For these questions, use the simulation “Scattering from a finite-depth square potential well” and work through the simulation, including the step-by-step exploration.

Assume that the incident wave function is a plane wave $\psi_i(x) = e^{ikx}$ with wavenumber $k = \frac{\sqrt{2mE}}{\hbar}$. The transmitted wave function is given by $\psi_t(x) = Te^{ikx}$. The transmitted intensity is $|T|^2 = \frac{16k^2q^2}{(q+k)^4+(q-k)^4-2(q-k)^2(q+k)^2\cos(2qa)}$, where a is the width of the well and $q = \frac{\sqrt{2m(E-V_0)}}{\hbar}$. ($m$ is the beam particle mass)

(a) Give an expression for the reflected intensity.

(b) Determine $|T|^2$ in the limit of very large particle energy, i.e. for $E \to \infty$. Verify your result qualitatively using the simulation.

(c) It can be seen in the simulation that there are special values of E for which there is zero reflection, so-called transmission resonances. Find all transmission resonances shown in the simulation. For each, note down how many wavelengths of ϕ (which is real in this case) fit into the region of the well, $0 \leq x \leq a$. Generalize your results to a formula relating the wavenumber $q = \frac{2\pi}{\lambda_{well}}$ in the region of the well and the length of the well a.

(d) Use your result from part (c) to show that $|T|^2 = 1$ at the transmission resonances.