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Slow light in photonic crystal waveguides

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Abstract
The physical principles behind the phenomenon of slow light in photonic crystal waveguides, as well as their practical limitations, are discussed and put into context. This includes the nature of slow light propagation, its bandwidth limitation, the scaling of linear and nonlinear interactions with the slowdown factor as well as issues such as losses, coupling into and the tuning of slow modes. Applications in all-optical signal processing appear to be the most promising outcome of the phenomena discussed.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The phenomenon of slow light has captured the imagination of many researchers in the last few years because it offers another level of control over light–matter interactions. So far, the slowdown of light has mainly been pursued not only in atomic media, most notably via electromagnetically induced transparency (EIT) [1], but also in semiconductors featuring population oscillations (PO) [2], nonlinear gain and loss characteristics as well as in fibres [3, 4]. More recently, slow light has also been demonstrated in dielectric structures, especially coupled ring resonators [5, 6] and photonic crystals [7, 8] and it is the latter that this paper is focused on.

The keen interest in slow light in dielectrics is motivated by the fact that slow light adds functionality to a material by structuring alone. This is of great interest, as it allows to enhance the weak interaction in a material that may be of interest otherwise, such as silicon. Linear effects such as gain, thermo-optic and electro-optic interaction scale with the slowdown factor, whereas nonlinear effects may scale with its square [9, 10], as we shall discuss in more detail below. In comparison to single cavities, which are widely studied in the photonic crystal community and which also offer sizeable enhancements, slow light structures offer additional control over the spectral bandwidth of interaction and the phase change achievable in a device.

The slowdown factor S is defined as the ratio of the phase velocity over the group velocity, $S = v_\phi/v_g$. If one also takes bandwidth and dispersion into account [11], one finds that the performance of dielectric slow light devices scales as the refractive index contrast. Therefore, high refractive index structures such as photonic crystals appear particularly promising.

While slow light in photonic crystals has already been observed by a variety of authors [7, 8, 12], the operating point is typically on a near-parabolic dispersion curve near the edge of the Brillouin zone. Therefore, slow light in photonic crystals tends to coincide with high dispersion, which removes most of the advantages of operating in the slow light regime and severely limits the bandwidth that can be utilized [13]. This is not an intrinsic property of the structure, however, but subject to design. Designs based on a better understanding of slow light operation can overcome this limitation, as already shown by several authors [14–16]. Another approach to overcome the bandwidth/dispersion issue is to dynamically tune the structure, as proposed by Yanik and Fan [17]. Losses are another issue currently being debated. It has been proposed that losses in photonic crystal waveguides scale as the square of the slowdown factor [18], but it is not obvious that this holds as a general rule.

We will discuss the suitability of photonic crystal slow light waveguides for enhanced functional devices such as switches, optical delay lines and all-optical storage. A key aspect offered by the photonic crystal system is that it offers a large bandwidth; this is critical for processing ultrashort optical signals. Photonic crystals also offer wavelength flexibility, as their operating point is defined by the lattice constant, unlike slow light schemes that interact with a material resonance.

2. The nature of the delay and its limits

The nature of the delay in a photonic crystal waveguide is easily understood by invoking the familiar ray picture

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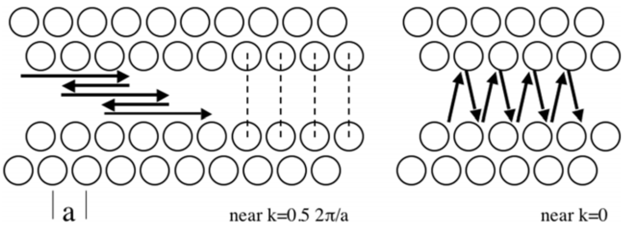

Figure 1. Illustration of the two possible mechanisms for achieving slow light in photonic crystal waveguides, namely coherent backscattering (left) and omnidirectional reflection (right).

commonly used to describe light propagation in a dielectric waveguide. Compared with total internal reflection alone, however, photonic crystals offer two additional features that can lead to the formation of slow modes.

