

CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
WORKING PAPER SERIES



CDMA12/09

E-stability in the Stochastic Ramsey Model*

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SEPTEMBER 20, 2012

ABSTRACT

Analytical expectational stability results are obtained for both Euler-equation and infinite-horizon adaptive learning in a simple stochastic growth model. The rational expectations equilibrium is stable under both types of learning, though there are differences in the learning dynamics.

JEL Classification: E62, D84, E21, E43.

Keywords: Euler-equation learning, Infinite-horizon learning, expectational stability.

*The first author acknowledges support from National Science Foundation Grant no. SES-1025011 and the second author acknowledges support from ESRC Grant RES-062-23-2617. Corresponding author: Kaushik Mitra, email: Kaushik.Mitra@st-andrews.ac.uk.

1 Introduction

Adaptive learning has been increasingly employed in macroeconomic models to supplement and extend the analysis under rational expectations (RE). On this approach agents are modeled as econometricians who estimate and update a forecasting rule that nests the REE (RE equilibrium) of interest. Adaptive learning is used, firstly, to determine whether RE can arise as the outcome of a boundedly rational learning process. Because the model under learning is self-referential, stability of RE under learning is not automatic. Second, when stability obtains, learning dynamics based, for example, on discounted least-squares learning, appear to be able in some cases to improve the fit of macroeconomic models to the data. Tools to establish stability under learning, based on the E-stability principle, are well-developed. See, e.g., Evans and Honkapohja (2001).

Macroeconomic models typically assume representative economic agents that solve stochastic infinite-horizon optimization problems. Adaptive learning can then be implemented in more than one way. Under “Euler-equation” learning (EE-learning), agents make decisions based on the relevant Euler equations, arising from their dynamic first-order conditions for optimization, combined with their forecasts of next period’s variables.¹ As discussed by Evans and McGough (2012), such agents make boundedly optimal decisions as well as boundedly rational forecasts.

Under what is sometimes called “infinite-horizon” learning (IH-learning) agents choose their optimal decisions based on forecasts of the entire future trajectory of variables exogenous to their decisions.² In principle EE-learning and IH-learning can have different stability conditions and in practice can yield different dynamics under discounted least-squares learning.³

IH-learning stability is substantially more difficult to analyze than EE-learning, and hence its analysis often employs numerical calculations of E-

¹For examples of EE-learning, see Howitt (1992), Evans and Honkapohja (2001), Bullard and Mitra (2002), Milani (2007), Eusepi (2007), McCallum (2007), Milani (2011) and Slobodyan and Wouters (2012). Multi-step Euler learning approaches can also be developed, as in Branch and McGough (2011) and Branch, Evans, and McGough (2012).

²This approach was emphasized in Preston (2006). Examples of IH-learning include Evans, Honkapohja, and Mitra (2009), Eusepi and Preston (2010), Eusepi and Preston (2011), Mitra, Evans, and Honkapohja (2011) and Evans, Honkapohja, and Mitra (2012).

³IH-learning uses the “anticipated utility” approach discussed by Cogley and Sargent (2008), who compare this to Bayesian learning. For additional discussion of EE vs. IH learning see Honkapohja, Mitra, and Evans (2012) and Evans and Honkapohja (2013).

stability conditions or direct stochastic simulations. While some analytical results have been obtained for purely forward-looking models under IH-learning, such as New Keynesian models without capital, there appear to be few, if any, formal results in models like the RBC model that have predetermined endogenous variables.

The current paper partially fills this gap in the literature for a central building block of macroeconomics, the stochastic Ramsey model. We demonstrate stability under least-squares learning for EE-learning and we formally show stability under IH-learning for a plausible subset of parameter values.

