For these questions, use the simulation "Quantum tunelling" and work through the simulation, including the step-by-step exploration (click on the "Step-by-step Exploration" tab).

1) A quantum particle of mass m is incident from the left onto a potential energy barrier of width a and height V_0 as shown in the simulation. The particle has energy E with $0 < E < V_0$.

a) Obtain the spatial part of the wavefunction in the three different regions $x \leq 0$ (region I), $0 \leq x \leq a$ (region II) and $x \geq a$ (region III) by solving the time-independent Schrödinger equation separately in the three regions. Give explicit expressions for the wavenumbers that appear in your solutions. State which parts of the general solution are zero. You do not need to find explicit expressions for the coefficients in your solution.

b) Explain how the wavefunctions derived in part a) for the three regions relate to the graphs shown in the simulation.

c) Outline how you would proceed to determine the coefficients in the wavefunction derived in part a). You do not need to find explicit expressions for these coefficients.

2) a) Explain in your own words what is meant by the transmission probability T.

b) What value would T have for a *classical* particle with energy i) $E = V_0/2$ and ii) $E = 2V_0$? Explain what values T would have if the energy E of the *classical* particle were gradually increased from $E = 0$ to $E = 2V_0$.

3) Choose settings in the simulation that lead to a relatively large transmission probability. Explain how you could approximately determine the value of the the transmission probability just from the graphs shown.

4) a) Starting from the de Broglie relation between momentum and wavelength, derive an expression for the kinetic energy as a function of wavelength. How can you graphically see that a quantum particle has high kinetic energy?

b) Do you agree or disagree with the following statement:

The quantum particle loses energy as it passes through the barrier.

Use your result from part a), and explain your reasoning.

5) a) Investigate qualitatively how the transmission probability depends on various quantities. Write down your findings, including an explanation of how you are seeing these relations in the simulation. How can you make the transmission probability as large as possible?

b) Inside the potential energy barrier, the wavefunction has the form $\psi(x) = Ae^{qx} + Be^{-qx}$, where the exponent $q = \sqrt{2m(V_0 - E)}/\hbar$ and A and B are constants. The exponentially decaying part dominates ($|B| > |A|$). Using this formula, explain your relations from part a) between the transmission probability and various quantities.