(a) Backscattering. Light is coherently backscattered at each unit cell of the crystal, so the crystal acts as a one-dimensional grating (indicated by the vertical lines on the left-hand side of figure 1). If the forward propagating and the backscattered light agree in phase and amplitude (as they do at the Brillouin zone boundary for $k = 0.52\pi/a$), a standing wave results, which can also be understood as a slow mode with zero group velocity. If we move away from the Brillouin zone boundary, we enter the slow light regime; the forward and backward travelling components begin to move out of phase but still interact, resulting in a slowly moving interference pattern: the slow mode. Further from the Brillouin zone boundary, the forward and backward travelling waves are too much out of phase to experience much interaction and the mode behaves like a regular waveguide mode that is dominated by total internal reflection. In figure 1, the slow light regime is depicted by arrows pointing right and left, for the forward and backward travelling components, respectively. The forward arrows are longer, as if the mode was making three steps forward, two steps back—a slow forward movement. This explanation suggests that the slow light effect is limited to the Brillouin zone boundary. This is not true, however; the key point is that the optical mode is close to a resonance with the structure. Other resonances may be created, such as anticrossing points, where slow light effects also occur [14]. Therefore, by balancing multiple resonances carefully, one may create a slow light regime that spans a considerable fraction of the Brillouin zone.

(b) Omnidirectional reflection. The other unique feature offered by the photonic crystal environment is that there is no cut-off angle; if a photonic bandgap is present, light propagating at any angle is reflected. Even light propagating at or near normal incidence may therefore form a mode, as indicated by the steep zigzag on the right of figure 1. In bandstructure terms, this corresponds to propagation at or near the Γ-point, i.e. $k \approx 0$. It is obvious that such modes have very small forward components, i.e. they travel as slow modes along the waveguide, or for $k = 0$, form a standing wave.

These two effects also represent the two limiting cases for slow light propagation in photonic crystal waveguides; the bandwidth is ultimately limited by the size of the Brillouin zone. In order to achieve a group velocity of c/n_g, with n_g the group index, the maximum bandwidth can then be determined as follows:

$$v_g = \frac{d\omega}{dk} = \frac{c}{n_g} \Rightarrow \Delta\nu = \frac{1}{2\pi}\frac{c}{n_g}0.5\frac{2\pi}{a} = \frac{c}{2n_g a}. \quad (1)$$

As an example, for $n_g = 100$ and a typical period of $a = 0.4\,\mu$m at $1.55\,\mu$m wavelength, one obtains a bandwidth of $\Delta\nu = 3.75$ THz. In practice, and subject to good design and operation below the light line, it may be possible to achieve 20–30% of this bandwidth, which corresponds to around 1 THz. A value of $n_g = 100$ corresponds to a slowdown factor of $S \approx 50$, given the typical phase index in a semiconductor photonic crystal material of around 2. Recent papers have already shown structures approaching this performance [14–16]. Please note that the above discussion holds for symmetric structures, which indeed represent the majority of structures studied in the literature. Introducing asymmetry offers another design parameter, as shown by Ibanescu *et al* [19] and Figotin and Vitebsky [20].

For comparison, the interaction with material resonances that underlies EIT, etc is intrinsically connected with the material gain or absorption, i.e. slowdown and gain/absorption are locked into a Kramers–Kronig type relationship. In photonic crystals, gain and absorption are independent parameters, which allows, for example, to include gain into the structures as an additional feature. There is also complete design freedom in terms of the operation wavelength, which is set by the lattice period a.

