Option Pricing When the Regime-Switching Risk is Priced

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Abstract

Recently, there has been considerable interest in investigating option valuation problem in the context of regime-switching models. However, most of the literature consider the case that the risk due to switching regimes is not priced. Relatively little attention has been paid to investigate the impact of switching regimes on the option price when this source of risk is priced. In this paper, we shall articulate this important problem and consider the pricing of an option when the price dynamic of the underlying risky asset is governed by a Markov-modulated geometric Brownian motion. We suppose that the drift and volatility of the underlying risky asset switch over time according to the state of an economy, which is modeled by a continuous-time hidden Markov chain. We shall develop a two-stage pricing model which can price both the diffusion risk and the regime-switching risk based on the Esscher transform and the minimization of

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the maximum entropy between an equivalent martingale measure and the real-world probability measure over different states. The latter is called a min-max entropy problem. We shall conduct numerical experiments to illustrate the effect of pricing regime-switching risk. The results of the numerical experiments reveal that the impact of pricing regime-switching risk on the option prices is significant.

Keywords: Option valuation; Regime-switching risk; Two-stage pricing procedure; Esscher transform; Martingale restriction; Min-max entropy problem.

§1. Introduction

Option valuation is an important topic in modern financial economics. Since the seminal works of Black and Scholes (1973) and Merton (1973), there has been extensive research on both the theoretical and practical aspects of option valuation. The original works of Black and Scholes (1973) and Merton (1973) assume that the price dynamic of the underlying risky asset is governed by a geometric Brownian motion (GBM). Under the assumptions of a perfect market and the absence of arbitrage, they are able to derive a closed-form pricing formula for a standard European call option. The pricing formula is preference-free and has widely been adopted by market practitioners. It has been coined as one of the most important formulas in economics. Despite its compact form and popularity, the Black-Scholes-Merton pricing formula is obtained under the GBM assumption, which cannot explain some important empirical features of financial time series, such as heavy-tailness of the unconditional return's distribution and time-varying conditional volatility. The Black-Scholes-Merton pricing model also cannot explain some stylized empirical behavior of option prices, namely, implied volatility smile or smirk. Many models that extend the celebrated Black-Scholes-Merton have been proposed and tested empirically in the literature. Some important models include the jump-diffusion model of Merton (1976), the stochastic volatility models of Hull and White (1987) and Wiggins (1987) and the regime-switching models by Niak (1993) and Buffington and Elliott (2002a, b), etc. These models provide a relatively realistic way to explain the empirical behaviors of both the option prices and their underlying assets' prices compared with the Black-Scholes-Merton model.

Recently, regime-switching models become more and more important in different branches in modern financial economics. The origin of regime-switching models can track back to the original work of Hamilton (1989) in which a discrete-time Markov-switching autoregressive time series models was proposed. Applications of regime-switching models penetrate different areas in modern financial economics. Some works on these applications include Elliott and van der Hoek (1997) for asset allocation, Pliska (1997) and Elliott, Hunter and Jamieson (2001) for short rate models, Elliott and Hinz (2002) for portfolio analysis and chart analysis, Niak (1993), Guo (2001) and Buffington and Elliott (2002a,b) for option valuation and Elliott, Malcolm and Tsoi (2003) for volatility estimation.

In this paper, we shall consider the pricing of an option when the price dynamic of the underlying risky asset is governed by a Markov-modulated geometric Brownian motion. We suppose that the drift and the volatility of the underlying risky asset switch over time according to the state of an economy, which is modeled by a continuous-time hidden Markov chain. Most of the literature on option pricing under regime-switching models consider the case that the risk due to switching regimes is not priced. In this case, the risk premium due to switching regimes is assumed to be zero. However, in practice, the risk premium due to switching regimes can be substantial, especially for some long-lived options, since there can be significant structural change in the state of an economy over a long period. Guo (2001) uses Arrow-Debreu securities related to the cost of switching to complete the market by hedging away the additional uncertainty induced by regime switching. Elliott et al. (2005) introduce the use of the Esscher transform for determining an equivalent martingale measure under the Markov-modulated geometric Brownian motion in which the risk due to switching regimes is not priced. In our case, we price the regime-switching risk. We shall develop a two-stage pricing model which can price both the diffusion risk and the regime-switching risk. The first stage of the method involves the use of a well-known tool in actuarial science, namely, the Esscher transform to determine a set of equivalent martingale measures satisfying a martingale restriction. In the second stage, we determine an equivalent martingale pricing measure by minimizing the maximum entropy between an equivalent martingale measure and the real-world probability measure over different states. The latter is called a min-max entropy problem. We shall conduct numerical experiments to illustrate the effect of pricing regime-switching risk.

