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Endogeneous Risk in Monopolistic Competition

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ABSTRACT

We consider a model of financial intermediation with a monopolistic competition market structure. A non-monotonic relationship between the risk measured as a probability of default and the degree of competition is established.

JEL Classification: G21, G24, D43, E13, E43.

Keywords: Competition and Risk, Risk in DSGE models, Bank competition; Bank failure, Default correlation, Risk-shifting effect, Margin effect.

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1 Introduction

There is growing evidence, both theoretical and empirical, of a non-monotonic relationship between competition and the risk undertaken by financial institutions. According to so-called traditional views, banks have incentives to take more risk as competition increases since in less competitive markets, there is no need to take on more risk due to a high monopoly rent (Keeley, 1990). However, there is also evidence of the negative relationship between bank risk taking and competition as in Boyd and de Nicolo (2005) and Boyd et al. (2009). There are a few papers where a U-shape relation between bank risk taking and the degree of competition is predicted: in Boyd and De Nicoló (2003), the effect of competition on bank risk taking is investigated when a bankruptcy cost is allowed; in an MMR due to the common shocks there is a default correlation between loans which leads to a U-shape relationship between risk and competition.

We find that a U-shape relationship between probability of default and the degree of competitiveness exists in a monopolistically competitive market as well. This is important since first, in our case, the nature of competition is quite different since FIs compete by differentiated products in contrast to an MMR setting, where they compete by a single product and, second, we have a continuum of banks.¹

A U-shape relationship between competition and risk has been found in Martinez-Miera and Repullo (2010, MMR hereafter). They consider the case of imperfectly correlated loan defaults where the probability of default is endogenously derived by entrepreneurs. The supply side is characterized by a finite number of banks engaged in Cournot competition for entrepreneurial loans. However, it is well known that banks do not supply identical financial products so as a more realistic case, we consider monopolistic competition between a continuum of financial intermediaries, while keeping imperfect correlation in loan defaults as an important and realistic feature. In our setting, the entrepreneurs purchase a basket of differentiated financial products, characterized by constant elasticity of substitution, each of them supplied by a single bank. In effect, entrepreneurs have to solve the portfolio problem by deriving how much of each differentiated product they have to purchase in order to minimize the borrowing cost.

2 Model

There is a continuum of entrepreneurs, financial intermediaries (FIs) and depositors. Financial intermediaries are monopolistic competitors and provide loans to entrepreneurs. For simplicity, loans are financed by a perfectly elastic supply of funds from depositors at zero price. We built on MMR by adding a continuum of monopolistically competitive banks which provide a variety of intermediary financial products (credits) characterized by their prices (interest rates) r_i .

¹A model with a continuum of banks seems to be more appropriate for the US banking system.

2.1 Entrepreneurs

There is a continuum of penniless risk-neutral entrepreneurs of measure one, indexed by $i \in [0, 1]$. To run the investment project, one unit of capital is needed and the revenue R generated by entrepreneur's i th investment project is a binomial random variable defined as:

$$R = \begin{cases} 1 + \zeta(p_i) & \text{with probability } 1 - p_i \\ \lambda & \text{with probability } p_i \end{cases} \quad (1)$$

where $\zeta(p_i)$ is an increasing and concave function of p_i , reflecting the fact that a project with a higher revenue has a higher probability of default and $\lambda < 1$. When the investment project is undertaken, the probability of its default p_i is endogenously chosen by the entrepreneur.²

There is a continuum of banks of measure one indexed by $j \in [0, 1]$ whose market power in a loan market is modeled in a Dixit-Stiglitz framework: one unit of capital purchased by the entrepreneur is a basket of differentiated financial products with a constant elasticity of substitution $\theta > 1$ – each supplied by bank j

$$1 = \left(\int l_j^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \quad (2)$$

where l_j is a quantity purchased of product j . This approach³ may be a realistic way of capturing competition between FIs at the aggregate level.

