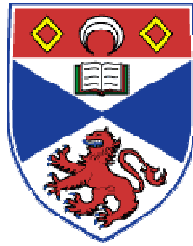


CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS  
CONFERENCE PAPERS 2007



CDMC07/03

# Inflation, Financial Development and Human Capital -Based Endogenous Growth: an Explanation of Ten Empirical Findings\*

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JUNE 2007

## ABSTRACT

The paper presents a general equilibrium that can explain ten related sets of empirical results, providing a unified approach to understand usually disparate effects typically treated separately. These are grouped into two sets, one on financial development, investment and inflation, and one on inflation's effect on other economy-wide variables such as growth, real interest rates, employment, and money demand. The unified approach also contributes a systematic explanation of certain nonlinearities that are found across these results, as based on the production function for financial intermediary services and the resultant money demand function.

**JEL:** C23, E44, O16, O42

**Keywords:** Inflation, financial development, growth, exchange credit production

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\* We are grateful to Szilard Benk for research assistance, and to Toni Braun, Mark Harris, Patrick Minford, Akos Valentinyi, the Cardiff Business School seminar series, CERGE seminar series, CEU seminar series, the European Monetary Forum, York, the SED Conference, Budapest, the ESEM-EEA Meetings, Vienna, the MMF Conference, York, and Cardiff University and Central European University for financial support. Corresponding author: Max Gillman, Cardiff University, Colum Dr., Cardiff Wales, CF10 3EU, UK; tel: 0044-2920-876547; fax: same.

## 1. Introduction

The paper defines financial development within an exchange economy with human-capital based growth and with investment included in the exchange constraint. It brings microeconomic theory and evidence in the finance sector to bear upon how inflation and financial development affect the economy, and interact with each other. This provides a perspective for interpreting related econometric evidence in unison, rather than treating each part in isolation. It shows how far a neo-classical theory can go in unifying the explanation of diverse phenomena, with its limitations providing a basis for further developing the theory of this interaction.

The model is consistent with ten distinct strands of empirical evidence, and explains these in a cohesive way. The first set of five concerns inflation, investment and financial development. 1) A negative effect of inflation on investment has been found empirically for example by Madsen (2003), for panel OECD data. By including investment in the exchange constraint (Stockman 1981), the model shows that inflation causes investment to decrease, and to decrease at a nonlinear diminishing rate as the inflation rate rises.<sup>1</sup> 2) Khan, Senhadji, and Smith (2006), Boyd, Levine, and Smith (2001), and Rousseau and Wachtel (2002) show how inflation decreases measures of financial depth that are related to investment,<sup>2</sup> and that this effect is to some extent of diminishing magnitude as the inflation rate rises. The model gives this result by assuming that all saving and investment is costlessly intermediated; this makes the degree of such intertemporal financial intermediation, which is consistent with what is called "financial depth", directly linked to the amount of investment. Inflation negatively effects financial depth as it affects investment, at a diminishing rate. 3) At the same time, Aiyagari, Braun, and Eckstein (1998) show empirically that the finance sector increases in

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<sup>1</sup>And given the well-known link between inflation rate levels and inflation uncertainty levels, also relevant is Byrne and Davis (2004), who finds US evidence in support of a negative effect on inflation uncertainty on investment.

<sup>2</sup>In Boyd et al (2001), for example, inflation decreases the quantity of loans that the financial sector provides to the private sector.

size as inflation increases. The model shows this by assuming a costly production of exchange credit, as in Aiyagari et al, whereby the agent chooses more exchange credit to avoid a rising inflation rate. 4) Levine, Loayza, and Beck (2000) shows how financial development can increase growth. The model produces through an increase in the productivity of the exchange credit production causing unambiguously a higher balanced-path growth rate. 5) Evidence has shown mixed effects of the interaction of inflation and financial development on growth, such as in Rousseau and Wachtel (2002)<sup>3</sup>. The model shows conditions under which this interaction can produce different effects on growth, based on a categorization of channels as in Rousseau and Vuthipadadorn (2005).

A second set of five facts concern the effect of inflation on other variables. 6) Inflation in itself has been found by many to decrease growth (Barro 1995), and again this is a nonlinear effect, occurring at a decreasing rate as the inflation rate rises (see Ghosh and Phillips 1998, Judson and Orphanides 1999, Gillman, Harris, and Mátyás 2004). The model produces this when an inflation increase causes goods to leisure substitution, at a decreasing rate, that lowers the capacity utilization rate of human capital, the return to human capital, and so the growth rate, at a decreasing rate.<sup>4</sup> 7) Inflation has also been found to be cointegrated with the unemployment rate, in the US by Ireland (1999) and in the UK by Shadman-Mehta (2001), with the latter finding Granger causality from inflation to unemployment. While strictly the model has only a rate of productively employed time and a leisure rate ("unproductive time"), empirically the movement in the employment rate and the unemployment tends to be highly negatively correlated. Therefore the model's inflation-induced decrease in leisure yields a decrease in the rate of productively employed time that is consistent with this cointegration evidence, and its causality. 8) Empirical evidence shows a positive Tobin (1965) effect of inflation decreasing the return to capital (Rapach 2003), and increasing

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<sup>3</sup>See Aghion, Howitt, and Mayer-Foulkes (2005) for the effect of the interaction of financial development with initial income, on growth.

<sup>4</sup>Insignificant but positive empirical effects of inflation on growth, for low inflation rates, are not robust to using instrumental variables to control for endogeneity; see Gillman et al (2004).

the capital to effective labor ratio across sectors (Gillman and Nakov 2003). The model produces this when inflation-induced substitution towards leisure causes an increase in the real wage and decrease in the return to human capital, thereby inducing substitution from expensive labor to less expensive capital, even while the investment rate decreases.<sup>5</sup> 9) Evidence supports a Cagan (1956) type of money demand that has a rising magnitude of the interest elasticity as the inflation rate rises (Mark and Sul 2003). The model's money demand is a general equilibrium version of such a rising interest elasticity model (Gillman and Otto 2007) 10) The production of financial intermediary services has been found empirically to exhibit constant returns to scale in the three factors of labor, capital, and deposited funds. The model uses this same CRS specification for exchange credit production, and so is consistent with this evidence.

There are two main extensions of the model from Gillman and Kejak (2005) that enable our results (while keeping the money demand functional form the same). First we include investment in the exchange constraint, so that the inflation tax falls on investment as well as on consumption (Stockman 1981). This is important because it results in a negative effect of inflation on investment, even while maintaining key Tobin (1965) effects. Second, we decentralize the finance sector in order to define its development in a standard fashion using parameters of the production function.

The equilibrium is presented in Section 2 and developed through propositions related to a closed form solution, and through simulations of the full model; these results are used to explain the evidence. It is established how financial development affects the marginal cost per unit of credit (Section 3.1), the average cost of credit (Section 3.2) and the growth rate (Section 3.3). Simulations show the effects of inflation and financial development on investment, growth and other variables (Section 4). The Discussion (Section 5) explains how the model's simultaneous results provide a interpretation of related econometric findings.

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<sup>5</sup>In contrast, this investment rate increases with inflation theoretically in Gillman and Kejak (2005) and empirically in Ahmed and Rogers (2000).

## 2. Representative Agent Model

### 2.1. Consumer Problem

The representative agent's discounted utility stream depends on the consumption of goods  $c_t$  and leisure  $x_t$  in a constant elasticity fashion:

$$\sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t (\log c_t + \alpha \log x_t). \quad (2.1)$$

Exchange is required for both consumption and investment goods, denoted by  $i_t$ , whereby the consumer uses either nominal money,  $M_t$ , or credit from the credit card. Let  $q_t$  denote the real quantity of credit, and  $P_t$  denote the nominal goods price. This makes the exchange constraint:

$$M_t + P_t q_t \geq P_t c_t + P_t i_t. \quad (2.2)$$

It is assumed that all expenditures are sourced from deposits, denoted by  $d_t$ , held at the financial intermediary. The consumer buy shares in the intermediary by making a deposit, whereby the price per share is given by the intermediary at a fixed price of one, so there is no possibility of a capital gain. However the share, or unit deposit, yields a dividend that is paid by the intermediary to the consumer, so that the intermediary has no remaining profits after the dividend distribution; the intermediary is a "mutual bank" owned by the consumer. The per unit dividend is in essence the payment of a nominal interest rate on deposited funds. Denote the per unit nominal dividend as  $R_{qt}$ ; total dividends are then  $R_{qt}d_t$ .