This paper is structured in the sequel. Section two presents the price dynamics in the model. In section three, we present the two-stage pricing method. We shall present and discuss the results of numerical experiments in section four. The final section concludes the paper.

§2. The Price Dynamics

We consider a financial model consisting of two primary assets, namely, a money market account B and a stock S, that are tradable continuously. In the sequel, we shall describe the price dynamics of these two assets.

First, fix a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where $\mathcal{P}$ is a real-world probability measure. Let $\mathcal{T}$ denote a finite time horizon $[0, T]$, where $T < \infty$. Define a hidden Markov chain $\{X_t\}_{t \in \mathcal{T}}$ on $(\Omega, \mathcal{F}, \mathcal{P})$ with a finite state space $\mathcal{X} := (x_1, x_2, \ldots, x_N)$. Following Elliott et al. (1994), we identify the state space of $\{X_t\}_{t \in \mathcal{T}}$ by a finite set of unit vectors $\mathcal{E} := \{e_1, e_2, \ldots, e_N\}$, where $e_i = (0, \ldots, 1, \ldots, 0) \in \mathcal{R}^{\mathcal{N}}$. This is called the canonical representation of the state space of $\{X_t\}_{t \in \mathcal{T}}$. Write Q for the generator or rate matrix $[q_{ij}(t)]_{i,j=1,2,\ldots,N}$ for the hidden Markov chain $\{X_t\}_{t \in \mathcal{T}}$. Then, with the canonical representation of the state space, Elliott (1993) and Elliott et al (1994) provide the following semi-martingale decomposition for $\{X_t\}_{t \in \mathcal{T}}$:

$$X_t = X_0 + \int_0^t Q X_s ds + M_t \ , \tag{2.1}$$

where $\{M_t\}_{t \in \mathcal{T}}$ is an $\mathcal{R}^N$-valued martingale with respect to the filtration generated by $\{X_t\}_{t \in \mathcal{T}}$ and the measure $\mathcal{P}$.

Let r_t denote the instantaneous market interest rate of the money market account at time t.

We suppose that

$$r_t = r(t, X_t) = \langle r, X_t \rangle \ , \tag{2.2}$$

where $r := (r_1, r_2, \ldots, r_N) \in \mathcal{R}^N$ with $r_i > 0$, for each $i = 1, 2, \ldots, N$.

Then, the price dynamic of the money market account $\{B_t\}_{t \in \mathcal{T}}$ is governed by:

$$B_t = \exp\Big(- \int_0^t r_u du \Big) \ , \quad B_0 = 1 \ . \tag{2.3}$$

Let $\{\mu_t\}_{t \in \mathcal{T}}$ and $\{\sigma_t\}_{t \in \mathcal{T}}$ denote the appreciation rate and the volatility of the stock S, which are assumed to be governed by:

$$\mu_t := \mu(t, X_t) = \langle \mu, X_t \rangle \ , \quad \sigma_t := \sigma(t, X_t) = \langle \sigma, X_t \rangle \ , \tag{2.4}$$

where $\mu := (\mu_1, \mu_2, \ldots, \mu_N) \in \mathcal{R}^N$ and $\sigma := (\sigma_1, \sigma_2 \ldots, \sigma_N) \in \mathcal{R}^N$ with $\sigma_i > 0$ for each $i = 1, 2, \ldots, N$.