3. Scaling of linear interactions with the slowdown factor: switching example

Any type of optical switching device, linear or nonlinear, operates on the basis of a relative phase change, typically expressed as ΔkL. In a Mach–Zehnder interferometer, for example, $\Delta kL = \pi$ for full on/off operation, whereas in a directional coupler, the condition is $\Delta kL = \sqrt{3}\pi$. One usually assumes that Δk can be expressed as $\Delta n k_0$, with Δn the refractive index change induced by an external effect or by the nonlinearity of the material and k_0 the wavevector in vacuum. The slowdown factor S does not seem to enter the discussion, so it appears as if the switching device derives no benefit from the slow light regime. On closer inspection, however, the assumption that Δn is given by the material index turns out to be false, given that one operates with guided modes. One then needs to distinguish between the material index n_{mat} and the effective modal index n_{eff}. This is illustrated in figure 2.

The dispersion curve of a given mode is described by the solid line. Since a mode represents a resonance condition of the waveguide, it can be depicted as a transmission peak, indicated to the right. If the material index is changed, the resonance moves accordingly, and the corresponding change in resonance frequency is proportional to the change in index, so $\Delta\omega/\omega \approx \Delta n/n \Rightarrow \Delta\omega_{12} \propto \Delta n_{mat}$. This is indicated by the dotted line. The change experienced by the guided mode is different, however. It is given by the change in effective modal index and can be derived from the wave equation as follows: $c = \omega/k \Leftrightarrow c_0/n_{eff} = \omega/k \Rightarrow \Delta n_{eff} = (c_0/\omega)\Delta k$, with c_0 the speed of light in vacuum. It is immediately obvious from figure 2 that where the dispersion curve is flat, i.e. in

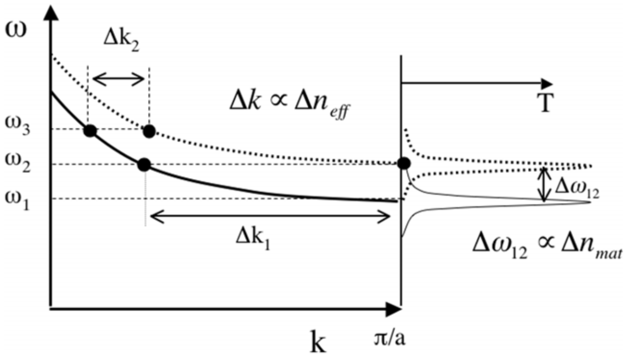

Figure 2. Illustration of the difference between the effect of the material index n_{mat} and the effective modal index n_{eff} in the slow light regime.

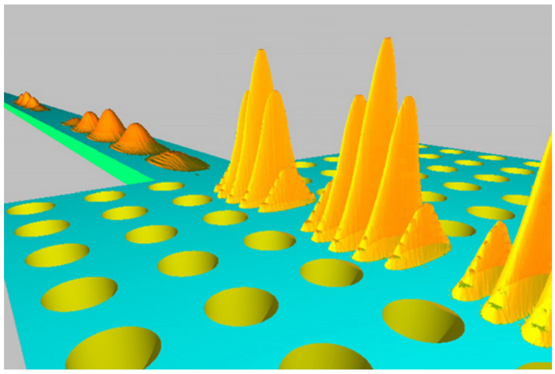

Figure 3. Illustration of pulse compression and intensity increase of a pulse after entering the slow light regime.

the slow light regime, the Δk and therefore the Δn_{eff} is much larger than in the fast light regime where the dispersion curve is steeper, hence $\Delta k_1 > \Delta k_2$. So a slow mode experiences a larger change in effective index than a fast mode, despite the fact that the change in material index Δn_{mat} is the same in both the cases. Since Δk scales with the slope of the dispersion curve, it also scales with the slowdown factor, which allows us to write the condition for switching as

$$\Delta k L = \pi = k_0 S \Delta n_{\mathrm{mat}} L, \tag{2}$$

which now includes the slowdown factor. This highlights the fact that although the slowdown factor is defined in terms of group velocity, the slow light regime offers significant benefits for devices operating on the basis of phase velocity or effective index. A beautiful demonstration of this effect was recently provided by Vlasov *et al* [8]. Utilizing a dispersion curve similar to the one depicted in figure 2, they demonstrated experimentally that a thermo-optically tuned Mach–Zehnder modulator requires less energy when operating in the slow light regime than it does when operating in the fast light regime.

