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2010 J. Opt. 12 104004

(http://iopscience.iop.org/2040-8986/12/10/104004)

IOP Publishing Journal of Optics
J. Opt. **12** (2010) 104004 (10pp) doi:10.1088/2040-8978/12/10/104004

REVIEW ARTICLE

Dispersion engineered slow light in photonic crystals: a comparison

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Received 9 February 2010, accepted for publication 19 April 2010
Published 24 September 2010
Online at stacks.iop.org/JOpt/12/104004

Abstract
We review the different types of dispersion engineered photonic crystal waveguides that have been developed for slow light applications. We introduce the group index bandwidth product (GBP) and the loss per delay in terms of dB ns^{-1} as two key figures of merit to describe such structures and compare the different experimental realizations based on these figures. A key outcome of the comparison is that slow light based on photonic crystals performs as well or better than slow light based on coupled ring resonators.

Keywords: dispersion engineering, slow light, photonic crystal waveguide, slow light losses

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Slow light is a very promising concept in photonics because it enhances linear phase changes and optical nonlinearities, and offers the potential for storing light [1, 2]. Planar photonic crystals (see figure 1) are amongst the most suitable structures for implementing slow light in a waveguide configuration [1, 3, 4], as exemplified by recent demonstrations of optical switches with a reduced footprint [5, 6], the demonstration of third harmonic generation in silicon [7] or the achievement of substantial tunable delay [8].

The typical starting point is a W1 line defect waveguide [9] which has been the workhorse of photonic crystal waveguide research. A W1 waveguide can operate in a single transverse mode and below the light line, so it is well suited for routing optical signals; it is also intrinsically lossless. The W1 waveguide furthermore offers ultra-compact waveguide interconnects, sharp bends, and most importantly, a slow light regime. On the other hand, it has two major limitations, namely high group velocity dispersion (GVD) [10] and high extrinsic (i.e. imperfection-dependent) backscattering losses [11] in the slow light regime. The strong dispersion, by definition, introduces a strong wavelength dependence of the group velocity, leading to pulse broadening and distortion [9, 10], which is clearly unwanted in any communications system. In addition, broadening reduces the peak intensity of a pulse making it less effective for driving nonlinear effects. The high backscatter loss is due to the high density of states in the slow light regime, with an n_g^2 dependence reported [11], suggesting that the usefulness of slow light waveguides will be rather limited. Fortunately, this is not the end of the road, and both the dispersion and loss issues can be addressed by specially designing the dispersion of the waveguides.

Here, we first develop suitable figures of merit to evaluate the performance of slow light waveguides before discussing the different types of dispersion engineered photonic crystal waveguides. We then compare them to other types of slow light waveguides, such as coupled resonator optical waveguides (CROW) [12] (sometimes referred to as coupled cavity waveguides (CCW) [13]). We finally discuss the dispersion and loss issues.

2. Figures of merit

The key aspects relevant to the design of slow light photonic crystal waveguides include the group velocity, bandwidth, propagation loss and dispersive properties. The first two

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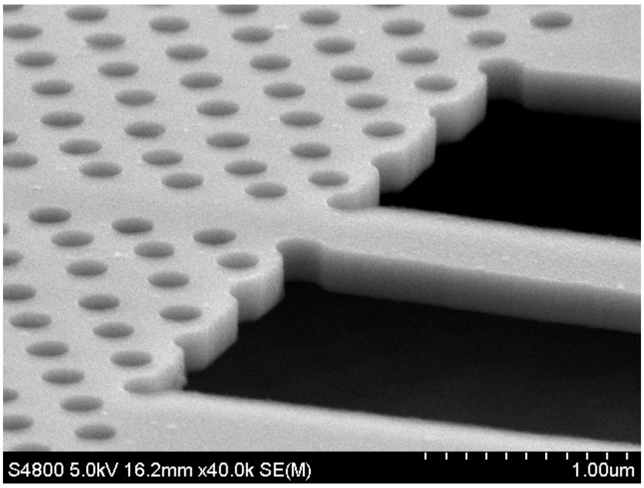

Figure 1. Scanning electron microscope image of a planar photonic crystal waveguide, etched from a silicon slab. The holes comprising the photonic crystal guide the light via the photonic bandgap effect in the row of missing holes, whereas total internal reflection confines the light to the plane of the slab. The dispersion of the waveguide modes can be altered by controlling the size and/or configurations of the holes around the waveguide.

aspects are already encapsulated by the commonly used figure of merit, i.e. the delay-bandwidth product, but the latter two are ignored. This may be a subtle point, but it is rather important; the total delay achievable with a waveguide clearly depends on its length, we can always make a waveguide longer in order to increase the delay, in contrast to single cavity systems, where both delay and bandwidth are intrinsically linked to the quality factor (increasing one automatically decreases the other). Increasing the waveguide length increases the loss as well, however, so the loss should either be included in the figure of merit, or a different figure should be used that is loss independent. We therefore propose to use the group index bandwidth product as an intrinsic figure of merit, as it does not depend on the technology used nor on the length of the device. In combination with this, we propose to use the loss per delay time (in dB ns^{-1}) to quantify the losses in a meaningful way. Using this approach allows us to separate the design (via the group index bandwidth product) from the technology (expressed in loss per delay time).