The cost of borrowing for the entrepreneur is given by:

$$\int (1 + r_j) l_j dj. \quad (3)$$

where $1 + r_j$ is the price of financial product j .

Combining (1) and (3), the i th entrepreneur's problem can be written as:

$$\bar{u} = \max_{p_i, l_j} (1 - p_i)(1 + \zeta(p_i) - \int (1 + r_j) l_j dj). \quad (4)$$

$$s.t. \quad (5)$$

$$1 = \left(\int l_j^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}. \quad (6)$$

Apart from choosing the probability of default p_i , the entrepreneur i will also choose fractions l_j to minimize the repayment cost subject to (2).

2.1.1 Demand

The FOC of the problem (4) gives us a down-sloping demand that bank j faces from a single entrepreneur i

$$l_j = \left(\frac{1 + r_j}{1 + r} \right)^{-\theta}$$

²See, for example, Alen and Gale (2001), Vereshchagina and Hopenhayn (2009) and Martinez-Miera and Repullo (2010) for further references.

³Some authors use this approach. See, for example, Gerali et al. (2008).

where

$$1 + r = \left[\int (1 + r_j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

is the aggregate gross rate.

Since all entrepreneurs who are in need of investment demand the same amount of capital l_j from bank j , the total demand faced by bank j is:

$$L_j = \left(\frac{1 + r_j}{1 + r} \right)^{-\theta} L(r) \quad (7)$$

where total demand $L(r)$ is exogenously given and is a decreasing function of r .

2.1.2 Distribution of Default rate

As in MMR, we assume that each investment project i is characterized by a latent random variable y_i so that whenever $y_i < 0$, the project is in default state. y_i is defined as

$$y_i = -\Phi^{-1}(p_i) + \sqrt{\rho}z + \sqrt{1 - \rho}\varepsilon_i^4, \quad z, \varepsilon_i \sim \mathcal{N}(0, 1),$$

where z is a common shock, ε_i is an idiosyncratic shock, all independently and normally distributed from each other, $0 \leq \rho \leq 1$ is a parameter which measures the correlation in project defaults, and $\Phi^{-1}(p_i)$ stands for inverse standard normal cdf. Because $\sqrt{\rho}z + \sqrt{1 - \rho}\varepsilon_i \sim N(0, 1)$ we have that $\mathbb{P}(y_i < 0) = p_i$ where p_i is the expected probability of default which will be endogenously selected by the entrepreneur and, in equilibrium, will depend on the loan rate r .

Since, in equilibrium,⁵ all entrepreneurs will choose the same p , the fraction of projects in default (the default rate) conditional on the realization of z is given by

$$\gamma(z) = \mathbb{P}(y_i < 0 | z) = \Phi \left(\frac{\Phi^{-1}(p) - \sqrt{\rho}z}{\sqrt{1 - \rho}} \right)$$

from which it follows that a cumulative distribution of the default rate is given by:

$$F(x) = \mathbb{P}(\gamma(z) < x) = \mathbb{P}(z < \gamma^{-1}(x)) = \Phi \left(\frac{\sqrt{1 - \rho}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}} \right). \quad (8)$$

2.2 FI's problem

Here, we focus on FI's optimization problem assuming, for simplicity, that deposits are supplied at zero cost and fully insured. Given the default rate x , the j -th FI's profit is

$$\pi_j = \max [L_j(1 + r_j)(1 - x) + L_j\lambda x - L_j, 0] \quad (9)$$

$$= L_j \max [r_j - (r_j + 1 - \lambda)x, 0] \quad (10)$$

⁴See McNeil et al. (2005) for more details about deriving this relation.

⁵In what follows, the existence of a symmetric equilibrium where all FIs choose the same interest rate r is established.

where the revenue comes from two channels: full repayment from the fraction $1 - x$ of entrepreneurs being in no default state and partial repayment from fraction x of entrepreneurs in default. The cost L_j is repayment to depositors.