2 The Model

We consider the standard discrete-time Ramsey model with representative agents forming expectations using an adaptive learning rule. Each household maximizes utility subject to a flow budget constraint:

$$\max E_t^* \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \ln(c_s) \right\}, \text{ where } 0 < \beta < 1, \text{ subject to} \quad (1)$$

$$a_{s+1} = w_s + r_s a_s - c_s, \text{ for all } s \geq t, \quad (2)$$

for consumption $c_s \geq 0$ for all $s \geq t$. Here a_t, w_t and r_t are treated as given at t . w_t is the real wage rate, r_t is the gross real rate of return on assets and, since loans are in zero net supply, $a_t \equiv k_t$, i.e. per capita household wealth equals holdings of capital k_t . It is convenient to normalize per capita labour supply to be equal to 1. There is no government spending or taxes. Households are also subject to a “No Ponzi Game” condition that prevents unlimited borrowing. E_t^* denotes the household’s subjective expectations at time t for the future, which under learning may deviate from RE.

The Euler equation for consumption is

$$c_t^{-1} = \beta E_t^*(r_{t+1} c_{t+1}^{-1}).$$

We assume that exogenous shocks have small compact support, which justifies the use below of linearizations in our analysis. It is therefore valid, to first order, for household to have point expectations, so that

$$c_t = \beta^{-1} (r_{t+1}^e(t))^{-1} c_{t+1}^e(t), \quad (3)$$

where $c_{t+1}^e(t)$ and $r_{t+i}^e(t)$ are short-hand for $E_t^* c_{t+1}$ and $E_t^* r_{t+i}$.

The production function is given by $Y_t = f(K_t)v_t$, where Y_t is aggregate output, K_t is aggregate capital, and labor has been normalized to unity. Here v_t is a positive *iid* productivity shock with mean 1. For simplicity we assume no depreciation of capital. Profit maximization by firms entails that w_t and r_t satisfy

$$w_t = (f(K_t) - K_t f'(K_t))v_t \text{ and } r_t = 1 + f'(K_t)v_t. \quad (4)$$

Market clearing determines K_{t+1} from

$$K_{t+1} = f(K_t)v_t - C_t + K_t, \quad (5)$$

where C_t is aggregate consumption. Since we have a representative agent, in equilibrium $K_t = k_t$, $C_t = c_t$ and $Y_t = y_t = f(k_t)$ where y_t is household income. We assume Cobb-Douglas production, so that $f(k) = k^\alpha$, where $0 < \alpha < 1$.

For the non-stochastic case $v_t \equiv 1$ the model has a unique steady state defined by $1 + f'(\bar{k}) = \bar{r} = \beta^{-1}$, i.e. $\bar{k} = (\alpha\beta(1-\beta)^{-1})^{1/(1-\alpha)}$ with $\bar{w} = (1-\alpha)\bar{k}^\alpha$ and $\bar{y} = \bar{c} = \bar{k}^\alpha$. We employ the linearization of the model, as developed below. The RE solution to the linearized model is standard and takes the form

$$k_t - \bar{k} = \lambda(k_{t-1} - \bar{k}) + h\varepsilon_{t-1}$$

where $\varepsilon_t = v_t - 1$ and $0 < \lambda < 1$. The variables w_t, r_t and c_t are linear functions of the state (k_t, ε_t) .

3 Euler-Equation Learning

Linearizing (3) around $k = \bar{k}$ and $r = \bar{r}$ we get

$$c_t - \bar{c} = E_t^*(c_{t+1} - \bar{c}) - \beta\bar{c}E_t^*(r_{t+1} - \bar{r})$$

and $r_t - \bar{r} = f''(\bar{k})(k_t - \bar{k})$. Linearizing also the market clearing equation, and letting $\hat{c}_t = c_t - \bar{c}$ and $\hat{k}_t = k_t - \bar{k}$, it follows that

$$\hat{c}_t = E_t^*\hat{c}_{t+1} - \beta\bar{c}f''E_t^*\hat{k}_{t+1} \quad (6)$$

$$\hat{k}_{t+1} = -\hat{c}_t + \beta^{-1}\hat{k}_t + \bar{c}\varepsilon_t. \quad (7)$$