2.1. Group index bandwidth product

The group index n_g of a waveguide is given by the inverse of the slope of its dispersion curve at the operating frequency or wavelength, since the group velocity $v_g = c/n_g = d\omega/dk$. The range of frequencies over which the group index remains constant is considered the useful bandwidth of the device, and is often (somewhat arbitrarily) defined as the range over which the group index remains constant within ±10% [14]. Using such a criterion may seem rather simplistic, and a limitation based on acceptable levels of higher-order dispersion appears more appropriate. However, in practice, it turns out that the 10% criterion is sufficient for most cases, as most experimental situations tend to be limited by propagation losses rather than by dispersive broadening.

The group index bandwidth product (GBP), given by GBP $= n_g\Delta\omega/\omega$, can be linked to the more established delay-bandwidth product via the device length L. It is itself independent of L, however, and therefore independent of the technological realization and the acceptable loss. Typical values are in the range of 0.1–0.4, e.g. a 1% bandwidth (15 nm at 1550 nm wavelength) and a group index of 30 would yield a value of 0.3.

2.2. Loss per unit time

Next, the question arises whether to describe the losses occurring in the slow light regime in terms of loss per unit time or in terms of loss per unit length. Currently, the loss of a device is expressed almost exclusively as a loss per unit length, i.e. in dB cm^{-1}. Assessing slow light waveguides in this fashion misses the main point of their deployment, however, which is to increase the delay or to increase the light–matter interaction in a shorter device by increasing the group index. Instead, the loss per unit time seems to be a more suitable measure, since it refers the loss to the achievable delay. Expressed differently, as the group index n_g increases, the undesired loss increases, but so does the desired delay/enhancement; hence, the loss per unit time may be constant, which represents a constant cost/benefit analysis. Therefore, the total delay is one of the key goals of a slow light structure, making loss per unit time a more sensible figure of merit. This value of the loss per delay is expressed in dB ns^{-1}, and typical values are between 10 and 100 dB ns^{-1} for the waveguide-type devices discussed here.

3. Dispersion engineering in photonic crystal waveguides

In order to address the dispersion problem of W1 waveguides, a number of dispersion engineering methods have been developed [14–17] that allow a limited level of control over the shape of the dispersion curve. These methods all achieve the desired ‘flat-band’regions, i.e. a section of the dispersion curve where its slope is low and constant. This means that the group index seen by an optical signal is constant across the signal’s bandwidth, and the signal is not distorted. Achieving flat-band slow light with a sizeable bandwidth (values of 5–15 nm @1550 nm are typically being achieved) is clearly beneficial for a number of applications. Modulators benefit as they can accommodate a large signal bandwidth and are tolerant to technological and environmental fluctuations; nonlinear effects benefit because a larger bandwidth means that shorter duration pulses can be accommodated, leading to higher available peak powers; delay lines benefit because a larger bandwidth results in larger storage capacity. Naturally, and in common with all other types of slow light systems, the bandwidth can only be extended at the cost of the group index, so the two are inversely related, which again supports the notion of the group index bandwidth product as the suitable figure of merit.

3.1. Mechanism for slow light in photonic crystal waveguides

The mechanism for generating slow light in photonic crystal waveguides is relatively straightforward. The holes that form the boundary of the waveguide on either side can be understood as periodic constrictions; where there is a hole, the waveguide is narrow, and where there is not, the waveguide is wider; this narrow-wide-narrow periodic structure forms a Bragg mirror. If the Bragg condition is fulfilled, which corresponds to the Brillouin zone boundary of the waveguide (i.e. $\lambda/2 = na$, where a is the period), a standing wave pattern forms. By operating away from the Bragg condition, the light that is coherently scattered by the mirror planes and the incoming light together form an interference pattern that is slowly moving forward. This interference pattern is commonly referred to as the 'slow mode' [1].

So far, we have only considered the periodicity in the propagation direction, which leads to the 'Bragg mirror' effect that causes the interference pattern. The fact that the photonic crystal is periodic in two dimensions, however, means that there is also a Bragg mirror perpendicular to the propagation direction; this perpendicular periodicity can also confine light and support additional modes [18]. The modes of a photonic crystal waveguide can therefore be grouped into either index guided or bandgap guided modes.

Because both types of modes may have the same symmetry, they can anticross; such an anticrossing clearly alters the shape of the dispersion curve of a given mode, and it is this anticrossing that is being exploited for dispersion engineering. The anticrossing and therefore the dispersion curve can be altered by either changing the index guided mode (e.g. by altering the width of the line defect) or by changing the gap-guided mode (by changing the nature of the lattice), which is the common theme of all dispersion engineering methods.

3.2. Different dispersion engineering methods

The first dispersion engineered slow light waveguides were realized by reducing the width of the line defect [11, 17, 18]. In their theoretical work, Petrov *et al* achieved a group index of 50 and a group index bandwidth product of 0.16 [17], thereby setting the first important benchmark. The problem with the chosen method of reducing the waveguide width is that the scattering losses are increased as the overlap of the optical mode with the hole surfaces increases. Such increased loss was later demonstrated experimentally by Kuramochi *et al*, who demonstrated a loss-figure of 5 dB cm^{-1} for W1 waveguides and a loss-figure of 30 dB cm^{-1} for the narrower W0.7 waveguides [11].

