For these questions, use the simulation "Comparison of energy eigenfunctions for the one-dimensional potential wells of finite and infinite depth" and work through the simulation, including the step-by-step exploration (click on the "Step-by-step Exploration" tab).

1) Compare the energy eigenvalues and eigenfunctions for the infinite well and the finite well shown in the simulation, and list qualitative differences. Which of these differences become more pronounced with increasing bound state energy?

2) For the finite well, the form of the energy eigenfunction in the classically forbidden region is

$\psi = Ne^{qx}$ for $x < 0$ and

$\psi = Ne^{-qx}$ for $x > L$, where N is a normalization constant and $q = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}$ with m as particle mass.

a) Using these expressions, explain the energy dependence of the "leakage" of the wave function into the classically forbidden region. Compare with the simulation.

b) Show that the "leakage" of the wave function into the classically forbidden region vanishes in the limit $V_0 \to \infty$. Compare with the graphs shown in the simulation.

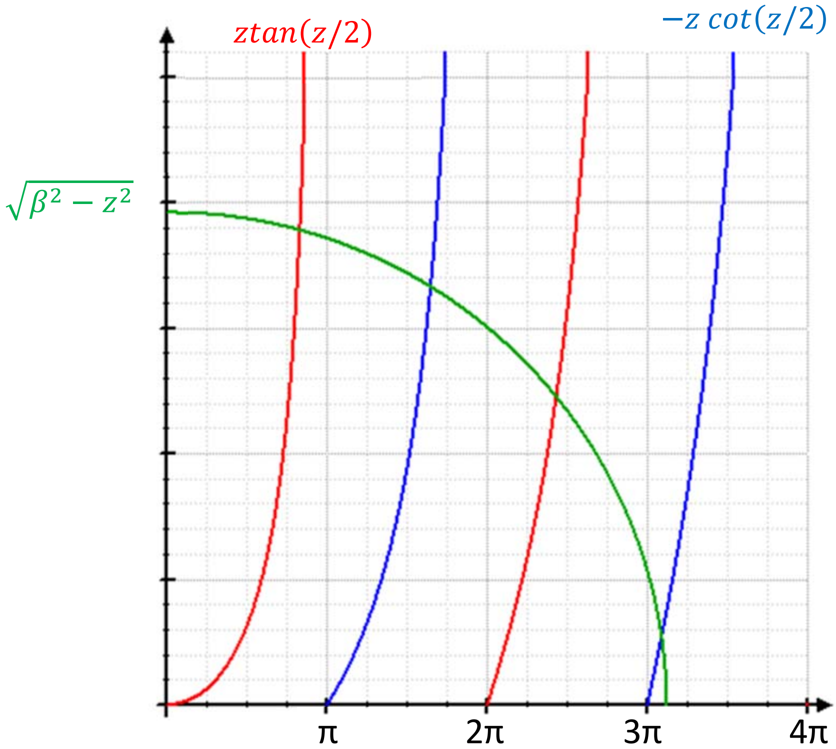
3) The finite well energy levels are found as solutions of the equations

$z \tan\left(\frac{z}{2}\right) = \sqrt{\beta^2 - z^2}$ and

$-zcot\left(\frac{z}{2}\right) = \sqrt{\beta^2 - z^2}$, where $\beta^2 = \frac{2mV_0L^2}{\hbar^2}$ and the energy eigenvalue is $E = \frac{\hbar^2 z^2}{2mL^2}$.

The sketch below shows graphs of $z \tan\left(\frac{z}{2}\right)$, $-zcot\left(\frac{z}{2}\right)$ and $\sqrt{\beta^2 - z^2}$ as a function of z

for the well depth shown in the simulation. Explain using the graph why the energy level in a finite well is lower than the corresponding energy level in an infinite well of the same width, and why the difference is most pronounced for the highest-lying energy level. Compare with the graphs shown in the simulation.



$z\tan(z/2)$
$-z\cot(z/2)$
$\sqrt{\beta^2 - z^2}$
z
π
2π
3π
4π