Now with the aim of (7) and (8), the expected profit can be written as

$$E(\pi_j) = \frac{L(r^*)}{(1+r^*)^\theta} (1+r_j)^{-\theta} \int_0^{\widehat{x}(r_j)} (r_j - (r_j + 1 - \lambda)x) dF(x, r^*) \quad (11)$$

subject to

$$x \leq \widehat{x}(r_j) = \frac{r_j}{r_j + 1 - \lambda} \quad (12)$$

where r^* is an equilibrium interest rate still to be determined.

In a symmetric equilibrium,⁶ all FIs set the same interest rate r so after dropping subscript j the bank's problem is to set r so as to maximize the function

$$\Psi(r, \theta) = (1+r)^{-\theta} \int_0^{\widehat{x}} [r - (r + 1 - \lambda)x] dF(x, r^*) \quad (13)$$

subject to

$$\widehat{x}(r) = \frac{r}{r + 1 - \lambda}. \quad (14)$$

Proposition 1 *There is an internal solution to the bank's problem (13).*

Proof. The function $\Psi(r, \theta)$ is continuous for all $r \geq 0$ starting from zero since $\widehat{x}(0) = 0$. On the other hand, when $r \rightarrow \infty$, we have $\widehat{x} \rightarrow 1$ and $\Psi(r \rightarrow \infty, \theta) \rightarrow 0$. ■

Now from the first-order condition $d\Psi(r, \theta)/dr = 0$, we have that the equilibrium interest rate r is given as a solution to the following equation:

$$\int_0^{\widehat{x}} [\theta[(r + 1 - \lambda)x - r] + (1+r)(1-x)] dF(x, r) = 0. \quad (15)$$

For a wide range of parameters ρ and θ , the numerical computations show that $r'(\theta) < 0$ as shown in Figure 1 (a).

3 Risk and Competition

In this section, we establish the existence of a U-shape relationship between the risk of the financial institution measured as the probability of default and the degree of competition. First, note that since in symmetric equilibrium all banks set the same interest rate r , the entrepreneur's problem can be rewritten as:

⁶Here, we implicitly assume that there is no social cost related to FIs. In other words, the total amount of available loans is equal to the sum of granted loans that is $\left(\int l_j^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}} = \int l_j dj$ which is possible only in a symmetric equilibrium.

$$u(r) = \max_p (1 - p)(\zeta(p) - r)^7 \quad (16)$$

where r is calculated from (15) for a given θ . Solving (16) gives us $p(r)$ which enters into the distribution of default rates (8).

As follows from (11), the risk or the probability of bank failure is the probability that the default rate x exceeds the threshold $\hat{x}(r)$ given by (14), that is

$$Risk(r) = \mathbb{P}(x > \hat{x}(r)) = \Phi \left(\frac{\Phi^{-1}(p(r)) - \sqrt{1 - \rho} \Phi^{-1}(\hat{x}(r))}{\sqrt{\rho}} \right) \quad (17)$$

where $p(r)$ is the solution of (16).

Differentiating *Risk* with respect to θ , we get:

$$Risk'(\theta) = \frac{\Phi'(\cdot)}{\sqrt{\rho}} \frac{d\Phi^{-1}(p)}{dp} p'(r) r'(\theta) - \frac{\Phi'(\cdot)}{\sqrt{\rho}} \sqrt{1 - \rho} \frac{d\Phi^{-1}(\hat{x})}{d\hat{x}} \hat{x}'(r) r'(\theta). \quad (18)$$

As competition rises, the interest rate falls which means that always $\Phi'(\cdot) > 0$, so that the sign of the first term (risk shifting effect) is negative since $d\Phi^{-1}(p)/dp > 0$, $p'(r) > 0$, and $r'(\theta) < 0$ while the second term (margin effect) is positive since first $d\Phi^{-1}(\hat{x})/d\hat{x} > 0$ and $\hat{x}'(r) > 0$.