Alternatively, dispersion engineering can be realized by selectively altering the size or position of individual rows of holes (see figure 2). For example, Frandsen *et al* changed the hole size [16] and achieved a group index bandwidth product of 0.24. Li *et al* [14] changed the hole position perpendicular to the waveguide and achieved a group index bandwidth product of around 0.3. Non-intuitively, Baba *et al* [19] changed the hole position longitudinally, i.e. they shifted rows of holes in the propagation direction, and achieved similar performance.

Changing hole size or position is similar in principle, but the hole position is easier to control technologically. Detailed studies have shown that the hole position can be changed much more accurately (<1 nm [20]) than the hole size (2–4 nm) [21]. This increased accuracy is a consequence of the electron beam fabrication process; while the lithography system allows an equally high control of hole position and size, other factors such as variations in the electron beam dose, the resist thickness, development time and temperature affect the hole size, but have very little effect on the hole position.

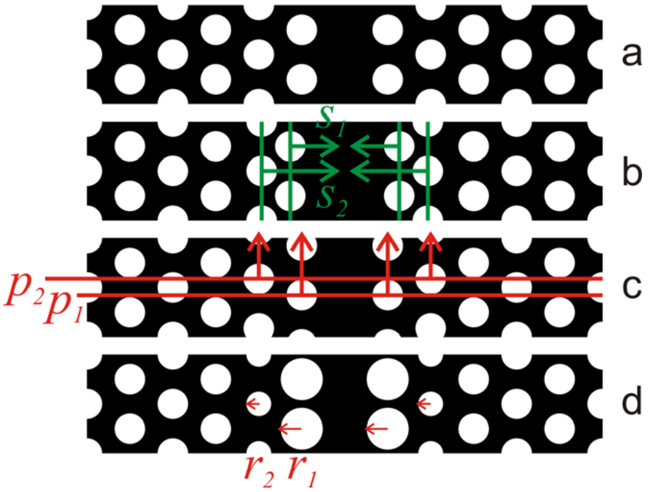

Figure 2. Illustration showing the different mechanisms of dispersion engineering. (a) Standard W1 waveguide. (b) Dispersion engineering through lateral hole position shift, where the position of selected rows of holes (a row running parallel to the defect) is shifted perpendicular to the defect (denoted by s1, s2 etc). (c) Longitudinal hole position shift, where again the position of selected rows of holes is shifted, this time parallel to the defect (denoted by p1, p2, etc). (d) Dispersion engineering through the radius variation method, where the radii of selected rows of holes are altered (with the radii being denoted as r1, r2, etc).

As a result, the hole shift method is proving to be more popular and has been studied more thoroughly. In particular, the lateral hole shift method offers a wide range of group indices, and also tends to have a better GBP (0.3 for [14] compared to 0.24 for [16]). The wide range of group indices and the fact that the GBP is constant over a wide range is demonstrated in figure 3, showing experimental results for laterally hole shifted slow light waveguides. The group index of the flat-band region can range from 23 to 111 with a near constant group index bandwidth product of around 0.28. To our knowledge, group indices in excess of 100 have not been demonstrated experimentally in other dispersion engineered, flat-band slow light waveguides. There is also a growing number of variants of this type of engineering—see [22, 23] for some examples. Also this approach has been extended to slotted waveguides [24–26] and Khayam and Benist [27] have examined the theoretical basis for dispersion engineering, extending it to wider defect waveguides.

3.3. Dispersion compensation

The third method to achieve flat-band slow light is through dispersion compensation. Here, a structure with anomalous dispersion is followed by a structure with normal dispersion of a suitable length, such that the overall group velocity dispersion is zero. This is best implemented using chirped

