For these questions, use the simulation “Superposition of energy eigenstates in the one-dimensional infinite square well” and work through the simulation, including the step-by-step exploration (click on the “Step-by-step Exploration” tab).

A quantum particle of mass m moves in one dimension under the influence of an infinitely deep square well potential with impenetrable walls at $x = 0$ and $x = a$. The energy eigenfunctions and energy eigenvalues are $u_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$ and $E_n = \frac{\pi^2\hbar^2}{2ma^2} n^2$ respectively.

Assume the particle is in an equally weighted superposition of the ground state u_1 and the excited state u_n with $n > 1$.

a) Write down expression for $\psi_{1n}(x, 0)$, the wave function at time $t = 0$, and for $\psi_{1n}(x, t)$.

b) By integration, determine the expectation value of position, $< x(t) >$ with respect to $\psi_{1n}(x, t)$.

c) Using your expression from part b), for which values of n will the expectation value of position be time-independent? Verify your result using the simulation. If $< x >$ is independent of time, does this imply that the probability density does not vary with time?

d) Determine the oscillation amplitude of $< x >$ in units of $a/2$ with respect to ψ_{12}, an equally weighted superposition of the ground state and the first excited state. Verify your result using the simulation.