Since all expenditures come out of the deposits, this means that

$$P_t d_t = P_t (c_t + i_t). \quad (2.3)$$

The fractions of capital allocated across the three sectors, of goods ( $G$ ), human capital ( $H$ ) and credit ( $Q$ ), add up to 1:

$$1 = s_{Gt} + s_{Ht} + s_{Qt}; \quad (2.4)$$

the fractions of labor add up to the total productively utilized time, or  $1 - x_t$  :

$$1 - x_t = l_{Gt} + l_{Ht} + l_{Qt}. \quad (2.5)$$

Physical capital,  $k_t$ , changes according to

$$k_{t+1} = i_t + (1 - \delta_K) k_t. \quad (2.6)$$

Human capital,  $h_t$ , is accumulated through a CRS production function using effective labor and capital; with  $A_H > 0$ , and  $\epsilon \in [0, 1]$ ,

$$h_{t+1} = A_H (s_{Ht} k_t)^{1-\epsilon} (l_{Ht} h_t)^\epsilon + (1 - \delta_H) h_t. \quad (2.7)$$

Using the time and goods constraints, equations (2.4) and (2.5) to substitute in for  $l_{Ht}$  and  $s_{Ht}$ ,

$$h_{t+1} = A_H [(1 - s_{Gt} - s_{Qt}) k_t]^{1-\epsilon} [(1 - l_{Gt} - l_{Qt} - x_t) h_t]^\epsilon + (1 - \delta_H) h_t. \quad (2.8)$$

The change in the nominal money stock,  $M_{t+1} - M_t$ , is equal to income minus expenditure. The nominal income received from capital and labor, with  $P_t$  denoting the price of goods, and  $r_t$  and  $w_t$  denoting the real rental and wage rates, is  $P_t r_t (s_{Gt} + s_{Qt}) k_t$  and  $P_t w_t (l_{Gt} + l_{Qt}) h_t$ . Also there is a lump sum government transfer  $V_t$ , and the dividend distribution from the intermediary of  $R_{qt} d_t$ . Expenditures are on consumption and investment,  $P_t (c_t + i_t)$ , and for the payment of the fee for credit services; with  $P_{qt}$  denoting the nominal price per unit of credit, this fee is  $P_{qt} q_t$ . Together these items make the income constraint:

$$\begin{aligned} M_{t+1} &= M_t + P_t r_t (s_{Gt} + s_{Qt}) k_t + P_t w_t (l_{Gt} + l_{Qt}) h_t + V_t + R_{qt} d_t \\ &\quad - P_t c_t - P_t [k_{t+1} - (1 - \delta_K) k_t] - P_{qt} q_t. \end{aligned} \quad (2.9)$$

During the subperiod in which trading for goods takes place, the consumer uses the credit card from the financial intermediary for the credit purchases, and withdraws the cash (or equivalently uses a debit card to transfer the cash), at the point of purchase.

## 2.2. Financial Intermediary Problem

The intermediary is assumed to operate competitively. It sets the price of deposits, and then the consumer determines the quantity of deposits it wants to hold,  $d_t$ , as with a mutual bank. The production function for credit services is CRS in effective labor, capital and the deposited funds  $d_t$ . With  $A_Q \in (0, \infty)$ ,  $\gamma_1 \in [0, 1]$ ,  $\gamma_2 \in [0, 1]$ , and assuming that  $\gamma_1 + \gamma_2 \leq 1$ , the production is given by

$$q_t = A_Q (l_{Qt} h_t)^{\gamma_1} (s_{Qt} k_t)^{\gamma_2} d_t^{1-\gamma_1-\gamma_2}, \quad (2.10)$$

Dividing equation (2.14) by  $d_t$ , and defining normalized variables as  $l_{qt} \equiv \frac{l_{Qt} h_t}{d_t}$ ,  $s_{qt} \equiv \frac{s_{Qt} k_t}{d_t}$ , and  $q_t^* \equiv \frac{q_t}{d_t}$ ; the production function can be written as

$$q_t^* = A_Q l_{qt}^{\gamma_1} s_{qt}^{\gamma_2}. \quad (2.11)$$

For  $\gamma_1 = 0.15$ ,  $\gamma_2 = 0.15$ , and  $A_Q = 0.62$ , Figure 1 illustrates the graph of the production function (labels: lq for  $l_{qt}$ , sq for  $s_{qt}$ , and  $q^*$  for  $q_t^*$ ).

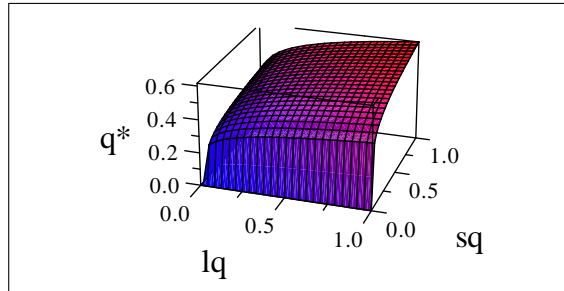


Figure 1. Production of Credit per Unit of Deposits,  $q^*$

The solvency restriction that assets equal liabilities is given by

$$P_t q_t + M_t = P_t d_t. \quad (2.12)$$

The liquidity constraint is that money withdrawn by the consumer is covered by deposits:

$$P_t d_t \geq M_t. \quad (2.13)$$

When no credit is used, the liquidity constraint holds with equality and is equal to the solvency constraint.

The intermediary's competitive profit maximization problem is the maximization of the per period nominal dividends:

$$\underset{l_{qt}, s_{qt}}{Max} R_{qt} d_t P_t = P_{qt} q_t - w_t l_{qt} h_t P_t - r_t s_{qt} k_t P_t, \quad (2.14)$$

subject to the production function for  $q_t$  in equation (2.10), or more simply with normalized variables and with  $\frac{P_{qt}}{P_t} \equiv p_{qt}$ , the firm's problem can be stated as

$$\underset{l_{qt}, s_{qt}}{Max} R_{qt} = p_{qt} A_Q l_{qt}^{\gamma_1} s_{qt}^{\gamma_2} - w_t l_{qt} - r_t s_{qt}. \quad (2.15)$$

The solvency and liquidity constraints in equations (2.12) and (2.13) are always satisfied in this simple problem. Zero profit results through the distribution of the dividends according to the number of shares of bank ownership that the consumer has, as given by the real quantity of deposits  $d_t$ , at the dividend rate of  $R_{qt}$ .

The first order conditions can be written as in terms of average and marginal products: with  $AP_{l_{qt}} \equiv \frac{q_t^*}{l_q}$ ,  $AP_{s_{qt}} \equiv \frac{q_t^*}{s_{qt}}$ , and  $MP_{l_{qt}} \equiv \gamma_1 AP_{l_{qt}}$ ,  $MP_{s_{qt}} \equiv \gamma_2 AP_{s_{qt}}$ , and the marginal cost per unit of credit, denoted by  $MC_t$ ,

$$p_{qt} = \frac{w_t}{\gamma_1 \left( \frac{q_t^*}{l_q} \right)} \equiv \frac{w_t}{\gamma_1 AP_{l_{qt}}} = \frac{w_t}{MP_{l_{qt}}} = MC_t; \quad (2.16)$$

$$p_{qt} = \frac{r_t}{\gamma_2 \left( \frac{q_t^*}{s_{qt}} \right)} \equiv \frac{r_t}{\gamma_2 AP_{s_{qt}}} = \frac{r_t}{MP_{s_{qt}}} = MC_t. \quad (2.17)$$

These Baumol (1952) conditions equate the marginal cost of credit funds to the value of the marginal products of effective labor and capital in producing the credit, the standard price theoretic conditions for factor markets; the marginal products are fractions,  $\gamma_1$  and  $\gamma_2$ , of the average products.