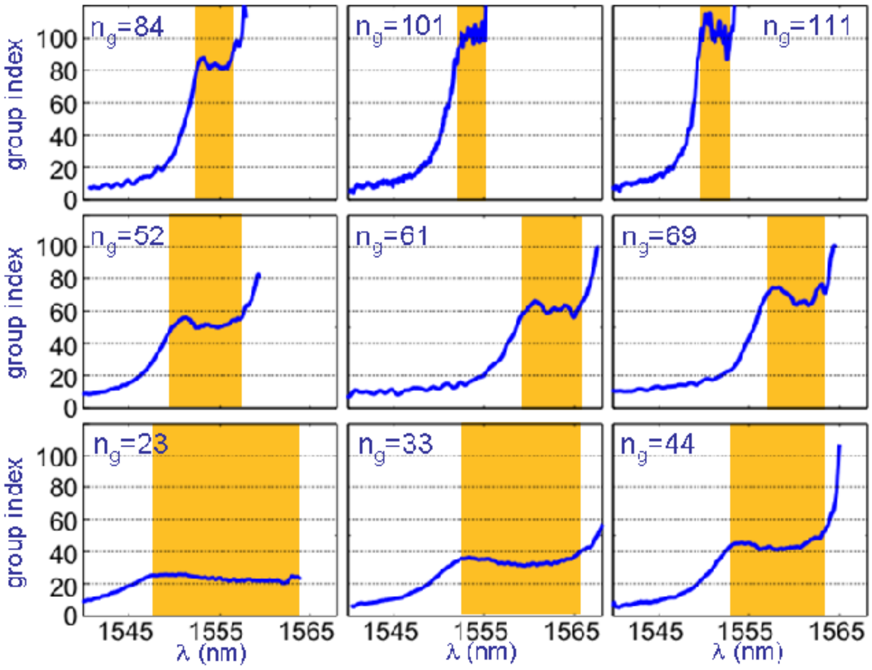

Figure 3. Experimental results on s-shifted photonic crystal waveguides with group indices ranging from 23 to 111 and GBP = 0.28. Shaded areas indicate the flat-band slow light region.

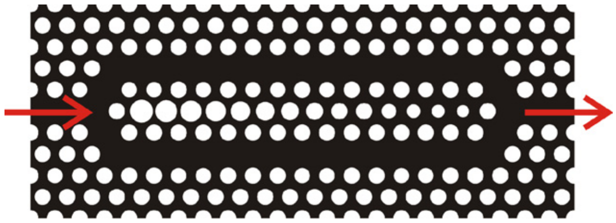

Figure 4. Outline of the PCCW waveguide design. Several different chirping mechanisms exist, of which one (the change in radius of the central row of holes) is shown here.

waveguides, where one or more of the parameters are tuned adiabatically, leading to a shift of the bandstructure, as shown in [15, 28].

This method has been implemented in a photonic crystal coupled waveguide (PCCW), that consists of two identical side-coupled waveguides separated by three rows of holes (figure 4), as first proposed by Mori and Baba in 2005 [29] and demonstrated experimentally by both Notomi *et al* [30] and Baba *et al* [31] in 2007. Each of the waveguides is dispersion engineered such that the PCCW has an s-shaped bandstructure that provides the normal and anomalous dispersion required for the group velocity dispersion compensation.

The PCCW is operated in the even supermode, and adiabatic chirping can be provided by tuning the size of the holes in the central row or by tuning the refractive index through external heating or a combination of both. As a pulse propagates along the chirped waveguide, it first experiences anomalous dispersion, leading to an increase in the temporal pulse width. In the latter section of the waveguide, it experiences normal dispersion, re-compressing the now expanded and chirped pulse, thus recovering the original pulse shape.

Assigning our figure of merit to these structures is a little more complicated, as the local properties of the waveguide vary significantly; an average value across the length of the structure should be used. Using such average values, a group index bandwidth product of 0.38 [31] can be determined, which is the best experimentally achieved value for the GBP to our knowledge.

While the adiabatic chirping and high group index bandwidth product makes these structures very suitable for tunable optical delay lines, they are less suited for nonlinear enhancement, since the variable dispersion inside the structure leads to an increase in the spatial pulse duration, reducing the peak intensity with respect to other slow light waveguides of the same group index.

4. Losses in the slow light regime

Losses in the slow light regime of photonic crystals and other slow light structures, can be decomposed into two main contributions. First there is the coupling loss at the fast-slow light interfaces and second, there is the propagation loss inside the device.

4.1. Coupling loss

Coupling from an access (ridge) waveguide directly into the slow mode of a W1 PhC waveguide is difficult and is associated with a high interface loss penalty [32]. Several schemes have been developed to address this issue, and slow light coupling loss can now be considered a resolved issue. Initial studies by Vlasov *et al* demonstrated that the termination of the photonic crystal plays an important role on the input coupling efficiency. By optimizing this termination, they were able to reduce the loss by as much as 20 dB for high group indices [32]. The dependence of loss on the termination is believed to relate to the excitation of surface states that help to couple light into the slow mode. Yang *et al* have taken this further [33] and used a topology optimization algorithm to find a termination that deliberately creates suitable surface states, achieving improvement of up to 5 dB close to the mode cut-off.

A second approach is to reduce the coupling loss via an adiabatic taper [34]. This was first demonstrated for width modulated waveguides [35]. The key point of this approach is that, in principle, it scales badly with the group index contrast; the larger the difference in group index between the two waveguides concerned, the longer the taper has to be.

It was then discovered in theory [36] and experiment [14], that very good coupling into a slow photonic crystal could be achieved by inserting a fast light section of photonic crystal waveguide between the ridge waveguide and the slow light section of photonic crystal. The principle is based on the understanding that a ridge waveguide consists of a mode with a single k-vector, whereas the periodic photonic crystal waveguide supports Bloch modes that are interference patterns between forward and backward propagating k-vector components. Inserting a section of a fast periodic waveguide facilitates the transition and allows the Bloch mode to build up. Hugonin *et al* [36] then extended this approach, adding an additional transition layer between the fast and slow light photonic crystals and demonstrating coupling efficiency as high as 99% at $n_g = 400$ in a W1. The basis of this improvement is the excitation of evanescent modes that facilitate the build up of power in the slow mode. For dispersion engineered photonic crystals, the additional transition layer is not necessary, as suitable evanescent modes are already available, giving the remarkable result that 90% coupling may be achieved at $n_g > 1000$ with only the simple mode conversion interface [37]. In practice, the mode conversion interface is very straightforward to implement. The first lattice constant of the 10 periods of the photonic crystal is extended by 30 nm providing the fast light section [14], see figure 5. Due to the simplicity of this approach, it is now the preferred solution, dramatically improving the fabrication tolerances in this very critical region.

