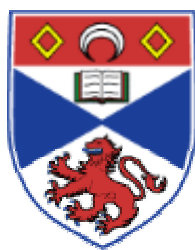


CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
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Optimal Simple Rules for the Conduct of
Monetary and Fiscal Policy*

TECHNICAL ANNEX

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0.1 Annex A: Solution Method

A linearized version of the model developed in sections 2 and 3 can be represented in compact form with all variables in percentage deviation from the steady state as

$$AE_t y_{t+1} = B y_t + C x_t \quad \forall t \geq 0, \quad (1)$$

where y_t is a vector of endogenous variables comprising both predetermined and non-predetermined variables including policy rules for the nominal interest rate and taxes. x_t is a vector of exogenous variables and A, B and C are matrices of fixed, time-invariant coefficients. E_t is the expectations operator conditional on information available at time t . King and Watson (1997) demonstrate that if a solution to (31) exists and is unique then we may write such a solution in state-space form as follows:

$$\begin{aligned} y_t &= \Pi s_t, \\ s_t &= M s_{t-1} + G e_t, \end{aligned} \quad (2)$$

where the s_t matrix includes the state variables of the model, e_t is a vector of shocks to the state variables and Π, M and G are coefficient matrices. The y_t matrix has also been augmented to include the model's exogenous state variables. We can use equations (32) to calculate the asymptotic variance-covariance matrix for the model's endogenous variables.⁴ To proceed, iterate on the second set of equations. Since there are a sufficient number of stable roots, we have

$$s_t = G \sum_{j=0}^{\infty} M^j e_{t-j}. \quad (3)$$

Using this result in the first set of equations in (32) we find that

$$y_t = \sum_{j=0}^{\infty} \Phi M^j e_{t-j}, \quad (4)$$

⁴ Let n_{pd} denote the number of predetermined variables, n_x the number of exogenous state variables and let $n_{n_{pd}}$ denote the number of non-predetermined variables. The dimensions of our system are as follows: y_t is $[(n_{n_{pd}} + n_{pd} + n_x) \times 1]$, s_t is $[(n_{pd} + n_x) \times 1]$, Π is $[(n_{n_{pd}} + n_{pd} + n_x) \times (n_{pd} + n_x)]$, M is $[(n_{pd} + n_x) \times (n_{pd} + n_x)]$, and G is $[(n_{pd} + n_x) \times (n_{pd} + n_x)]$. The interested reader should consult Hansen and Sargent (1998) for further details.

where $\Phi \equiv \Pi G$.⁵ As the stochastic shocks to the economy are assumed to be covariance stationary, it then follows that we may write,

$$y_t y_t' \equiv \Sigma = \sum_{j=0}^{\infty} \Phi M^j \Omega M^{j'} \Phi', \quad (5)$$

where a prime denotes a transpose and $\Omega \equiv e_t e_t'$.⁶ Using the relevant entries from the Σ matrix finally allows us to evaluate the policymaker's loss function.

Our problem, then, is to find the set of parameter values $\Gamma = (\phi_\pi^m, \phi_y^m, \phi_y^f, \rho^m, \rho^f)$, where the superscript m refers to the linearized interest rate rule, (25), the superscript f refers to the linearized fiscal rule, (26), and where ρ refers to the degree of autocorrelation in the two rules, such that the following welfare loss function is minimized:

$$E \{J_T\} = E \left\{ \sum_{t=0}^{T-1} \beta^t L_t(y_t; \Gamma) + \beta^T L_T(y_T; \Gamma) \right\}. \quad (6)$$

The policymaker's period loss function, L , is given by

$$L = \alpha_1 \Sigma_\pi + \alpha_2 \Sigma_y + \alpha_3 \Sigma_i, \quad (7)$$

where Σ_x denotes the asymptotic variance of the annualized value of x , given Γ .⁷

0.2 Annex B: Dynamics of Consumption and Debt

In this section, we consider the dynamic responses in this model for the optimized policy rules for two key state variables, consumption and debt, from a composite shock to consumption.⁸ The stable arms for $c_{t+1} - c_t = 0$ and $b_t - b_{t-1} = 0$ in Figure B1 are drawn for the structural parameters given in Table 2 (of the main paper) and for

⁵With dimensions given by is $[(nnpd + npd + nx) \times (npd + nx)]$.