**Proposition 1:** The marginal cost curve is upward sloping for all  $\gamma_1 + \gamma_2 < 1$ , and convex for  $\gamma_1 + \gamma_2 < 0.5$ .

**Proof:** From equation (2.16),  $MC_t = \left( \frac{w_t}{\gamma_1} \right) l_{qt} \left( \frac{1}{q_t^*} \right)$ . Substituting in for  $l_{qt} = A_Q^{-\gamma_1} (s_{qt})^{-\gamma_2/\gamma_1} (q_t^*)^{1/\gamma_1}$  from the production function in equation (2.11), gives that

$MC_t = \left(\frac{w_t}{\gamma_1}\right) \left[ A_Q^{-\gamma_1} (s_{qt})^{-\gamma_2/\gamma_1} (q_t^*)^{1/\gamma_1} \right] (q_t^*)^{-1}$ . Finally, substituting in for  $s_{qt}$  from the bank's first-order condition in equation (2.17), in which  $s_{qt} = \frac{\gamma_2 MC_t}{r_t} q_t^*$ , and simplifying

$$MC_t = \left[ w_t^{\left(\frac{\gamma_1}{\gamma_1+\gamma_2}\right)} r_t^{\left(\frac{\gamma_2}{\gamma_1+\gamma_2}\right)} \gamma_1^{\left(\frac{-\gamma_1}{\gamma_1+\gamma_2}\right)} \gamma_2^{\left(\frac{-\gamma_2}{\gamma_1+\gamma_2}\right)} A_Q^{\left(\frac{-1}{\gamma_1+\gamma_2}\right)} \right] (q_t^*)^{\left(\frac{1-\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\right)}. \quad (2.18)$$

The graph of the marginal cost per unit of  $q_t^*$ , or  $MC_t = B_t (q_t^*)^{\left(\frac{1-\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\right)}$ , where  $B_t \equiv w_t^{\left(\frac{\gamma_1}{\gamma_1+\gamma_2}\right)} r_t^{\left(\frac{\gamma_2}{\gamma_1+\gamma_2}\right)} \gamma_1^{\left(\frac{-\gamma_1}{\gamma_1+\gamma_2}\right)} \gamma_2^{\left(\frac{-\gamma_2}{\gamma_1+\gamma_2}\right)} A_Q^{\left(\frac{-1}{\gamma_1+\gamma_2}\right)}$ , is upward sloping for  $\gamma_1 + \gamma_2 < 1$  since  $\frac{1-\gamma_1-\gamma_2}{\gamma_1+\gamma_2} > 0$  for  $\gamma_1 + \gamma_2 < 1$ ; and it is convex for  $\gamma_1 + \gamma_2 < 0.5$ , since  $\frac{1-\gamma_1-\gamma_2}{\gamma_1+\gamma_2} > 1$  for  $\gamma_1 + \gamma_2 < 0.5$ .

In the equilibrium described below in Section 2.5, the  $MC_t$  intersects with the nominal interest rate  $R_t$  at the equilibrium quantity of credit output per unit of deposits,  $q_t^* \in [0, 1)$ , as Figure 2 illustrates (for  $\gamma_1 + \gamma_2 = 0.3$ ,  $B = 1.3541$  and  $R = 0.15$ ).

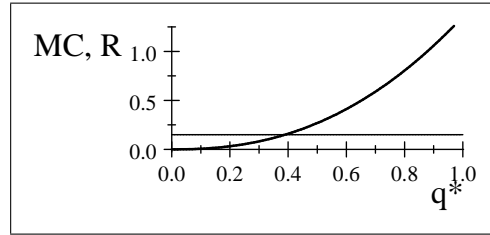


Figure 2. Marginal Cost of Credit per unit of  $q_t^*$

### 2.3. Goods Producer Problem

The goods producer competitively hires labor and capital for use in its Cobb-Douglas production function. Given  $A_G \in (0, \infty)$ ,  $\beta \in [0, 1]$ ,

$$y_t = A_G (l_{Gt} h_t)^\beta (s_{Gt} k_t)^{1-\beta}, \quad (2.19)$$

with the first-order conditions of

$$w_t = \beta A_G (l_{Gt} h_t)^{\beta-1} (s_{Gt} k_t)^{1-\beta}, \quad (2.20)$$

$$r_t = (1 - \beta) A_G (l_{Gt} h_t)^\beta (s_{Gt} k_t)^{-\beta}. \quad (2.21)$$

## 2.4. Government Financing Problem

The government money supply changes according to a lump sum transfer of cash,  $V_t$ , given to the consumer each period:

$$M_{t+1} = M_t + V_t. \quad (2.22)$$

## 2.5. Equilibrium

Given prices  $r_t$ ,  $w_t$ ,  $P_t$ ,  $P_{qt}$ , and  $R_{qt}$ , the consumer maximizes utility in equation (2.1) subject to the exchange, income, and human capital investment constraints, in equations (2.2), (2.3), (2.8), and (2.9), with respect to  $c_t$ ,  $x_t$ ,  $l_{Gt}$ ,  $l_{Qt}$ ,  $s_{Gt}$ ,  $s_{Qt}$ ,  $q_t$ ,  $d_t$ ,  $k_{t+1}$ ,  $h_{t+1}$ , and  $M_{t+1}$ . Given the prices  $r_t$ ,  $w_t$ ,  $P_t$ , and  $P_{qt}$ , the financial intermediary maximizes the dividend in equation (2.15) with respect to normalized inputs giving the equilibrium conditions (2.16) and (2.17). The goods producer maximizes profit subject to the CRS production function constraint (2.19), giving the marginal product conditions (2.20) and (2.21). And the government's budget constraint (2.22) provides the market clearing condition for the money market; the deposit condition (2.3) provides market clearing for the intermediary's deposit market; and goods market clearing of income equal to expenditure is given by equation (2.9).

Equilibrium conditions along the balanced-growth path are given here for log-utility, and used to describe how inflation affects the equilibrium.

$$p_{qt} = R_t, \quad (2.23)$$

$$R_t = R = \sigma + \rho + \sigma\rho, \quad (2.24)$$

$$q_t^* = A_Q^{\frac{1}{(1-\gamma_1-\gamma_2)}} \left( \frac{R_t\gamma_1}{w_t} \right)^{\frac{\gamma_1}{(1-\gamma_1-\gamma_2)}} \left( \frac{R_t\gamma_2}{r_t} \right)^{\frac{\gamma_2}{(1-\gamma_1-\gamma_2)}}. \quad (2.25)$$

$$\frac{x_t}{\alpha c_t} = \frac{1 + \tilde{R}_t}{w_t h_t}, \quad (2.26)$$

$$\tilde{R}_t = (1 - q_t^*) R_t + (\gamma_1 + \gamma_2) R_t (q_t^*), \quad (2.27)$$

$$r_{Ht} = \varepsilon A_H \left( \frac{s_{Ht} k_t}{l_{Ht} h_t} \right)^{(1-\varepsilon)} (1 - x_t), \quad (2.28)$$

$$1 + g_t = \frac{1 + r_{Ht} - \delta_H}{1 + \rho} = \frac{1 + \frac{r_t}{1 + \tilde{R}_t} - \delta_K}{1 + \rho}, \quad (2.29)$$