4.2. Propagation loss

Propagation loss in W1 waveguides has been studied extensively [11, 38–41] for some time and is still an active topic of research. While photonic crystal waveguides are intrinsically lossless below the light line, they do exhibit extrinsic loss due to fabrication disorder, which leads to scattering into radiation modes (out-of-plane loss) and into the backward propagating mode (backscattering loss). The out-of-plane component is proportional to the density of states at the site of the defect and thus scales with group index [40]. However, the backscattering component is proportional to the density of states in both modes and thus, scales with n_g^2. This n_g^2 dependence was first discussed by Hughes *et al* [41] and confirmed experimentally by Kuramochi *et al* [11] on state-of-the-art structures. These scaling relations are in agreement with most research on this topic, making it a serious issue for the practical use of slow light photonic crystals.

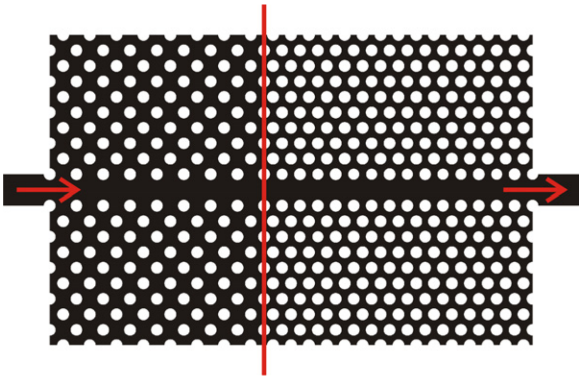

Figure 5. A mode conversion interface. Light is coupled from the access waveguide (on the left) into a 10 period PhC section where the hole spacing along the waveguide is increased, giving a fast mode. This is coupled directly to a slow light waveguide without intermediate transition layers. This design can be applied to standard W1 or dispersion engineered waveguides. The increase in hole spacing is typical 30 nm, however in this image it is exaggerated for ease of demonstration.

Fortunately, dispersion engineering adds another dimension to this problem. Given that the best devices appear to be close to the limits that are technologically achievable (very low sidewall roughness values have been reported, 3 nm RMS roughness with a 40 nm correlation length [11] and 1.4–1.7 nm RMS equivalent radial disorder [42]), then one should look towards waveguide design in order to improve losses further. Petrov *et al* were among the first to study this systematically by numerically demonstrating different levels of backscattering for different designs [22]. They compared devices with holes shifted laterally away from the waveguide with W0.8 structures and showed that the latter ones have up to three times higher backscattering when compared with the lateral shift design.

Preliminary experimental studies on engineered waveguides support this. With a device based on the slow light design of [14], O'Faolain *et al* were able to demonstrate a linear dependence of loss on group index in a delay line experiment (see figure 6) operating at 40 Gbit s^{-1} [43], at least within a limited operating window, and they managed to realize a loss of 35 dB ns^{-1}. This is the lowest value reported so far in any high refractive index contrast system, and it approaches the loss per unit time otherwise only achieved with lower contrast SiON structures [44]. These observations highlight the fact that substantial delay times may indeed be achieved with slow light photonic crystal waveguides and that the n_g^2-dependence does not hold universally.

4.3. Coupled resonator optical waveguide

So far, we have only discussed slow light in a photonic crystal waveguide geometry. A commonly used, and in fact more popular alternative is to create optical delay lines by coupling a series of resonators together. This geometry was first proposed for optical systems by Stefanou and Modinos in 1998 [13],

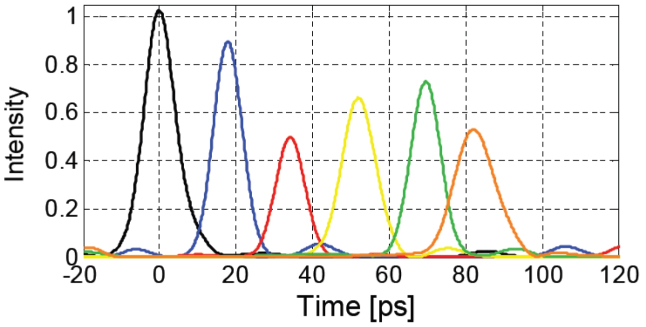

Figure 6. Pulse delay measurement for a dispersion engineered waveguide [45]. The delay is tuned by tuning the wavelength, and the propagation loss corresponds to 35 dB ns^{-1}. (Reprinted with permission from [45]. Copyright 2010 IEEE.)

then termed coupled cavity waveguide (CCW), with the name CROW being proposed shortly afterward by Yariv *et al* [12]; CROW has now become the de facto term used for this type of geometry. Both papers suggest that a CROW is best understood as the optical analog to the tight binding approach well known from solid state physics. A CROW consists of a series of cavities with light progressing along the chain by coupling from one cavity to the next, with each cavity producing a delay, thus creating slow light overall. The majority of CROWs demonstrated are based on ring resonators, and the first CROW-type structure in a photonic crystal waveguide was realized by Olivier *et al* in 2001 [46]. Although the focus of the present paper is on photonic crystal waveguides, we will also include some of the results on ring resonator based CROWs for comparison.