⁶To derive (6.5) we have used the result that for any two conformable matrices A and B , $(AB)' = B'A'$. Note also that since our shocks are serially uncorrelated, we have that $e_t e_{t-j}' = 0 \quad \forall j > 0$.

⁷Following Alexandre, Driffill and Spagano (2002), Söderlind (1999) and Williams (1999) we assume that $\alpha_1 = \alpha_2 = 1$, and $\alpha_3 = 0.25$. In sensitivity analysis, we have relaxed these relative weights in the loss function and examined the resulting loss functions, see Rotemberg and Woodford, 1997, for a discussion of this issue.

⁸Full details of the linearization of the model and the derivation of this phase diagram are available on request from the authors.

the policy criterion parameters derived in the previous section. The stable arm for consumption growth is positively sloped because any accumulation in current period debt raises current period consumption by its annuity value and this will mean that consumption in the next period will have to be similarly positive in order to ensure growth in consumption is zero. The extent of this impact on current consumption is tempered because the real interest rate simultaneously rises to offset any increases in current period consumption.⁹ The slope of the stable arm for consumption is therefore relatively inelastic with respect to debt accumulation.

Monetary and fiscal policy jointly determine the slope of the stable arm for debt. The action of monetary policy through real interest rates, which increases debt, and fiscal policy through the creation of a surplus, which reduces debt, in response to positive output and inflation means that current period debt is related to the current level of consumption. It turns out that current debt is positively related to consumption for the following reasons. Under a well defined steady-state for debt the term on lagged debt must be less than one and so the growth rate in debt will be negative unless consumption in the current period is positively related to debt. As long as the real interest rate impact on current debt outweighs the reduction in current debt, caused by running a surplus (where both are functions of consumption) current debt does in fact respond positively to current consumption.

The relative slopes in the two curves are thus determined by the arguments on debt for both stable arms and the relative strength of monetary and fiscal policy:

$$\frac{\left(\lambda \frac{b}{w}\right)}{r(c_t) \Big|_{c_{t+1}-c_t=0}} < \frac{(1+\gamma)\beta^{-1}-1}{r(c_t)-s(c_t) \Big|_{b_t-b_{t-1}=0}}, \quad (8)$$

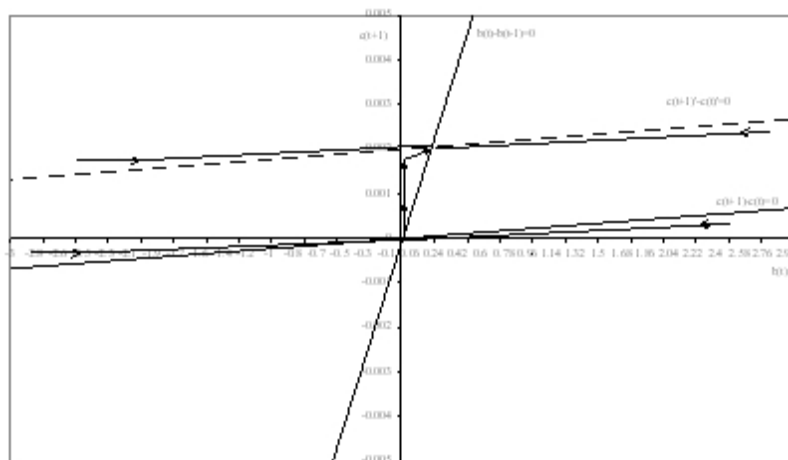
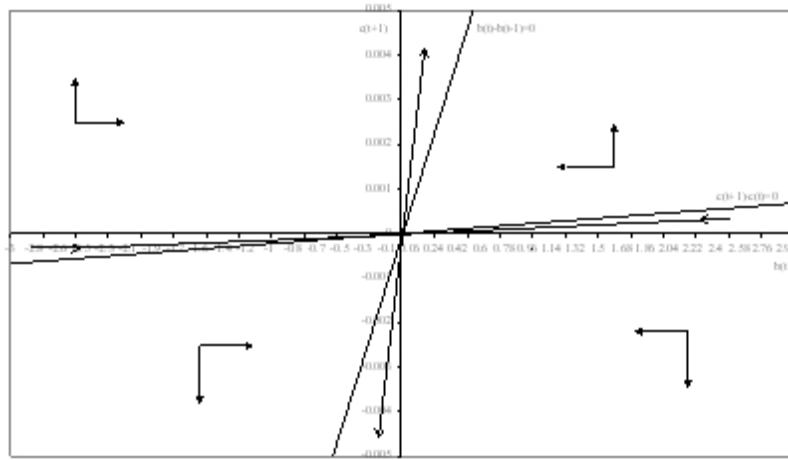
where λ is the probability of death, $\frac{b}{w}$ is the steady-state proportion of wealth held as bonds, $r(c_t)$ sets the real interest rate as an implicit function of consumption, γ is the rate at which debt is retired, β is the rate of time preference and $s(c_t)$ sets the budget surplus as an implicit function of consumption.