Under EE-learning agents use (6) with forecasts based on a Perceived Law of Motion (PLM) of the same form as the REE, i.e.

$$\hat{c}_t = b_c + a_{ck}\hat{k}_t + a_{c\varepsilon}\varepsilon_t, \quad (8)$$

$$\hat{k}_{t+1} = b_k + a_{kk}\hat{k}_t + a_{k\varepsilon}\varepsilon_t. \quad (9)$$

Although $b_c = b_k = 0$ in the REE, we assume agents do not know the steady state values of the variables of interest, and thus estimate the intercept as well as the other coefficients of (8)-(9). Under least-squares learning coefficients are estimated and updated each period using recursive least squares.⁴

Using (8)-(9) agents forecast their next period consumption as $E_t^*\hat{c}_{t+1} = b_c + a_{ck}E_t^*\hat{k}_{t+1}$, where $E_t^*\hat{k}_{t+1} = b_k + a_{kk}\hat{k}_t + a_{k\varepsilon}\varepsilon_t$. Then (6)-(7) yields the Actual Law of Motion (ALM)

$$\hat{c}_t = T_{b_c} + T_{a_{ck}}\hat{k}_t + T_{a_{c\varepsilon}}\varepsilon_t, \quad (10)$$

$$\hat{k}_{t+1} = T_{b_k} + T_{a_{kk}}\hat{k}_t + T_{a_{k\varepsilon}}\varepsilon_t. \quad (11)$$

See the Appendix for details. This system characterizes EE-learning.

Collecting the coefficients as $\theta' = (b_c, a_{ck}, a_{c\varepsilon}, b_k, a_{kk}, a_{k\varepsilon})$, the ALM (10)-(11) defines a map $T(\theta)$ from PLM to ALM parameters. The RE arises as a fixed point $\bar{\theta}$ of the map. Looking at the components $(\bar{a}_{ck}, \bar{a}_{kk})$ of $\bar{\theta}$ we see that \bar{a}_{kk} solves $\bar{a}_{kk}^2 + (\beta\bar{c}f'' - \gamma^{-1} - 1)\bar{a}_{kk} + \beta^{-1} = 0$, which has two positive roots, exactly one of which, denoted λ , satisfies $0 < \bar{a}_{kk} \equiv \lambda < 1$. For this solution $\bar{a}_{ck} = \beta^{-1} - \lambda > 0$. The other coefficients are then easily obtained.

Will EE-learning converge to RE? Using the E-stability principle, we know that this is governed by local stability of the E-stability ordinary differential equation

$$d\theta/d\tau = T(\theta) - \theta \quad (12)$$

where $\theta' = (b_c, a_{ck}, a_{c\varepsilon}, b_k, a_{kk}, a_{k\varepsilon})$. Local stability is in turn is determined by the Jacobian matrix $DT - I$ at the steady state. We have:

Proposition 1: The REE of the stochastic Ramsey model is locally stable under EE-learning for all $0 < \alpha, \beta < 1$.

⁴We are here making use of the representative agent assumption. In general an agent's consumption would depend on both their own capital and on aggregate capital. However, because agents are assumed to be *ex-ante* identical, to make identical forecasts, and to follow the same decision-making rules, aggregate and individual capital holdings are perfectly correlated.

Proofs of propositions are in the Appendix. Although numerical results for the (deterministic) Ramsey model are given in Ch. 4 of Evans and Honkapohja (2001), the analytical result in Proposition 1 is new. EE-learning can be viewed as a form of short-horizon decision-making. We now turn to the analysis of long-horizon decision-making for the same model.