Since the goal is to create a large delay, coupling a large number of cavities together is an obvious approach. CROWs consisting of large number of cavities have been demonstrated in both ring resonator based systems [47] and in photonic crystal cavity based systems [48, 49]. The first substantial chain of 10 photonic crystal cavities was discussed by O'Brien *et al* [48] and later Notomi *et al* [49] have shown that up to 200 photonic crystal cavities can be coupled together successfully. The first and last three cavities were apodized to improve the in- and out-coupling efficiency [50–52]. This resulted in no additional coupling loss associated with the PhC waveguide-cavity interface. The 200 coupled cavity system achieved very impressive results, such as a photon lifetime of the order of 1 ns, giving an out-of-plane loss of 14.5 dB mm^{-1} for $n_g = 100$, which corresponds to a best case loss of 4.5 dB ns^{-1}. The highest group index measured was $n_g = 170$ (directly measured from pulse delay measurements) with no apparent GVD. The bandwidth of this device was relatively low (<2 nm), however, resulting in a GBP of 0.22.

Despite being a very promising slow light structure, there is a major drawback to the photonic crystal CROW approach, namely the difficulty of obtaining a flat-top frequency response. This issue is clearly evident from the large variation of transmission (>20 dB) across the 2 nm bandwidth observed for the 200 cavity system. This variation is firstly due to the finite size of the structure, which means that each cavity has a spectral signature, i.e. it produces an individual peak in the spectrum. Secondly, even tiny imperfections on a nm-scale shift these individual resonances, thus unbalancing the chain. Careful apodization of the system can address this problem, but apodization is harder to obtain than may appear at first sight.

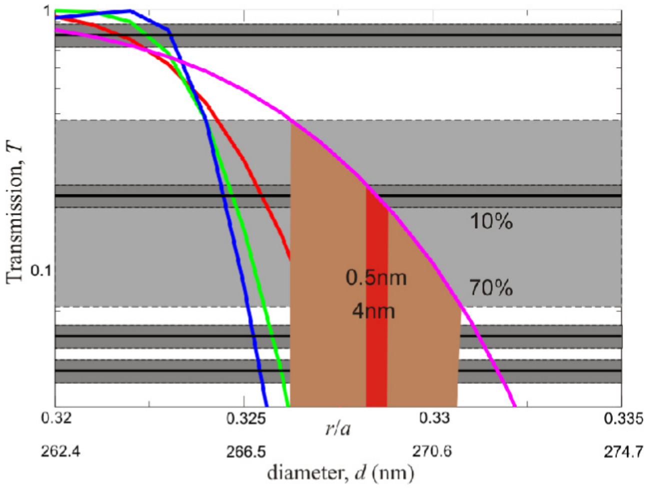

Figure 7. The black lines represent the required mirror reflectivity for an apodized six cavity CROW based on heterostructure cavities. The dark gray bands correspond to a 10% variation on the required values, which is the maximum acceptable variation, while the light gray band is an example of the actual variation that occurs with state-of-the-art fabrication technology. The graph shows that the required accuracy cannot be met in practice.

4.3.1. *Crow apodization.*

The process of apodization aims to optimize the transmission and phase-shift incurred by the coupling region between neighboring cavities in order to produce a flat-top transmission band for the entire structure. Apodization requires for the coupling coefficients to vary gradually between successive cavities, and a 10% deviation from the required mirror transmission already increases the ripple considerably [53]. This 10% value can therefore be taken as a guideline for the maximum variation in the mirror transmission from the prescribed value that can be accepted. In ring resonator CROWs, the 10% condition can be met, as it is possible to control the coupling coefficient between successive rings quite precisely. In PhC mirrors, this is not so simple, and it is difficult to change the coupling coefficient with the required precision. For example, in order to adjust the reflectivity of a photonic crystal mirror to within the allowed 10%, the hole size has to be controlled to an accuracy of better than 0.5nm (see figure 7). Given that state-of-the-art diameter control is on the order of 2–4 nm [21], this condition cannot be met at present, and a 70% variation in the mirror transmission would result instead. Therefore, a true flat-top transmission characteristic is almost impossible to achieve in this system.

The above discussion is valid for CROWs based on PhC heterostructure cavities. Other methods might offer a less critical way of altering the transmission of the individual coupling sections (e.g. waveguide width, as in [54]). However, design difficulties remain, as a PhC mirror is a distributed reflector—so the magnitude and phase of the reflection are entwined, and cannot easily be separated. This is not as

Table 1. Comparison of different dispersion engineered photonic crystal waveguides in terms of the group index bandwidth product (GBP).