⁹The real interest rate effect acts via the nominal rate, which responds to inflation and output above steady-state, rising by more than expected inflation. In this model, higher output is a positive function of consumption and a negative argument in expected inflation and so higher real rates are a positive function of consumption.

For our set of parameters, the stable arm for debt is thus steeper than that for consumption. Figure 3 depicts the phase diagram with lines of force and both stable and unstable saddle-paths drawn. The stable saddle path tells us that should consumption and debt be below (above) the steady-state, the stable saddle-path involves both debt and consumption both increasing (decreasing), where the consumption response is highly inelastic to the accumulation of debt.

The middle panel draws the trajectory of consumption following a positive composite shock to consumption. We note that consumption jumps nearly all the way to its new equilibrium but that the associated increase in real interest rates acts to increase debt and this debt then promotes some small further increase in the level of consumption. Fiscal stabilization of inflation and output acts to prolong the movement in consumption to a given shock as debt accumulation continues to increase consumption. The lower panel, drawn for $\phi^f = 0$, shows that the system moves to equilibrium more quickly when fiscal policy is not acting to any great extent to increase or decrease debt in response to shocks. But as fiscal stabilization of output acts to amplify the impact of monetary policy in stabilizing output¹⁰ and by accruing more debt allows a somewhat smoother response of consumption to a given shock, the policy criterion established in the previous section prefers some weight on output stabilization by the fiscal policy maker.

¹⁰This is one of the key monetary-fiscal interactions. Because output becomes a negative function of the fiscal stabilisation of output, monetary stabilisation of output is amplified.



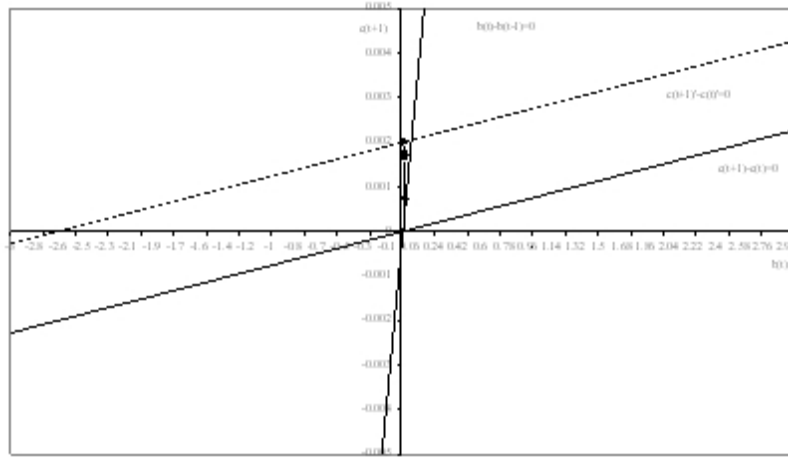


Fig. B1 - Consumption and Debt Dynamics