The price of credit per unit is simply the nominal interest rate (equation (2.23)); thus the marginal cost of money equals that of credit. At the Friedman optimum, the nominal interest  $R$  of equation (2.24) equals zero and no credit is used in equation (2.25). But as inflation rises, the agent substitutes from goods towards leisure while equalizing the margin of the ratio of the shadow price of goods to leisure,  $x_t/(\alpha c_t) = [1 + \tilde{R}_t] / (w_t h_t)$ , in equation (2.26). Here  $\tilde{R}_t$ , as given in equation (2.27), is the average exchange cost per unit of output; this equals the average cost of using cash,  $R_t$ , weighted by  $(1 - q_t^*)$  and the average cost of using credit,  $(\gamma_1 + \gamma_2)R_t$ , weighted by  $q_t^*$ . That  $(\gamma_1 + \gamma_2)R_t$  is an average cost can be verified by dividing the total cost of credit production by the total output of credit production. And this total exchange cost determines how much substitution there is from money to credit, as given in equation (2.25), and from goods to leisure. Substitution towards leisure causes a fall in the human capital return of  $r_{Ht} \equiv \varepsilon A_H (s_{Ht} k_t / l_{Ht} h_t)^{(1-\varepsilon)} (1 - x_t)$ , given in equation (2.28). The marginal product of physical capital  $r_t$ , in equation (2.21), also falls, while the real wage  $w_t$  in equation (2.20) rises, and there is a Tobin (1965)-type substitution from labor to capital across all sectors in response to the higher real wage to real interest rate ratio; the Tobin (1965) like rise in  $s_{Ht} k_t / l_{Ht} h_t$  mitigates but does not reverse the fall in the return to human capital  $r_{Ht}$  caused by the increase in leisure. The growth rate, in equation (2.29), falls as  $R_t$  rises since both  $r_{Ht}$  and  $r_{Kt} \equiv r_t / (1 + \tilde{R}_t)$  fall. As the inflation rate continues to rise, the credit substitution channel allows the growth rate to decline at a decreasing rate, as more credit and less leisure are used as the substitute for the inflation-taxed good (Gillman and Kejak 2005).

### 3. Financial Development

An increase in inflation causes more use of exchange credit, with a movement along the marginal cost curve up to a new higher  $MC_t$ . This can be viewed as an increase in the scope of the credit production, with a given degree of financial development; this is a wider use of finance, rather than a deeper use, such as when it is used for intermediating more investment. The effect of the degree of financial development itself can also be identified.

#### 3.1. Effects of Financial Development on Marginal Cost

One interpretation of development is that there is a simple productivity increase; applied to the finance sector here, this would be given by an increase in  $A_Q$ . Another interpretation of development given in Lucas (2002) is that there are greater returns to scale in a function with less than constant returns to scale; applied to  $q^*$  production function in equation (2.11), this would be given by an increase in  $\gamma_1$  or  $\gamma_2$ . Both of these parameter cause a shift in the marginal cost curve, and impact upon growth.

**Proposition 2.** For a given level of  $q_t^*$ , an increase in  $A_Q$  decreases the marginal cost.

**Proof:** From equation (2.18), for a given  $q_t^*$  and  $(\gamma_1 + \gamma_2) \in (0, 1]$ , it follows that  $\partial(MC_t)/\partial A_Q < 0$ .

Figure 3 graphs how an increase in  $A_Q$  pivots down the marginal cost (dotted line) from its baseline (solid line). In contrast, changes in the scale parameters  $\gamma_1$  and  $\gamma_2$  cause changes in the curvature of the marginal cost curve and in its level for a given  $q_t^*$ :

**Proposition 3.** For a given  $w$  and  $r$ , an increase in  $\gamma_1$  causes a decrease in the curvature of the marginal cost curve, and second, causes an increase in the level of marginal costs for a given level of credit output, given a sufficiently low quantity of credit output.

**Proof:** Define curvature as  $\eta \equiv \left(\frac{\partial MC_t}{\partial q_t^*}\right) / \left(\frac{MC_t}{q_t^*}\right)$ ; then with  $\gamma \equiv \gamma_1 + \gamma_2$ , it

follows that  $\eta = (1 - \gamma) / \gamma$ , and  $\partial\eta/\partial\gamma_1 < 0$ . Second, by equation (2.18),

$$\begin{aligned} \frac{\partial MC_t}{\partial \gamma_1} &= \frac{\partial}{\partial \gamma_1} \left\{ e^{\frac{1}{\gamma_1 + \gamma_2} [\gamma_1 (\log w_t - \log \gamma_1) + \gamma_2 (\log r_t - \log \gamma_2) - \log A_Q + (1 - \gamma_1 - \gamma_2) \log q_t^*]} \right\} \\ &= MC_t \frac{-\gamma_1 - \gamma_2 + \gamma_2 (\log w_t - \log r_t) - \gamma_2 (\log \gamma_1 - \log \gamma_2) + \log A_Q - \log q_t^*}{(\gamma_1 + \gamma_2)^2}. \end{aligned}$$

For ease of exposition, let  $\gamma_1 = \gamma_2$ . Then

$$\frac{\partial MC_t}{\partial \gamma_1} = MC_t \frac{-2\gamma_1 + \gamma_1 (\log w_t - \log r_t) + \log A_Q - \log q_t^*}{4\gamma_1^2} > 0$$

for  $q_t^* < e^{-2\gamma_1} \left(\frac{w_t}{r_t}\right)^{\gamma_1} A_Q$ .

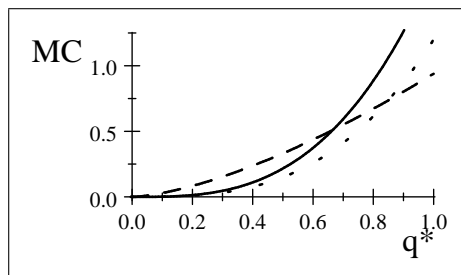


Figure 3. Marginal Cost with Changes in  $A_Q$  and  $\gamma$

Figure 3 illustrates the proposition. For  $MC_t = B (q^*)^{(1-\gamma)/\gamma}$ , where  $B$  is given by equation (2.18), it graphs an increase in  $\gamma$  from  $\gamma = 0.25$  (solid line) to  $\gamma = 0.40$  (dashed line), while  $B$  actually depends on  $\gamma$  and falls in turn in this example from 1.73 to 0.94. For  $MC_t = B (q_t^*)^{(1-\gamma)/\gamma}$ ; the increase causes less curvature and a higher marginal cost for a given, sufficiently low,  $q_t^*$ . The proposition indicates that a more financially developed economy has a less "curved" marginal cost. It is convex for  $\gamma < 0.5$ , linear for  $\gamma = 0.5$ , and concave for  $\gamma > 0.5$ . The marginal cost approaches a horizontal line for  $\gamma = 1$ ; the  $\lim_{\gamma \rightarrow 1} MC = B$ . It approaches a vertical one for  $\gamma = 0$ . Put differently, with a very financially undeveloped economy, as when  $\gamma \rightarrow 0$ , credit is "constrained" in that the  $MC_t$  curve approaches a reverse-L shape (∩).

It is important to note that if  $\gamma = 1$ , then there is no third factor, deposited funds, entering into the credit production function, and there is no equilibrium:

**Proposition 4:** Assume that  $\gamma_1 + \gamma_2 = 1$ , that both the credit sector and goods production sector are equally labor intensive ( $\gamma_1 = \beta$ ). Then there exists no equilibrium.

**Proof:** Please see the Appendix A.1.

The proposition shows the importance of deposited funds as a non-trivial factor: the marginal cost per unit of funds is upwards sloping, as in Figure 2 above, and there is a unique equilibrium of credit supplied, and of money demanded, at a given nominal interest rate.<sup>6</sup>

### 3.2. Interpretation of Economies of Scale

The economies of scale can be interpreted as the per unit cost of credit. Consider that the total financial intermediary dividends returned to the consumer are  $R_{qt}d_t$ . In per unit of credit terms these dividends are  $R_{qt}d_t/q_t$ , denoted by  $R_{qt}^* \equiv R_{qt}d_t/q_t$ .

**Proposition 5.** The proportional per unit cost of credit is equal to the degree of the economies of scale:  $(R_t - R_{qt}^*)/R_t = \gamma_1 + \gamma_2$ .