4. Scaling of the intensity with the slowdown factor: Kerr-effect example

If we assume a dispersion-free environment, i.e. one where the different spectral components of a pulse experience the same slowdown factor, a pulse will be spatially compressed when entering the slow light regime. The front of the pulse, entering the slow light regime first, will travel slower than the back of the pulse which therefore catches up. The resulting pulse will occupy less space, i.e. it will be 'spatially compressed' without changing its properties in terms of time and spectrum. If we further assume that no energy is lost at the interface, the same amount of energy is concentrated in a smaller volume, so the intensity of the pulse increases. This effect is illustrated in figure 3 and can be shown mathematically, using a Gaussian pulse as an example. The distribution of a Gaussian pulse can be described by the following functional dependence:

$$I(x) = I_0 \exp(-Sx)^2, \tag{3}$$

where I_0 is the peak intensity and S the slowdown factor, with x a spatial coordinate normalized to the size of the pulse. The full width half maximum (FWHM) of such a pulse scales inversely with S, which is the condition discussed above; a larger slowdown factor results in a shorter pulse. Given that the total pulse energy does not change, the integral over the pulse has to be a constant, which is achieved by the following expression:

$$\int_{-\infty}^{\infty} I_0 \exp(-Sx)^2 \,\mathrm{d}x = I_0 \frac{\sqrt{\pi}}{S}$$
$$\Leftrightarrow \int_{-\infty}^{\infty} I_0 \frac{S}{\sqrt{\pi}} \exp(-Sx)^2 \,\mathrm{d}x = 1. \tag{4}$$

This shows that the peak intensity scales linearly with the slowdown factor and inversely with the size of the pulse; as the pulse is spatially compressed, the peak intensity increases by the same factor in order to satisfy energy conservation. In this respect, slow light structures are similar to optical cavities, which enhance the intensity by a factor $Q/2(m+1)\pi$ for the case of cavities of mode order m.

Overall, nonlinear interaction benefits twofold from the slowdown factor, namely (a) via the enhanced phase change and (b) via the enhanced intensity, such that the interaction in a Kerr-medium, for example, can be written as

$$n_{\mathrm{mode}} = n_0 + n_2 I \Rightarrow n_{\mathrm{mode}} = n_0 + S\big(n_2(I\ S)\big) \Rightarrow \Delta n \propto S^2. \tag{5}$$

So $\chi(3)$-type nonlinearities scale with the square of the slowdown factor. This favourable scaling law is unique to slow light devices based on dielectric structures such as photonic crystals and ring resonators; slow light based on material resonances does not exhibit the same effect. There, the optical power is transferred to an electronic excitation of the medium, so the optical intensity does not increase. This effect was already described by some of the pioneers of EIT, e.g. Harris *et al* [21], who observed that '... there is an unusual type of spatial pulse compression.... As a pulse enters a medium with very slow group velocity ..., its peak electric field and power density are unchanged and the pulse compresses spatially by a factor of c/v_{g}.'

Naturally, there are limitations. The square scaling law assumes zero dispersion and no change of mode distribution. If the dispersion is nonzero, pulses will broaden, with drastic consequences such that the effect of dispersion may outweigh any benefit of slow light compression [13]. It is therefore important to design waveguides with low second and