Let $\{W_t\}_{t \in \mathcal{T}}$ denote a standard Brownian Motion on $(\Omega, \mathcal{F}, \mathcal{P})$. Then, we suppose that the price dynamic of the underlying stock $\{S_t\}_{t \in \mathcal{T}}$ is governed by the following Markov-modulated geometric Brownian motion with jumps:

$$dS_t = \mu_t S_t dt + \sigma_t S_t dW_t \ , \quad S_0 = s \ . \tag{2.5}$$

Let Y_t denote the logarithmic return $\ln(S_t/S_0)$ from S over the interval $[0, t]$. Then, the price dynamic of S can be written as:

$$S_t = S_u \exp(Y_t - Y_u) \ , \tag{2.6}$$

where

$$Y_t = \int_0^t \Big(\mu_s - \frac{1}{2} \sigma_s^2 \Big) ds + \int_0^t \sigma_s dW_s \ . \tag{2.7}$$

§3. The Pricing Method

In this section, we shall present a two-stage pricing method for an option under the regime-switching framework presented in the last section. Our goal is to develop a method which is able to price the risk due to switching regimes. In the first stage, we shall adopt the regime-switching Esscher transform to determine a martingale condition. The Esscher transform is a well-known tool in actuarial science. It has been adopted for developing premium rules and approximating aggregate claim distributions. The seminal work of Gerber and Shiu (1994) pioneers the use of the Esscher transform for option valuation in an incomplete market. Their work highlights the interplay between financial and actuarial pricing, which is an important topic of modern actuarial research as pointed out in Bühlmann et al. (1996). In fact, the Esscher transform is a convenient and flexible tool for option valuation. In general, we have more than one set of risk-neutral Esscher parameters satisfying the martingale condition implied by the regime-switching Esscher transform. In other words, the regime-switching Esscher transform is not enough to determine a unique martingale pricing measure. So, we require some additional conditions to determine an equivalent martingale pricing measure. We shall select a equivalent martingale pricing measure by minimizing the maximum entropy between an equivalent martingale measure and the real-world probability measure over different states. Some works on the minimization of entropy for option pricing include Buchen (1996), Avellaneda (1998), Frittelli (2000), Miyahara (2001) and McLeish and Reesor (2003), and others.

First, we shall describe the regime-switching Esscher transform. Let $\{\mathcal{F}_t^X\}_{t\in\mathcal{T}}$ and $\{\mathcal{F}_t^S\}_{t\in\mathcal{T}}$ denote the $\mathcal{P}$-augmentation of the natural filtrations generated by $\{X_t\}_{t\in\mathcal{T}}$ and $\{S_t\}_{t\in\mathcal{T}}$, respectively. Define $\mathcal{G}_t$ to be the σ-algebra $\mathcal{F}_t^X \vee \mathcal{F}_t^S$, for each $t \in \mathcal{T}$. Write θ_t for the regime switching Esscher parameter at time t, which can be written as follows:

$$\theta_t := \theta(t, X_t) = \langle \theta, X_t \rangle \ , \tag{3.1}$$

where $\theta := (\theta_1, \theta_2, \ldots, \theta_N) \in \mathcal{R}^N$.

Following Elliott (1982), write $(\theta \cdot Y)_t := \int_0^t \theta_u dY_u$, for each $t \in \mathcal{T}$. Then, we define the regime switching Esscher transform $\mathcal{Q}_\theta \sim \mathcal{P}$ on $\mathcal{G}_T$ as follows:

$$\frac{d\mathcal{Q}_\theta}{d\mathcal{P}} := \frac{e^{(\theta \cdot Y)_T}}{E[e^{(\theta \cdot Y)_T} | X_0]} = \Lambda_T \ , \tag{3.2}$$

where $E[\cdot]$ denotes an expectation under $\mathcal{P}$.

Note that the expectation in the denominator of the regime-switching Esscher transform is unconditional. This is different from Elliott et al. (2005) in which the corresponding expectation is conditional on $\mathcal{F}_t^X$. With the unconditional expectation, the risk due to switching regimes is priced.

Define a $(\mathcal{G}, \mathcal{P})$-martingale $\{\Lambda_t\}_{t \in \mathcal{T}}$ as below:

$$\Lambda_t := E[\Lambda_T | \mathcal{G}_t] \ , \quad t \in \mathcal{T} \ . \tag{3.3}$$

Lemma 3.1: Define

$$\lambda_i(\theta_i) := \theta_i \mu_i - \frac{1}{2} \theta_i \sigma_i^2 + \frac{1}{2} \theta_i^2 \sigma_i^2 \ , \quad i = 1, 2, \ldots, N \ , \tag{3.4}$$

and $\lambda(\theta) := (\lambda_1(\theta_1), \lambda_2(\theta_2), \ldots, \lambda_N(\theta_N)) \in \mathcal{R}^N$.