4 Infinite-Horizon Learning

The intertemporal budget constraint of the household, which can be derived from the no-Ponzi-game condition, is $PV_t^e(c) = r_t k_t + PV_t^e(w)$, where $PV_t^e(w) = \sum_{j=0}^{\infty} (D_{t,t+j})^{-1} w_{t+j}$ and $PV_t^e(c) = \sum_{j=0}^{\infty} (D_{t,t+j})^{-1} c_{t+j}$ and where $D_{t,t+j} = \prod_{i=1}^j r_{t+i}$, $j \geq 1$. Agents must plan to satisfy this in expectation, i.e.

$$PV_t^e(c) = r_t k_t + PV_t^e(w), \quad (13)$$

where $PV_t^e(c)$ and $PV_t^e(w)$ are expected $PV_t(c)$ and $PV_t(w)$.

With point expectations, we have, e.g., $PV_t^e(w) = \sum_{j=0}^{\infty} (D_{t,t+j}^e)^{-1} w_{t+j}^e$. Forward substitution of (3) gives

$$c_{t+j}^e(t) = c_t \beta^j \left(\prod_{i=1}^j r_{t+i}^e(t) \right) \equiv c_t \beta^j (D_{t,t+j}^e(t)).$$

Thus $PV_t^e(c) = c_t (1 - \beta)^{-1}$ and (13) yields

$$c_t = (1 - \beta) (r_t k_t + PV_t^e(w)). \quad (14)$$

As in (7) we have

$$k_{t+1} - \bar{k} = \bar{y} \varepsilon_t + \beta^{-1} (k_t - \bar{k}) - (c_t - \bar{c}) \quad (15)$$

The w_t and r_t equations are

$$w_t = b_w + a_{wk} k_t + a_{w\varepsilon} \varepsilon_t \quad \text{and} \quad r_t = b_r + a_{rk} k_t + a_{r\varepsilon} \varepsilon_t \quad (16)$$

where $b_w = (1 - \alpha)^2 \bar{k}^\alpha$, $a_{wk} = \alpha(1 - \alpha) \bar{k}^{\alpha-1}$, $a_{w\varepsilon} = (1 - \alpha) \bar{k}^\alpha$ and $b_r = 1 + \alpha(2 - \alpha) \bar{k}^{\alpha-1}$, $a_{rk} = \alpha(\alpha - 1) \bar{k}^{\alpha-2}$, $a_{r\varepsilon} = \alpha \bar{k}^{\alpha-1}$. For simplicity we assume agents know the coefficients of (16); because these equations are not self-referential this does not affect stability.

Linearizing (14) around the perceived steady state $(\bar{c}^e, \bar{w}^e, \bar{r}^e, \bar{k}^e)$ yields

$$(1 - \beta)^{-1}(c_t - \bar{c}^e) = \sum_{j=1}^{\infty} \beta^j [(w_{t+j}^e(t) - \bar{w}^e)] - \bar{w} \sum_{j=1}^{\infty} \beta^{j+1} \sum_{i=1}^j (r_{t+i}^e(t) - \bar{r}^e) + \bar{k}(r_t - \bar{r}^e) + \beta^{-1}(k_t - \bar{k}^e) + (w_t - \bar{w}^e), \quad (17)$$

where

$$\bar{c}^e = (1 - \beta) \left(\bar{r}^e \bar{k}^e + \bar{w}^e (1 - (\bar{r}^e)^{-1})^{-1} \right) \quad (18)$$

is obtained from evaluating (14) at the estimated steady state. The consumption function (17) is the basis of IH-learning in the linearized model.

To implement (17) agents need to forecast future wages and interest rates and also the means \bar{k} , \bar{r} , \bar{w} and \bar{c} . To do this agents estimate

$$k_{t+1} = b_k + a_{kk}k_t + a_{k\varepsilon}\varepsilon_t. \quad (19)$$

From (19) they compute $\bar{k}^e = (1 - a_{kk})^{-1}b_k$ and forecast $\tilde{k}_{t+j}^e = k_{t+j}^e - \bar{k}^e$ for $j = 1, 2, 3, \dots$. Using (16) they then construct forecasts of $w_{t+j}^e(t) - \bar{w}^e = a_{wk}\tilde{k}_{t+j}^e$ and $r_{t+i}^e(t) - \bar{r}^e = a_{rk}\tilde{k}_{t+j}^e$. Finally, \bar{w}^e and \bar{r}^e are obtained from (16) as $\bar{w}^e = b_w + a_{wk}\bar{k}^e$ and $\bar{r}^e = b_r + a_{rk}\bar{k}^e$ and (18) is used to obtain \bar{c}^e .

