For these questions, use the simulation "Time development of two-level quantum states in the Bloch sphere" in the QuVis HTML5 collection.
https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/bloch-timedev/bloch-timedev.html

1) Have a play with the simulation for a few minutes, getting to understand the controls and displays. Note down three things about the displayed quantities that you have found out.

2) Go to step 2 of the "Step-by-step Explanation". Explain each of the steps in the derivation of the expression for the time-dependent quantum state $|\psi(t)\rangle$. Describe how you can see this time-dependence in the Bloch sphere representation.

3) Investigate the theoretical measurement probabilities for measurements of S_z, the z-component of spin, and S_x, the x-component of spin, for the different input states. Write down your observations, and qualitatively try to explain them (no calculations are needed).

4) Consider the state $|\psi(t)\rangle = \frac{1}{\sqrt{2}}(\,|\uparrow\rangle\ +\ \exp(i\omega t)\,|\downarrow\rangle\,)$ shown in the simulation.

(a) Start from the general quantum state $|\psi\rangle\ = \cos\left(\frac{\theta}{2}\right)\ |\uparrow\rangle +\ \sin\left(\frac{\theta}{2}\right)\exp(i\phi)\,|\downarrow\rangle$. Determine the angles θ and ϕ for this state. Describe the motion of this state in the Bloch sphere representation. Verify your answer using the simulation.

(b) Calculate the theoretical measurement probabilities for a measurement of S_z on this input state. Verify your probability values using the simulation.

(c) The states $|+\rangle$ and $|-\rangle$ with $S_x = +\hbar/2$ and $S_x = -\hbar/2$ can be written in terms of $|\uparrow\rangle$ and $|\downarrow\rangle$ as follows:

$$|+\rangle\ = \frac{1}{\sqrt{2}}(|\uparrow\rangle +\ |\downarrow\rangle)$$

$$|-\rangle\ = \frac{1}{\sqrt{2}}(|\uparrow\rangle -\ |\downarrow\rangle)$$

Calculate the theoretical measurement probabilities as a function of time for a measurement of S_x on the input state $|\psi(t)\rangle$. Verify your result using the simulation, including at which positions the point on the Bloch sphere is located for special values of the probabilities. Show by calculation that the probabilities sum to one for all times.

5) Consider the state $|\psi(t)\rangle = \frac{1}{\sqrt{5}}(2|\uparrow\rangle + \exp(i\omega t)\,|\downarrow\rangle)$ shown in the simulation.

(a) Determine the angles θ and ϕ for this state. Describe the motion of this state in the Bloch sphere representation. Verify your answer using the simulation.

(b) Calculate the theoretical measurement probabilities for a measurement of S_z on this input state. Verify your probability values using the simulation.

(c) At what times is the theoretical measurement probability maximal for a measurement of S_x on this input state? Where on the Bloch sphere is the quantum state then located? Calculate this maximal probability, and verify your result using the simulation.