Slow light type	$n_{g\text{exp}}$	$n_{g\text{theory}}$	$n_g\Delta\omega/\omega_{\text{exp}}$	$n_g\Delta\omega/\omega_{\text{theory}}$
Lateral hole position shift [14]	111	140 [22]	0.31 [14]	0.32 [22]
Radius reduction [16, 57]	37 [57]		0.24 [16]	
Longitudinal hole position shift [19]	50		0.16	
Dispersion compensation (PCCW)	60 [31]	450 [29]	0.38 [31]	0.5 [29]
PhC CROW [49]	>170		0.22	
RR CROW [47]	35		0.05	
EIT [58]	31 250		0.17	

Table 2. Comparison of different optical delay lines highlighting the loss per delay in dB ns^{-1} as the key parameter. Loss per unit length values are for $n_g = 4$–5 unless stated otherwise. To our knowledge these values are the best reported in the literature so far. *Based on a photonic crystal line defect waveguide with the quoted loss. **Assuming out-of-plane loss only (represents a best case result for cavities with $Q = 1 \times 10^6$), corresponds to the loss of a single operating frequency only, not the full device bandwidth.

Slow light type	Loss dB ns^{-1}	dB cm^{-1}	Max delay (ps)	Max Bit rate (GBbit s^{-1})
Lateral hole position shift [14, 43]	25–40	5–10	200	100
Radius reduction [16, 57]	105	42 ($n_g = 17$)		
PCCW [8]	100	24	72	0.9 ps pulses
RR CROW (SOI)	60 [45]	1	220 [47]	100 [45]
RR APF (SOI) [47]	44	1.7	510	20
PhC CROW [49]	(4.5)**	2*	125	12.5
SiON CROW RR [44]	10	0.35	800	15–25

severe in the case of rings, where the magnitude and the phase of coupling can be controlled almost independently of one another.

CROWs based on ring resonators have been demonstrated with up to 100 cavities [47]. This system has produced a maximum delay of 220 ps, a bandwidth of under 2 nm and a loss of 104 dB ns^{-1}. Also in [47], an all pass filter based on 56 ring resonators was presented with slightly better properties, giving a delay of 500 ps with an associated loss of 44 dB ns^{-1}. The transmission spectra of these devices show much lower ripple than the PhC based CROWs, because the coupling strength of the ring resonators is less sensitive to fabrication disorder than in the case of photonic crystals. In ring resonator based CROWs, the propagation loss can be decomposed into three components, namely (a) the bending loss, (b) the out-of-plane and backscattering loss of the photonic wire on which the ring resonator is based and (c) the additional loss incurred by the coupler, which is visited on every round trip. The bending loss however is very small, typically <0.001 dB per 90° bend, even for ring resonators with only a few micrometer bend radius, so can be neglected. Out-of-plane loss depends on the refractive index contrast and is the main contributor to photonic wire propagation loss over backscattering loss [55]. Melloni *et al* have shown that increasing the delay via the addition of extra or larger rings is preferable to increasing the delay through additional round trips [56]. Additional round trips lead to coherent backscattering, because a given imperfection is visited many times by the mode, with the successive scattering events being in phase. This type of backscattering scales as n_g^2. In contrast, adding extra length to a given ring only increases the number of imperfections that do not necessarily add up coherently, and mainly increase the out-of-plane scattering that scales as n_g, thus leading to a lower additional propagation loss. Finally, they found that the coupling loss may add >0.1 dB per round trip [45], which, depending on the individual resonator Q, may add a substantial additional loss to the system. Therefore, this coupling loss is the reason for the fact that the photonic crystal delay lines may outperform ring-based CROWs, assuming that they are based on the same technology.

5. Discussion

As we have seen, the different approaches to dispersion engineered slow light in PhC waveguides all result in a group index bandwidth product that is much higher than that of a standard W1 waveguide (GBP = 0.01). We have now introduced the relevant figures of merit, the physical principles underlying the slow light designs and given some values for the figures of merits. For easier comparison, these values are all summarized in table 1.

As far as our second figure of merit, the loss per delay in dB ns^{-1}, is concerned, we have compiled table 2. The best loss per delay in silicon-based structures, as far as we are aware, is achieved with the dispersion engineered waveguides based on laterally shifted holes [43]. This is slightly surprising, as ring resonators would be expected to show better propagation loss values, since they are based on photonic nanowires that ultimately exhibit lower losses than photonic crystal waveguides. For example, wires achieve losses below 1 dB cm^{-1} for $n_g = 4$ [59] against 5–10 dB cm^{-1} for $n_g = 5$–6 for the photonic crystals in [43]. This low propagation loss does not guarantee a low delay loss, however, as illustrated by Vlasov *et al* [47], whose devices exhibit a loss per round trip deduced from the propagation and bending loss of 0.03 dB, yet the actual round trip loss is more than 0.1 dB. This suggests that the couplers add a significant fraction of the loss [45] as already mentioned in the previous section; clearly a CROW based on ring resonators is more than just a delay line made of wires with some bends.

5.1. Dispersion length versus attenuation length

So far, we have assumed that propagation loss is the limiting parameter for the dispersion engineered waveguides. Let us briefly verify the validity of this assumption by comparing the dispersion length versus the attenuation length. The dispersion length, L_D can be defined as the length over which the pulse width broadens by a factor of $\sqrt{2}$. This is given by: $L_D = \frac{\tau_0^2}{4\ln 2\beta_2}$ where τ_0 is the initial pulse duration and β_2 the group velocity dispersion. Thus, we see that the faster the bit rate and the higher the GVD, the quicker dispersive broadening will occur. To determine the limiting factor for a delay line, the dispersion length should be compared with the attenuation length defined as the maximum acceptable attenuation (assuming, somewhat arbitrarily, 10 dB of acceptable loss). If the attenuation length is longer than the dispersion length, then our delay line will be dispersion limited and vice versa.

