in the region dx. This is equal to the amount of time dt that the particle spends in the region dx, divided by the total time needed for one traversal (i.e., half a period). Using the fact that $dt = \frac{dx}{v(x)}$, determine the classical probability density of the half-harmonic oscillator and sketch P_{CL} as a function of position.