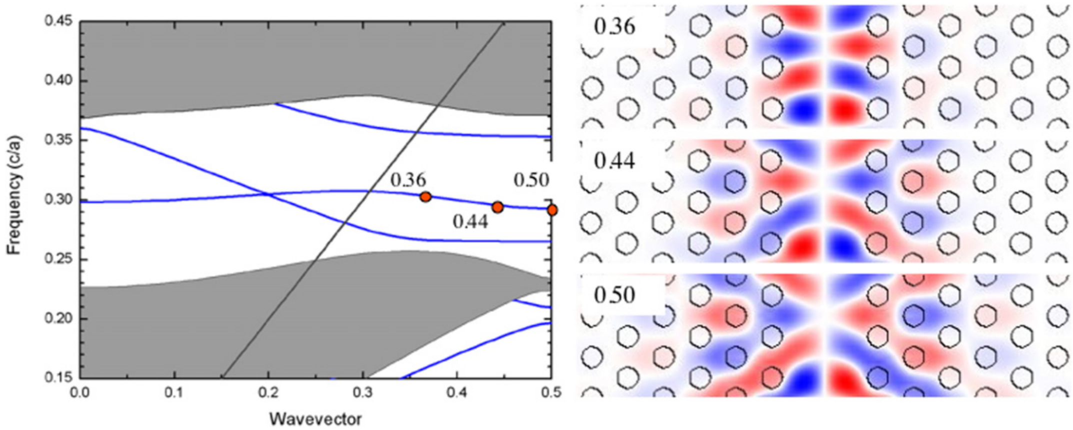

Figure 4. Comparison of the shape of a guided mode in the fast and the slow light regime, exemplified by the odd mode of the system. The odd mode was chosen as it demonstrates the effect more dramatically than the fundamental mode. The respective k-values are indicated both in the mode profiles and in the corresponding bandstructure. As k approaches the Brillouin zone boundary, the mode slows down and samples deeper into the photonic lattice.

higher order dispersion. This has already been proposed by several authors [14–16] who adjusted the dispersion curve of a W1-type waveguide, in particular the presence of an anticrossing with the lattice modes, by adjusting its width and/or the diameter of the innermost rows of holes. In coupled resonator systems, which have a cosine-type dispersion curve with low second order but high third order dispersion, the same effect can be achieved by sidecoupling additional resonators, as proposed by Khurgin [22]. It is interesting to note that in both cases the dispersion is controlled by involving additional resonances, either those of the photonic lattice or of additional resonators. This insight also suggests that the dispersion limitation can be addressed.

The second limitation arises from the assumption of unchanged modeshape. In photonic crystals, this assumption does not generally hold, as illustrated in figure 4. In the slow light regime, the mode samples more of the photonic lattice and therefore assumes a different shape.

So even though the mode may compress in propagation direction, the corresponding lateral spread dilutes some of the benefit one can expect. This leads to the conclusion that an enhancement proportional to $S^P(1 < P < 2)$ can be expected in reality, depending on how well the above conditions on dispersion and modeshape are met. Nevertheless, even for a modest slowdown factor of around 20, an overall enhancement of around 100 can be expected for $P = 1.5$. So the size of a Kerr-type optical switch would drop from centimetres to hundreds of micrometres, or the required switching power would be reduced by two orders of magnitude. This is also equivalent to increasing the nonlinear coefficient (the n_2 or the $\chi(3)$) by two orders of magnitude, simply by appropriately microstructuring the material: a tremendous achievement!

5. Coupling and propagation loss issues

An immediate consequence arising from figure 4 is that exciting a slow mode efficiently is a nontrivial issue. While a fast photonic crystal mode, e.g. the fundamental in a W1 away from the slow light regime, behaves essentially like a total internal reflection mode and can therefore couple efficiently with the fundamental mode of a ridge waveguide, the slow mode requires specially designed couplers. We believe that this is a mode matching rather than an impedance matching problem: the impedance of a given waveguide is given by its phase index and not by its group index; otherwise, it would be impossible to inject light into some of the atomic media that have been used to demonstrate slowdown factors of order 10^7 or higher [1]. This is good news, as a mode-matching problem can be solved by appropriate design. Vlasov and McNab [23] have approached the problem by controlling the termination of the photonic crystal waveguide with respect to the access waveguide, whereas Johnson *et al* [24] have suggested an adiabatically graded transition. A tapered mode transformer as used in [25] offers an alternative solution. A recent study by Biallo [16] compares the different approaches and proposes a high-efficiency injector based on a combination of these effects, so the coupling issue can indeed be addressed.