**Proof.** Since  $R_t = p_{qt}$  by equation (2.23), then, by use of the CRS property of the production function of equation (2.10),  $\frac{w_t l_{qt} h_t}{R_t q_t} = \gamma_1$  and  $\frac{r_t s_{qt} k_t}{R_t q_t} = \gamma_2$ . From

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<sup>6</sup>The nature of the mixed equilibrium problem was put forth by King and Plosser (1984) in terms of the assumption of constant returns to scale (CRS) in financial sector production: "The constant returns to scale structure implies that at given factor prices the finance industry supply curve is horizontal." This flat marginal cost schedule competes against a similarly flat price of money, being the nominal interest rate, and there is no equilibrium. Baltensperger (1980), in focusing on costly intermediation services, finds that the production function must be of decreasing returns to scale in capital and labor, or conversely that there needs to be a convex cost function, so that the constant marginal revenue per unit of funds equals the rising marginal cost per unit funds. Berk and Green (2004), in their study of mutual funds intermediation, specify a convex cost function, as does Wang, Basu, and Fernald (2004) for a variety of value-added bank services. Alternative approaches to this problem include Aiyagari, Braun, and Eckstein (1998) and Li (2000). Relatedly, Lucas (2000) and Canzoneri and Diba (2005) normalize their monetary transaction cost function by the quantity of exchange funds (equal to output). For an aggregate output application in which less than CRS in normalized labor and capital is central, Hansen and Prescott (2005) specify a third factor of production as the number of production plants, with a normalization per unit of plants.

equation (2.15), and using the definitions above of  $l_{qt} \equiv \frac{l_{Qt}h_t}{d_t}$ ,  $s_{qt} \equiv \frac{s_{Qt}k_t}{d_t}$ , and  $q_t^* \equiv \frac{q_t}{d_t}$ , then it follows that  $R_{qt} = R_t q_t^* - \gamma_1 R_t q_t^* - \gamma_2 R_t q_t^* = R_t q_t^* (1 - \gamma_1 - \gamma_2)$ . With the definition above that  $R_q^* \equiv R_{qt}/q_t^*$ , then  $R_q^* = R_t (1 - \gamma_1 - \gamma_2)$ , or  $R_t = R_q^* + (\gamma_1 + \gamma_2) R_t$ , and so  $(R_t - R_q^*)/R_t = \gamma_1 + \gamma_2$ .

The differential between the price of credit per unit of credit output,  $R_t$ , and the dividend rate of return per unit of credit,  $R_{qt}^*$ , gives the average cost of the resource use per unit of credit,  $(\gamma_1 + \gamma_2) R_t$ . This makes the degree of the returns to scale,  $\gamma_1 + \gamma_2$ , equal to the fraction of the nominal interest rate that are used up by the costs per unit of credit, which is the basis for calibration in Section 4.

### 3.3. The Effect of Financial Development on Growth

For the case of no physical capital, the analytic effect on growth of changes in financial development can be simply derived. With  $\beta = \varepsilon = 1$ , and  $\gamma_2 = 0$ , the production functions become  $c_t = A_G l_{Gt} h_t$ ,  $h_{t+1} = (1 + A_H l_{Ht} - \delta_H) h_t$ , and  $q_t^* = A_Q l_{qt}^1$ . Using the equilibrium conditions, (2.23)-(2.28), the solutions for normalized credit use, goods, leisure and the growth rate are given by  $q_t^* = q^* = (\gamma_1 R/A_G)^{\gamma_1/(1-\gamma_1)} A_Q^{1/(1-\gamma_1)}$ ,  $\frac{c_t}{h_t} = \frac{A_G \rho [1 + A_H (1-x) - \delta_H]}{A_H (1 + \gamma_1 A_G q^* R) (1 + \rho)}$ ,  $x = \frac{\frac{\alpha \rho}{1 + \rho} \Omega(R) \left(1 + \frac{1 - \delta_H}{A_H}\right)}{1 + \frac{\alpha \rho}{1 + \rho} \Omega(R)}$ , where  $\Omega(R) = \frac{1 + (1 - q^*)R + (\gamma_1)q^*R}{1 + \gamma_1 A_G q^* R}$  (ratio of shadow price of goods to social cost of goods), and finally  $1 + g = \frac{1 + A_H (1-x) - \delta_H}{1 + \rho}$ .

**Proposition 6.** With  $\beta = \varepsilon = 1$ , and  $\gamma_2 = 0$ , an increase in the credit sector productivity level,  $A_Q$ , causes an unambiguous decrease in leisure and increase in growth.

**Proof:** From the equilibrium solution given above, it is clear that  $\partial q^*/\partial A_Q > 0$ , and since  $\gamma_1 < 1$  that  $\partial \Omega(R)/\partial q^* < 0$ . With  $\delta_H < 1$ , it follows that  $\partial x/\partial \Omega(R) > 0$ . Consequently  $\partial x^*/\partial A_Q < 0$ ; with  $\partial g/\partial x < 0$ , then  $\partial g^*/\partial A_Q > 0$ .

This shows the intuitive result that greater productivity in producing credit results in a higher balanced-path growth rate, for a given rate of money supply growth. Increasing the degree of economies of scale gives the opposite result for sufficiently low nominal interest rates:

**Proposition 7.** With  $\beta = \varepsilon = 1$ , and  $\gamma_2 = 0$ , and given that  $R = \sigma + \rho + \sigma\rho < R' \equiv \frac{A_G}{A_Q} \frac{1}{\gamma_1} e^{-(1-\gamma_1)}$ , an increase in  $\gamma_1$  causes an increase in leisure  $x$  and a decrease in the growth rate  $g$ ; i.e.  $\partial x / \partial \gamma_1 > 0$  and  $\partial g / \partial \gamma_1 < 0$ .

**Proof:** Please see the Appendix.

Intuitively, an increase in the normalized returns to scale of labor (and capital) in credit production, while maintaining a given nominal interest rate, causes a lower marginal cost per unit of credit output for a high level of credit output, but a higher marginal cost for low credit outputs. The latter case results in less total credit production, a more inelastic money demand, and more leisure because of a greater use of the goods to leisure channel for avoiding inflation, instead of the money to credit channel for avoiding inflation. More leisure causes less growth.

#### 4. Investment Rate, Tobin Effects, and Employment

The effect of inflation and financial development on investment can be seen in the full model with physical capital. To illustrate this, we calibrate the economy and simulate the effects of inflation on investment and growth and other variables, and with the effects of changes in financial development also included. These are presented below in Figure 4-8.

The benchmark calibration uses realistic values so that the simulations are plausible. The parameter values are  $\rho = 0.04$ ,  $\delta_K = \delta_H = 0.1$ ,  $\theta = 1$ ,  $\beta = 0.64$ ,  $\varepsilon = 0.8$ ,  $\gamma = 0.13$ ,  $\gamma_1 = 0.083$ ,  $\alpha = 5$ ,  $A_G = 0.616$ ,  $A_H = 0.6$ ,  $A_Q = 1.31$ , and  $\gamma_1 + \gamma_2 = 0.13$ . The implied variable values are  $q^* = 0.776$ ,  $x = 0.64$ ,  $g = 0.02$ ,  $\pi = 0.05$ ,  $l_G = 0.14$ ,  $l_Q = 0.0016$ ,  $s_G = 0.59$ ,  $s_Q = 0.007$ . Also residually we then have  $\sigma = g + \pi = 0.02 + 0.05 = 0.07$ ;  $R = \rho + \sigma = 0.11$ ;  $h/k = 2.66$ ,  $r_H = 0.160$ ,  $l_G h / (s_G k) = 0.634$ ,  $r = 0.166$ ,  $w = 0.464$ . This is similar to the values used in the literature, where applicable, and for the credit sector, to Benk et al (2007).