In summary, given estimated coefficients b_k , a_{kk} and $a_{k\varepsilon}$ for the PLM (19), agents are able to estimate $\bar{c}^e, \bar{w}^e, \bar{r}^e, \bar{k}^e$ and to forecast future wages and interest rates. Then, using (17), agents determine their consumption. Note that, given the coefficient estimates, c_t depends linearly on k_t and ε_t .

Finally, inserting c_t into (15), we obtain

$$k_{t+1} = T_{b_k} + T_{a_{kk}}k_t + T_{a_{k\varepsilon}}\varepsilon_t. \quad (20)$$

This is the ALM implied by the PLM when agents make infinite-horizon consumption decisions. Under least-squares learning, agents update the estimated coefficients of (19) each period using recursive least squares.

As usual, local stability of the REE under learning is determined by E-stability. However, under IH-learning the map $(T_{b_k}, T_{a_{kk}}, T_{a_{k\varepsilon}})(b_k, a_{kk}, a_{k\varepsilon})$ is extremely complicated. Although the map can be written entirely in terms of the parameters α and β , the functions are analytically almost intractable. While full analytical results are unattainable, we nonetheless can obtain stability results for α near $1/3$, a value often used in the empirical literature. We have:

Proposition 2: There is an open interval B around $\alpha = 1/3$ such that the REE is locally stable under IH-learning for all $\alpha \in B$ and all $0 < \beta < 1$.

The proof relies on showing that $\det(J) > 0$ and $\text{tr}(J) < 0$, where $J = DT - I$ is the 2×2 Jacobian matrix of the E-stability differential equation for (b_k, a_{kk}) . We have also checked stability of IH-learning more generally by verifying that $\det(J) > 0$ and $\text{tr}(J) < 0$ for all $0 < \alpha, \beta < 1$ using a numerical grid of 0.01.

5 Numerical Illustration

The importance of stability of the REE under learning is partly that this carries over to “perpetual” learning in which agents use discounted least squares with small discount rates (“gains”). Such learning rules are widely used in the learning literature to generate boundedly rational learning dynamics that can sometimes improve the empirical fit to the data.

We conclude by noting that short-horizon EE-learning and long-horizon IH learning can generate different learning dynamics around the REE. To illustrate we set $\alpha = 1/3$, $\beta = 0.95$ and the learning “gain” parameter at $\gamma = 0.005$ or $\gamma = 0.02$. (Choosing $0 < \gamma < 1$ geometrically discounts past data with geometric discount factor $1 - \gamma$.) As in Eusepi and Preston (2011), we now assume that under learning agents cannot condition forecasts on current productivity level.

Table 1 reports the results under learning for two different choices of gain. Compared with the standard RE solution, output volatility is little affected. For the Ramsey model this is not surprising since employment is fixed. Both consumption and investment volatilities, however, are substantially higher, and, in contrast to Eusepi and Preston (2011), these are higher for both types of learning. In fact investment volatility is even higher under EE learning than under IH learning. Finally, we see that the size of the gain γ affects volatility more markedly under IH learning.

These results suggest that the planning horizons of agents, as well as the extent to which they discount past data, can be important parameters in fitting empirical learning models. A detailed investigation is left for future research.