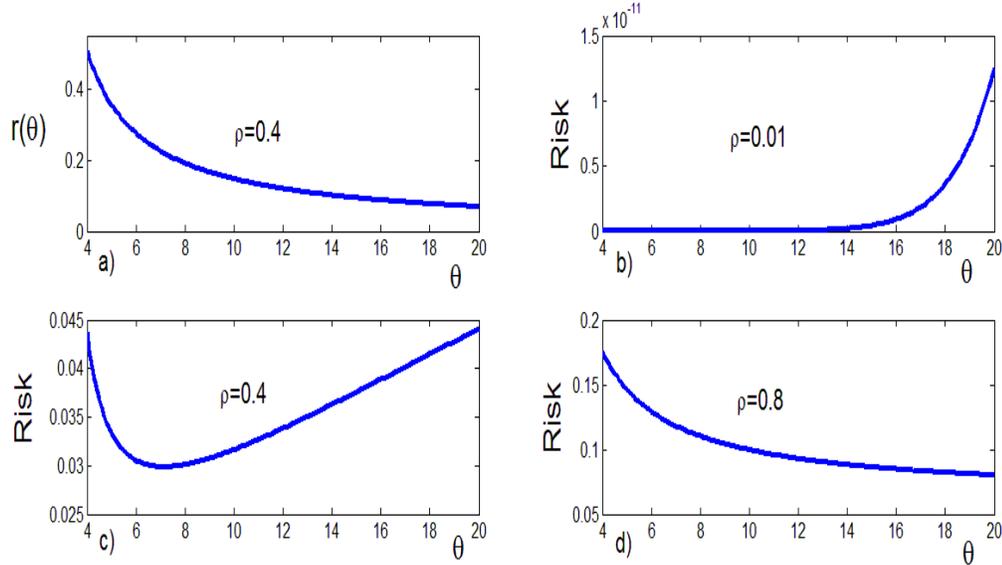
The discussion of (18) as a function of θ closely resembles that in MMR in the case of Cournot competition. The negative effect (first term) called the risk shifting effect (Boyd and DeNicolò) says that as competition increases, the interest rate goes down which, in turn, decreases the probabilities of default. The other side (second term) called marginal effect tells us the opposite: as competition rises, the revenue from performing loans goes down, thus making banks riskier.

The interplay between the risk shifting effect and the margin effect is reflected in a U-shape relation as shown in subplot a) in Figure 1. A simple sensitivity analysis shows that the U-shape relationship between risk and competition is sensitive with respect to the correlation coefficient ρ . Depending on which of the two above mentioned effects dominates, the impact of competition on the risk of bank failures may be positive or negative as demonstrated on subplots b) and d) in Figure 1. When loan defaults are perfectly correlated in the absence of idiosyncratic shocks, the margin effect disappears completely.

4 Conclusion

We showed the existence of a non-monotonic (U-shaped) relationship between the risk taken by FIs and competition in a monopolistically competitive market. This is relevant since first, our finding is evidence that an MMR result holds for a wider spectrum of market structures. Second, the existence of an 'optimal' level of competition with respect to the default rate is something that must be accounted for in any macroeconomic model with financial frictions. Especially an U-shaped relation between competition and risk should be taken into consideration by policy makers.

⁷As in Martínez-Miera and Repullo (2010), in order to have an interior solution $0 < p < 1$ the condition $\zeta(0) - \zeta'(0) < r < \zeta(1)$ is imposed.



1.pdf

Figure 1: $r'(\theta) < 0$ as shown in a) for $\rho = 0.4$. For a sufficiently small ρ , there is only a marginal effect shown in b) as opposed to risk shifting when ρ is sufficiently large, shown in d). Both effects can be seen in subplot c). In all graphs $p = 0.01 + 0.5r$ as in MMR (2010) and $\lambda = 0.6$.

5 Acknowledgment

Part of this work was done at the Department of Economics, NYU, during my visit in September 2012. I am grateful to Tatiana Damjanovic, Douglas Gale and Boyan Jovanovic for useful comments and suggestions. All remaining errors are mine.

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