The basis for the calibration for  $\gamma_1 + \gamma_2 = \gamma$  is the interest differential formula of Proposition 5, whereby  $\gamma_1 + \gamma_2 = (R - R_q)/R$ . It is calibrated using financial industry data at  $\gamma = 0.13$ . First note that the Cobb-Douglas function implies

that  $R_q d = Rq(1 - \gamma)$  is the total dividend returned to the consumer (interest dividend on deposits); this makes  $\gamma Rq$  the resource cost of the credit. Per unit of credit this is  $\gamma R$ , so  $\gamma$  is the per unit cost of credit divided by  $R$ . To calculate this, we use the M1 US average annual income velocity for 1972-2005, equal to  $1/0.15$ ; so  $q/d = q^* = [1 - (m/y)] = 1 - 0.15 = 0.85$ . Then  $\gamma = (\text{total credit cost})/Rd(0.85)$ . For a rough estimate, we use for the basis the average annual fee for a American Express credit card, for how much interest is paid on average; it is assumed to reflect the total interest costs of using the annual exchange credit, also called a "charge card" rather than a roll-over intertemporal credit. For an average person this is calculated as \$170, comprised of the basic \$125 Gold card annual fee plus other ad-on charges of \$45 such as for late payment penalties and risk premiums for less credit worthy purchasers. Then  $\gamma = 170/Rd(0.85)$ . With GDP per-capita at 25127, at 2006 prices, then we set  $y = d = 25127$ ; and  $R = 0.0606$  the 3-month Treasury Bill interest rate (annual basis), then  $\gamma = 170/[(0.0606)25127(0.85)] \simeq 0.13$ .

Using the calibrated values, instead of a general illustration as in Figure 1, Figure 4 shows a cross-section of the equilibrium production of credit  $q^* \in [0, 1]$  in equation (2.11), graphed with respect to the  $l_q$  axis and including the (2.15) line. The equilibrium point (circle) shows where the marginal product of labor in credit production equals the real wage divided by the market price of credit,  $R$ .

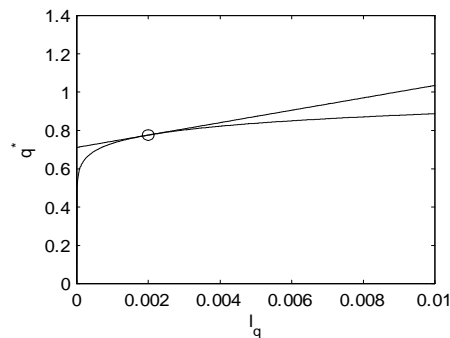


Figure 4. Equilibrium Credit Production

Figures 5-7 present the baseline simulation (solid lines) along with changes to

the baseline (dashed line). The changes are a 5% rise in  $A_Q$ , in Figure 5, from 1.31 in the baseline to 1.376, and a 20% rise in  $\gamma$ , in Figure 6, from 0.13 to 0.156.

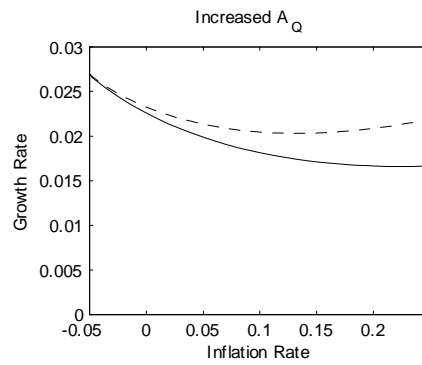


Figure 5. Inflation, Growth, and Changes in  $A_Q$

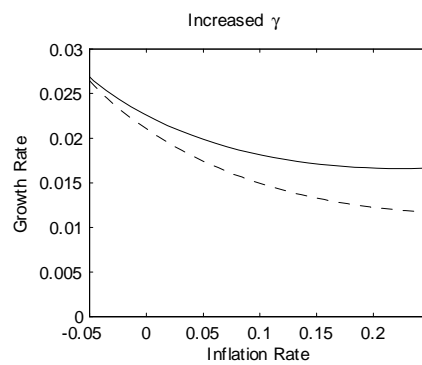


Figure 6. Inflation, Growth, and Changes in  $\gamma$

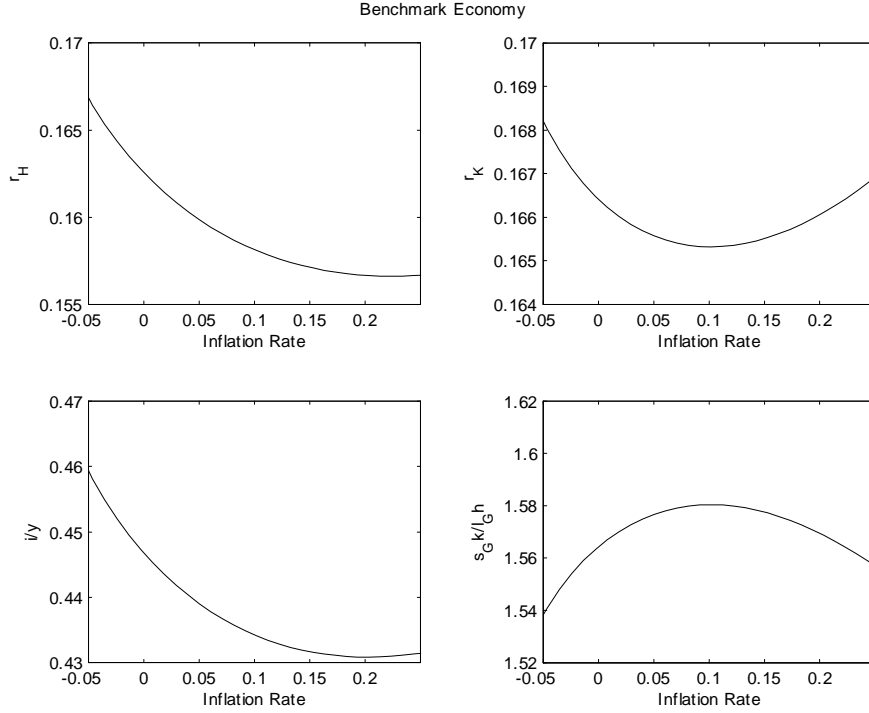


Figure 7. Inflation vs Returns on Capital, Investment Rate, Capital/Effective Labor

The baseline simulation in Figures 5 and 6 shows that as the inflation rate rises, the balanced-path growth rate falls at a decreasing rate, for moderately high inflation rates. The comparative statics show this effect has less curvature when  $A_Q$  increases, with a higher growth rate for each inflation rate, and more curvature for an increase in  $\gamma$ , with a lower growth rate for each inflation rate.

Figure 7 shows how inflation similarly affects other variables: the return on human capital,  $r_H$ , the investment to output ratio,  $i/y$ , and the return on physical capital,  $r_K \equiv r_t / (1 + \tilde{R}_t)$  all fall as the inflation rate increases, at low levels of inflation; and the capital to effective labor ratio in goods production rises with inflation, at low levels of inflation. The first two variables,  $r_H$  and  $i/y$ , both decrease nonlinearly as the inflation rate increases, until the decrease approaches zero and even starts to increase. Meanwhile  $r_K$  falls and then rises as inflation

increase, and  $s_G k / l_G h$ , moves as a mirror reflection in opposite of that of  $r_K$ . The capital return  $r_K$  and the capital to effective labor ratio  $s_G k / l_G h$  begin inflecting at lower inflation rates, than do  $r_H$  and  $i/y$ , because of the inclusion of investment within the exchange constraint.

The Section's last Figure 8 indicates that the rate of productively employed labor, in the three sectors of goods, human capital and credit production, or  $1 - x$ , falls at a decreasing rate as the inflation rate rises. This rate of employed time is also what can be called the human capital utilization rate. It exhibits a similar nonlinearity as seen for other variables.

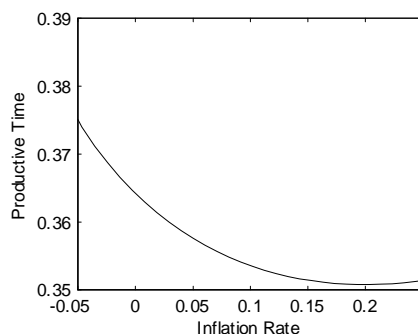


Figure 8. Employment Rate and Inflation

## 5. Discussion: Explaining the Evidence

Driffill (2003), Trew (2006), and Rousseau and Vuthipadadorn (2005) for example, describe many of the difficult issues involved in determining how finance affects growth, and these can become more complex when bringing in the interaction of finance with inflation, for example as in Aiyagari, Braun, and Eckstein (1998), Boyd, Levine, and Smith (2001), Rousseau and Wachtel (2001) and Rousseau and Wachtel (2002).