6 Conclusions

In representative agent models with adaptive learning, both short-horizon EE-learning and long-horizon IH-learning approaches are widely used. For stochastic growth models, in which the dynamics depend on a predetermined endogenous state variable as well as on expectations of future variables, there have to date been virtually no available analytical stability results, and researchers have relied on numerical techniques. In the current paper we have shown that it is possible to obtain analytical stability results for both EE and IH-learning in the simplest, and most central, model in this class, the stochastic Ramsey model.

Appendix

Proof of Proposition 1: The coefficients of (10)-(11) are given by

$$\begin{aligned} T_{b_c} &= b_c + (a_{ck} - \beta \bar{c} f'') b_k, & T_{b_k} &= -b_c - (a_{ck} - \beta \bar{c} f'') b_k, \\ T_{a_{ck}} &= (a_{ck} - \beta \bar{c} f'') a_{kk}, & T_{a_{kk}} &= \beta^{-1} - (a_{ck} - \beta \bar{c} f'') a_{kk}, \\ T_{a_{c\varepsilon}} &= (a_{ck} - \beta \bar{c} f'') a_{k\varepsilon}, & T_{a_{k\varepsilon}} &= \bar{c} - (a_{ck} - \beta \bar{c} f'') a_{k\varepsilon}. \end{aligned}$$

The 6×6 Jacobian matrix of (12) at a fixed point has a triangular block structure, with stability of $\bar{a}_{kk} \equiv \lambda, \bar{a}_{ck}$, determined by the 2×2 matrix

$$DT_{\bar{a}_{ck}, \bar{a}_{kk}} - I = \begin{pmatrix} \bar{a}_{kk} - 1 & \bar{a}_{ck} - \beta \bar{c} f'' \\ -\bar{a}_{kk} & -(\bar{a}_{ck} - \beta \bar{c} f'') - 1 \end{pmatrix}.$$

Since $0 < \bar{a}_{kk} \equiv \lambda < 1$ and $\bar{a}_{ck} > 0$ this matrix has negative trace and positive determinant, and thus has eigenvalues with negative real parts. It can then easily be verified that the remaining four eigenvalues have negative real parts.

We remark that Proposition 1 does not require Cobb-Douglas production.

Proof of Proposition 2: Stability is determined by the two-dimensional subsystem $(T_{b_k}, T_{a_{kk}})(b_k, a_{kk})$. Let $J(\alpha, \beta)$ denote the Jacobian matrix of the corresponding E-stability differential equation, at the REE, for given $0 < \alpha, \beta < 1$. For E-stability we need both eigenvalues of $J(\alpha, \beta)$ to have negative real parts, i.e. $J(\alpha, \beta)$ to have a positive determinant and negative trace. Using Mathematica it can be verified that (i) $\det(J(\alpha, \beta))$ and $\text{tr}(J(\alpha, \beta))$ are continuous over $0 < \alpha, \beta < 1$, (ii) $\det(J(1/3, \beta)) > 0$ and $\text{tr}(J(1/3, \beta)) < 0$ for all $0 < \beta < 1$ and (iii) that $\lim_{\beta \rightarrow 1}(\text{tr}(J(1/3, \beta))) = -3/2$, $\lim_{\beta \rightarrow 1}(\det(J(1/3, \beta))) = 1/2$, $\lim_{\beta \rightarrow 0}(\text{tr}(J(1/3, \beta))) = -4$ and also $\lim_{\beta \rightarrow 0}(\det(J(1/3, \beta))) = 4$. Therefore $\det(J(\alpha, \beta)) > 0$ and $\text{tr}(J(\alpha, \beta)) < 0$ for α sufficiently close to $\alpha = 1/3$ and all $0 < \beta < 1$, and the result follows.

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Table 1						
Excess volatilities under learning, in %						
gain	IH learning			EE learning		
γ	C	Y	I	C	Y	I
0.005	3.2	0.0	5.3	6.3	0.2	10.7
0.02	6.2	-0.1	5.8	7.8	0.3	10.8

Note: C, Y and I denote consumption, output and investment. Excess volatilities are standard deviations relative to those of the REE in which ε_t is observed.

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