Using the best dispersion engineered delay line from table 2 [43], which has $n_g = 35$ and a loss of 40 dB cm^{-1}, yields an attenuation length of 2.5 mm. Using the value of $\beta_2 = 10$ ps nm^{-1} mm^{-1} exhibited by this device, we calculate a dispersion length of 390 mm at 10 Gbit s^{-1}, 25 mm at 40 Gbit s^{-1} and 3.5 mm at 100 Gbit s^{-1} (assuming non return to zero pulses). Up to the very highest bit rates, it is clearly the attenuation length that dominates the operation of the device. In fact, the measurement in [43], which was conducted at 40 Gbit s^{-1}, was set up such that the operation was attenuation-limited; if higher group indices had been used, dispersion would have had a more noticeable impact. This discussion highlights that both dispersion and attenuation need to be considered, but that typically, attenuation is the more severe limitation.

Ease of fabrication and achievable tolerances are also important considerations, although not easily quantifiable. Here, the dispersion engineered waveguides based on shifting the hole position also appear to have an advantage. Compared to other approaches, positional shifting of holes is relatively easy to implement, e.g. it is easier to control than apodizing a PhC CROW or creating the chirping typically needed for the dispersion compensation approach. Current state-of-the-art technology gives a <1 nm error in the hole position [20], a level that comfortably allows the reliable fabrication of even advanced designs. In terms of the target wavelength response, i.e. achieving dispersion control in a given wavelength window, we have previously established a figure of approx. 3.5 nm change in wavelength for a 1 nm change in hole size [21]. This is a very tight requirement, but slow light operation introduces no additional complexity.

Tolerance to operating conditions is an often forgotten advantage of flat-band slow light. A flat-band region with 15 nm bandwidth at $n_g = 33$, (as in figure 3), gives a slowdown of approx 10 that is temperature insensitive over a range greater than 100 K. For practical applications, this is an extremely useful feature avoiding the need for the compensation techniques such as thermal control required for ring resonator based systems [60], thereby making complex photonic integration circuits more practical. Similarly, a larger operating window reduces the sensitivity critical dimension control, increasing device yields.

6. Conclusion

We have developed improved figures of merits for slow light photonic crystal waveguides and shown that they enable a fair comparison between different types of delay lines, including different types of dispersion engineered waveguides and coupled resonator optical waveguides (CROWs) based on microrings. The first figure of merit is the group index bandwidth product (GBP) ($n_g\Delta\omega/\omega$) to replace the commonly used delay-bandwidth product. The GBP allows a fair comparison of the different types of dispersion engineered waveguides, with the result that the highest GBP observed so far of 0.38 has been achieved with the dispersion-compensated waveguides developed by Baba *et al* [31], followed by a value of 0.31 achieved with the lateral hole shifted structures developed by Li *et al* [14]. The main reason to replace the delay with the group index is that the total delay of a structure is both a function of the group index and of the length of the structure; hence the total delay achieved depends on the acceptable loss, which is both a function of the technology used and the other system parameters, such as detector sensitivity. The group index bandwidth product does not depend on these external parameters, with the only practical limitation that very high group indices can only be observed for short structures as propagation losses quickly become excessive.

The second figure of merit is the loss per delay time (typ. dB ns^{-1}) rather than the loss per unit length (typ. dB cm^{-1}). The argument is that delay rather than length is the parameter that describes the desired function, so one might as well use it to compare the different structures. Here, we have also compared the different types of photonic crystal waveguides with CROWs based on microrings and found that the best performance is achieved by photonic crystals, with the best value of 35 dB ns^{-1} for dispersion engineered waveguides based on shifting rows of holes [14]. It is remarkable that these values are better than the best microring CROWs realized in the same SOI technology with 44 dB ns^{-1} [47], which we explain with the excess coupling loss incurred by the coupled rings [45]. We also discuss the fact that the photonic crystal CROW is difficult to apodize and thus achieve a flat-top frequency response, which is unfortunate as it is the highest performing geometry in terms of losses.

Finally, we highlight that the types of delay lines discussed in this paper are typically limited by propagation loss rather than by dispersive broadening, at least for bit rates up to 100 Gbit s^{-1}. Reducing the propagation loss is the major remaining challenge for slow light. The origins of this loss and ways to reduce it are topics of active research. While it is not yet fully understood, there are indications, e.g. work by Melloni *et al* [56] and O'Faolain *et al* [43], that losses can be much reduced by suitable design, at least within a limited operating window. We look forward to such improved designs, and believe that the figures of merit presented here will provide a suitable framework to assess these.

Acknowledgments

S A Schulz is supported by a EPSRC postgraduate scholarship. L O'Faolain was supported by the EPSRC UK Silicon Photonics and the Nanoscience ERA LECSIN projects. D M Beggs is supported by the EU-FP6 SPLASH project.

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