### 5.1. Financial Development, Investment, and Growth

Rousseau and Vuthipadadorn (2005), using VAR and VECM time series methods, find that financial development robustly is associated with increases in investment

in several countries; this is consistent with the our model's comovement of "financial depth" with the amount of investment, which results from our assumption of no intertemporal intermediation cost. However, Rousseau and Vuthipadadorn (2005) also find Granger causality from financial development to investment; to show this we would need to model the intertemporal intermediation process explicitly, with a costly process (in addition to the existing costly exchange credit intermediation). Then it would be clear that a decrease in the cost of such intermediation would cause more investment to be intermediated.

Rousseau and Vuthipadadorn (2005) categorize the effects of finance on growth into two channels. First, 'the "total factor productivity" channel operates through innovative financial technologies' (quotes in original). This exactly describes our model's increase in the  $A_Q$  parameter, which causes an unambiguous increase in growth. 'The second "factor accumulation" channel focuses on the spread of organized finance in place of self-finance and the resulting improvement to the ability of intermediaries to mobilize otherwise unproductive resources and help firms to overcome project indivisibilities' (quotes in original). This fits well a description of increasing economies of scale, which in our model is characterized by an increase in  $\gamma_1$  and  $\gamma_2$ . Their results indicate little effect of financial development on output which they interpret, in combination with their investment effects, as implying that financial development worked primarily through the factor accumulation channel.

With panel data, Rousseau and Wachtel (2002) find that finance positively affects growth only for low levels of the inflation rate. Gillman and Harris (2004a) and Gillman and Harris (2004b) find that financial development interacts with inflation and causes a negative effect on growth. Our model is consistent with both of these results if at higher levels of inflation the "factor accumulation" channel is operative and the scale of the financial sector ( $\gamma_1$  or  $\gamma_2$ ) is induced to increase with a negative effect on growth that is sufficiently large to dominate any simultaneous positive total factor productivity increases to  $A_Q$ .<sup>7</sup> At low levels of inflation, if

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<sup>7</sup>Other explanations of how the increase in exchange credit can cause lower growth rates can

total factor productivity increases dominate any increase in scale, then the growth effect of financial intermediation would be positive. This is plausible in that the scale of operations might more likely remain relatively fixed when inflation is low and inflation-tax avoiding demands on financial intermediation are less; then total factor productivity increases can dominate the effect on growth. However, in the model the finance sector's total factor productivity and scale economies are exogenous, and so can only explain the evidence in a comparative static fashion, for a given level of inflation.

## 5.2. Inflation Effects on Economy-wide Variables

In the model, with the extent of costless intertemporal credit intermediation viewed as a measure of financial depth, inflation causes a decrease in financial depth at the same time as it decreases investment. This is consistent with Boyd, Levine, and Smith (2001) and Rousseau and Wachtel (2001) who find empirically that an increase in inflation causes decreases in various measures of financial depth that are related to investment;<sup>8</sup> Boyd and Champ (2006) discuss this in a related context. And inflation decreases the growth rate in a nonlinear way as found empirically and explained in Gillman and Kejak (2005). The employment rate decreases in the same nonlinear fashion.

At the same time that the model can explain how inflation decreases the investment rate, the related financial depth, the growth rate, and the employment rate, it also explains why the scope of finance expands as an inflation-avoidance instrument, as in the evidence presented by Aiyagari, Braun, and Eckstein (1998). There they find that countries such as Israel experienced increases in the size of their financial sector as the inflation rate increased. Our model produces this in

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be found in the literature: for example in Ireland (1994), transitionally, the inflation-induced investment of capital in producing financial intermediation is a diversion of resources that causes a temporarily lower growth rate.

<sup>8</sup>A close relation between financial depth and investment is put forth in Boyd, Levine, and Smith (2001) in the following way: "...as inflation rises... the financial sector makes fewer loans, resources allocation is less efficient, and intermediary activity diminishes with adverse consequences for capital investment."

that the use of exchange credit rises, and money demand falls, as the inflation rate goes up. Thus the model can be viewed as explaining this phenomenon by the substitution from intertemporal financial intermediation towards exchange financial intermediation when the inflation-tax rises and decreases the return to both human and physical capital.

In addition, the inflation-induced decreased return to physical capital has been found empirically in a series of papers (Ahmed and Rogers (2000), Rapach (2003), Rapach and Wohar (2004)), and this has been viewed to be indicative of an operative Tobin (1965) effect. In our model, inflation causes a decreased return to physical capital, and also a rise in the capital to effective labor ratio in all sectors, since the ratio of the wage rate to the real interest rises. We view the capital to effective labor increase as the key part of the Tobin effect: instead of the capital stock rising as in Tobin, here in general equilibrium it is the capital to effective labor ratio that rises.<sup>9</sup> Thus our model is consistent with this positive Tobin (1965) effect, even though it includes a Stockman (1981) exchange constraint that causes an inflation tax on investment.

In Stockman (1981), this constraint causes the capital stock to fall, which is a reverse Tobin effect that occurs in a framework with no labor, leisure, or endogenous growth, and so no reallocation from labor to capital as a result of the inflation tax causing substitution from goods to leisure. Within our framework, the Stockman (1981) constraint is the reason why the investment ratio declines when inflation goes up, even as the real wage to real interest ratio rises, along with the capital to effective labor ratio. So the extension plays a critical role in the model's ability to explain evidence that inflation causes financial depth and investment ratio to decline. Without the extension, the investment ratio rises with the inflation rate.

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<sup>9</sup>Gillman and Nakov (2003) present US cointegration evidence of inflation with the capital to effective labor ratio, with Granger causality from inflation to this input ratio.

### 5.3. Source and Pervasiveness of Nonlinearity

Rousseau and Wachtel (2001) specifically identify nonlinear relations in the inflation-finance-growth nexus and consider these a challenge to explain. Our model shows that this nonlinearity is not an isolated event. It is found in the effect of inflation on all variables. Its source is the demand for money that is residual from the credit supply, since they are perfect substitutes in exchange. The demand for exchange means affects the equilibrium through the marginal rate of substitution between normalized goods and leisure, in equation (2.26). To see this, for simplicity assume that  $\gamma_1 = \gamma_2$ , and  $\gamma = \gamma_1 + \gamma_2$ . As the inflation rate goes up, the shadow price of goods in equation (2.26),  $1 + \tilde{R}_t \equiv 1 + (1 - q_t^*)R_t + \gamma q_t^* R_t$ , rises *but at a decreasing rate*. This follows directly from the solution for  $q_t^*$  in equation (2.25); in particular,  $\partial (1 + \tilde{R}_t) / \partial R_t > 0$ , and  $\partial^2 (1 + \tilde{R}_t) / \partial R_t^2 < 0$ . Therefore the shadow price of goods rises and induces substitution toward leisure, but since it rises at a decreasing rate, it induces a decreasing amount of additional leisure.

The pervasiveness of this nonlinearity in the model provides an explanation of why the nonlinearity has been found empirically with respect to the effect of inflation on financial depth and growth. It also gives a foundation for finding such a nonlinearity in the empirical study of the other effects of inflation, such as the investment, Tobin effects and employment effects, although these nonlinearities apparently have been little studied as yet.