Propagation loss is another critical issue; any benefit due to slowdown may be lost if the corresponding propagation loss penalty is too high. Recent reports have proposed a square dependence of the propagation loss on the slowdown factor [18]. This is due to the increased intensity already discussed in section 4 above (roughness scattering loss scales as the square of the electric field at the interface [26]), as well as an increase in backscattered light. While the scaling of the loss is still a subject of debate, we believe that the backscattered component can possibly be avoided; in the study by Hughes [18], the occurrence of slow light and the proximity to the Brillouin zone boundary were closely linked. If this link was broken, i.e. if slow light was created away from the band-edge, backscattering may have less impact.

Assuming that the tunability issue can be solved and that loss indeed scales linearly with the slowdown factor, how many bits of information could one then delay or store in a photonic crystal waveguide? State-of-the-art devices exhibit losses of order 2–4 dB cm^{-1} [27, 28]. For a 40 Gbit s^{-1} signal, a single bit is 25 ps or 2.5 mm long (assuming a modal index of $n = 3$), so 4 bits could fit into one centimetre and would suffer approximately 3 dB loss. The situation improves if we move to 160 Gbit s^{-1}, as the individual bits become shorter, while the system can clearly accommodate the bandwidth, so more bits could be stored. Please note that for a given loss and linear scaling, the slowdown factor does not influence the storage capacity of the system as it only reduces the size of each bit but not the associated loss. Even if 10 or 20 bits could be stored, however, such a capacity is very far from the requirement of storing multiple packets (1 packet = 100s of bits) which is demanded by modern telecommunications systems. This suggests that memory applications in telecommunications are doubtful for slow light devices unless systems and networks designers find architectures that benefit from such low capacity, yet all-optical, memories. However, the situation is different in the case of quantum information processing (QIP) where, due to entanglement and the utilization of the phase information, more information can be stored per bit [29], which explains the interest of the QIP community in slow light for optical storage.

6. Discussion and conclusion

In a photonic crystal, the phenomenon of slow light is created by the resonant interaction of the guided mode with the periodic lattice, resulting in the formation of a slow moving interference pattern (the 'slow mode'). By controlling the different resonances contributing to this effect through appropriate design, it is possible to control the resulting slowdown factor and the bandwidth over which it occurs, within the limitations set by the size of the Brillouin zone. Tunability may then be achieved by detuning these resonances, although it is far from obvious how exactly to proceed—tunability is clearly one of the major challenges facing photonic crystals for memory/delay type applications. Ideally, one wants to tune the delay from near zero to the maximum achievable. In EIT and similar media, this is feasible, as the delay directly depends on the pump power.

It is clear from the above discussion that slow light effects in photonic crystal waveguides are most advantageously deployed to enhance (a) linear effects, such as electro-optic and thermo-optic tuning as well as gain, and (b) nonlinear effects such as Kerr-based switching, Raman amplification and possibly parametric effects such as wavelength conversion. Delay lines, e.g. for dynamic time-slot interleaving, are also possible, but true all-optical memory applications appear unrealistic at this stage.

Any all-optical device has to accommodate a sizeable bandwidth, and slow light based devices are no exception; otherwise, they will easily be outperformed by electronic devices. The typical bandwidth achieved with photonic crystals, namely hundreds of gigahertz or even terahertz, is therefore an essential requirement. Hence, we believe that broadband and slow light enhanced all-optical functions such as switches and modulators are the most likely outcomes of this promising research. In order to realize this promise, however, careful device design is required to overcome the dispersion limitation by creating a broadband linear response, to inject light efficiently and to avoid the backscattering losses encountered at the band-edge.

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