The money demand itself, at the basis of the nonlinearity, is supported directly by evidence. As the inflation-induced substitution is less from goods to leisure, it is more from money to credit, giving a rising interest elasticity demand for money. Such Cagan forms have been verified as holding in international evidence both for individual countries and for panel data (Mark and Sul 2003).<sup>10</sup>

Since the money demand is a residual of the credit supply, it is also important that the credit supply evidence be supportive. The standard assumption in the financial intermediation services literature, coming from Sealey and Lindley (1977)

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<sup>10</sup>This Cagan elasticity feature has been shown elsewhere in a related economy, in Gillman and Kejak (2005) and Gillman and Otto (2007).

and Clark (1984), is to assume constant returns to scale in labor, capital and deposited funds. This form repeatedly has been supported empirically, back as far as Hancock (1985).<sup>11</sup>

Now, since our model assumes this CRS feature, and this underlies the nonlinearity, the paper does not explain it per se. However it does show that with CRS in only labor and capital, without deposited funds, there is no equilibrium. With the third factor of deposited funds, then the normalization of the cost of credit production in terms of units of deposited funds puts the marginal cost of credit into the same units of the marginal cost of money. And it is this normalized marginal cost that is monotonically upward sloping, as a unique equilibrium requires, and that forms the basis of the empirically consistent nonlinearity.

## 6. Conclusion

The theory of how inflation interacts with financial development in the model is novel. It contributes a perspective for interpreting empirical research into the determinants of economic growth and other economy-wide variables. It explains simultaneously ten different empirical findings, how these findings are related theoretically, and why a nonlinearity is systemic.

The model is an endogenous growth monetary economy in which inflation causes the investment rate to decrease. With the savings-investment intermediation process assumed to be costless, financial development is defined relative to the parameters of the production of exchange credit that enables inflation-tax avoidance. Similar to Rousseau and Vuthipadadorn (2005), there are two channels for change in financial development: the total factor productivity in the finance sector, and the degree of scale economies for normalized capital and labor in producing exchange credit. An increase in the former causes an increased growth rate while an increase in the latter leads to the opposite. We simulate how inflation causes investment, financial depth, and growth to decrease nonlinearly, while the

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<sup>11</sup>For a recent example, see Wheelock and Wilson (2006).

scope of finance increases through greater inflation-avoidance activity. The Tobin (1965) effect is positive, as in evidence, even while the investment rate declines. The money demand function of the model underlies the nonlinearities that are present, and this function, in itself, is consistent with evidence in support of the Cagan function; the financial intermediation production is also consistent with empirical evidence.

An extension that makes costly the investment-savings intermediation process, and that also made stochastic the model by introducing standard (goods productivity and money shocks) plus shocks to the intermediation sector, could potentially be a platform for studying the effects from financial crises. And business cycle effects might also be useful to examine, as in Benk, Gillman, and Kejak (2007), but with a focus on investment. The deterministic framework makes for perfect foresight which certainly does not apply. Episodes when inflation was clearly not expected would be better examined stochastically. In this sense the paper only provides a suggestive framework for explaining what has been found in longer run empirical studies.

## A. Appendix

### A.1. Proof of Proposition 1

From equation (2.11),  $q_t/d_t = A_Q \left(\frac{l_{Qt}h_t}{d_t}\right)^{\gamma_1} \left(\frac{s_{Qt}k_t}{d_t}\right)^{\gamma_2}$ , and with  $\gamma_1 + \gamma_2 = 1$ , then  $1 = A_Q (l_{Qt}h_t/q_t)^{\gamma_1} (s_{Qt}k_t/q_t)^{\gamma_2}$ . Using equations (2.11) and (2.25), it can be shown that  $l_{Qt}h_t/q_t = \gamma_1 R_t/w_t$ , and  $s_{Qt}k_t/q_t = \gamma_2 R_t/r_t$ ; substituting these relations back into the previous equation, it results that  $1 = A_Q (\gamma_1 R_t/w_t)^{\gamma_1} (\gamma_2 R_t/r_t)^{\gamma_2}$ , or  $R_t = A_Q^{-1} (\gamma_1/w_t)^{-\gamma_1} (\gamma_2/r_t)^{-\gamma_2}$ . Substituting in for  $w_t$  and  $r_t$  from the equations (2.20) and (2.21),  $R_t = \left(\frac{\beta}{\gamma_1}\right)^{\gamma_1} \left(\frac{1-\beta}{\gamma_2}\right)^{\gamma_2} \left(\frac{A_G}{A_Q}\right) \left(\frac{l_{Gt}h_t}{s_{Gt}k_t}\right)^{(\beta-\gamma_1)}$ . With  $\gamma_1 = \beta$ , the last expression becomes  $R = \frac{A_G}{A_Q}$ . With log-utility, the nominal interest rate is a constant independent of the growth rate:  $R = \sigma + \rho + \sigma\rho$ , which in general is not equal to  $\frac{A_G}{A_Q}$ , giving a contradiction. In the case when  $\frac{A_G}{A_Q} = \sigma + \rho + \sigma\rho$ , then there is no equilibrium

since  $A_G > 0$  implies that  $R > 0$ ; then equation (2.25) implies that  $q_t = \infty$ , which violates that  $q_t/d_t \in [0, 1)$ , derived by combining equations (2.2) and (2.3).<sup>12</sup>

## A.2. Proof of Proposition 7

From the solution given in Section 3.3,  $1 + g_t = \frac{1 + A_H(1 - x_t) - \delta_H}{1 + \rho}$ , and so  $\frac{\partial g_t}{\partial \gamma_1} = -\frac{A_H}{1 + \rho} \frac{\partial x_t}{\partial \gamma_1}$ . The sign of this  $\left(\frac{\partial g_t}{\partial \gamma_1}\right)$  (using  $sign()$  as notation) depends negatively upon the  $sign\left(\frac{\partial x_t}{\partial \gamma_1}\right)$ , which in turn is equal to  $sign\left(\frac{\partial \Omega}{\partial \gamma_1}\right) = -sign\left(\frac{\partial(\gamma_1 q_t^*)}{\partial \gamma_1}\right)$ . Evaluating  $\frac{\partial(\gamma_1 q_t^*)}{\partial \gamma_1}$ , we find that

$$\begin{aligned} \frac{\partial(\gamma_1 q_t^*)}{\partial \gamma_1} &= \frac{\partial}{\partial \gamma_1} \left\{ e^{\frac{1}{1-\gamma_1} [\log \gamma_1 + \log A_Q + \gamma_1 \log(\frac{R}{A_G})]} \right\} \\ &= \gamma_1 q_t^* \frac{1 - \gamma_1 + \gamma_1 \log\left(\frac{R}{A_G}\right) + \gamma_1 \log \gamma_1 + \gamma_1 \log A_Q}{(1 - \gamma_1)^2} < 0 \end{aligned}$$

if  $R < R'(\gamma_1, A_G, A_Q) \equiv \frac{1}{\gamma_1} \frac{A_G}{A_Q} e^{-(1-\gamma_1)}$ . Therefore  $\frac{\partial x_t}{\partial \gamma_1} > 0$  and  $\frac{\partial g_t}{\partial \gamma_1} < 0$  for  $R < R'$ .

With the additional assumption that  $A_G = 0.616$  and  $A_Q = 1.31$ , as in the calibration in Section 4, Figure 8 below graphs  $R'$ . It shows that the resulting  $R'$  (curved line) is at hyperinflation ranges, above 50% (straight line) for all convex marginal cost of credit curves; that is for  $\gamma_1 < 0.5$ ,  $R' > 0.50$ . At the limit of  $\gamma = 1$ ,  $R' = 0.34$  and so  $R' > 0.35$  for all  $\gamma$ .

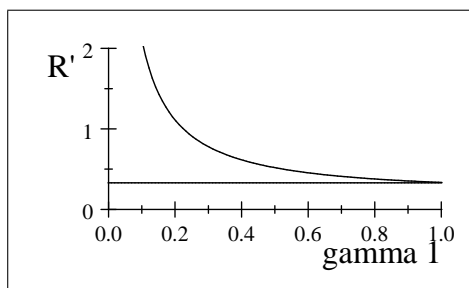


Figure 8.  $R'$  Bounds of Proposition 7

<sup>12</sup>This proof is a version of that found in Braun and Gillman (2006).

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