Then,

$$\Lambda_t = e^{(\theta \cdot Y)_t} \frac{\left\langle e^{(Q + diag(\lambda(\theta)))(T-t)} X_t, \mathbf{1}_N \right\rangle}{\left\langle e^{(Q + diag(\lambda(\theta)))T} X_0, \mathbf{1}_N \right\rangle} \ . \tag{3.5}$$

Proof: First, note that $\int_0^t \theta_u dY_u | \mathcal{F}_t^X \sim N(\int_0^t \theta_u (\mu_u - \frac{1}{2}\sigma_u^2) du, \int_0^t \theta_u^2 \sigma_u^2 du)$ under $\mathcal{P}$. Then,

$$E[e^{(\theta \cdot Y)_T} | \mathcal{F}_T^X] = \exp\Big[\int_0^T \theta_t \Big(\mu_t - \frac{1}{2} \sigma_t^2 \Big) dt + \frac{1}{2} \int_0^T \theta_t^2 \sigma_t^2 dt \Big] \ . \tag{3.6}$$

Then,

$$E[e^{(\theta \cdot Y)_T} | X_0] = E\Big\{ \exp\Big[\int_0^T \theta_t \Big(\mu_t - \frac{1}{2} \sigma_t^2 \Big) dt + \frac{1}{2} \int_0^T \theta_t^2 \sigma_t^2 dt \Big] \Big\} \ . \tag{3.7}$$

Let J_i denote the occupation time of $\{X_t\}_{t\in\mathcal{T}}$ in state i, for each $i = 1, 2, \ldots, N$. Write $J := (J_1, J_2, \ldots, J_N) \in \mathcal{R}^N$.

The generalized moment generating functional $E[e^{(\theta\cdot Y)_T}|X_0]$ of the process Y with respect to θ under $\mathcal{P}$ can be written as:

$$E[e^{(\theta\cdot Y)_T}|X_0] = E\Big[\exp\Big(\sum_{i=1}^{N}\lambda_i(\theta_i)J_i\Big)\Big|X_0\Big] = E\Big[\exp(\langle\lambda(\theta), J\rangle)|X_0\Big] \tag{3.8}$$

Following the method in the proof of Proposition 2 in Elliott and Osakwe (2006) (see Pages 261-262 therein),

$$E[e^{(\theta\cdot Y)_T}|X_0] = \left\langle e^{(Q+diag(\lambda(\theta)))T}X_0, \mathbf{1}_N\right\rangle \ , \tag{3.9}$$

where $\mathbf{I}_N := (1, 1, \ldots, 1) \in \mathcal{R}^N$.

For each $i = 1, 2, \ldots, N$, write

$$(\theta\cdot Y)_{t,T} := \int_t^T \theta_u dY_u \ . \tag{3.10}$$

Let $J_i(t, T)$ denote the occupation time of $\{X_t\}_{t\in\mathcal{T}}$ in state i. Write

$$J(t, T) := (J_1(t, T), J_2(t, T), \ldots, J_N(t, T)) \ . \tag{3.11}$$

Then,

$$E[e^{(\theta\cdot Y)_{t,T}}|\mathcal{G}_t] = E[e^{\langle\lambda(\theta),J(t,T)\rangle}|\mathcal{G}_t] \ . \tag{3.12}$$

Following the method in Elliott and Osakwe (2006),

$$E[e^{(\theta\cdot Y)_{t,T}}|\mathcal{G}_t] = \left\langle e^{(Q+diag(\lambda(\theta)))(T-t)}X_t, \mathbf{1}_N\right\rangle \ . \tag{3.13}$$

Note that

$$E[\Lambda_T|\mathcal{G}_t] = \frac{e^{(\theta\cdot Y)_t}E[e^{(\theta\cdot Y)_{t,T}}|\mathcal{G}_t]}{E[e^{(\theta\cdot Y)_T}|X_0]} \tag{3.14}$$

Hence, the result follows.

□

By the fundamental theorem of asset pricing (see Harrison and Kreps (1979), Harrison and Pliska (1981, 1983) and Delbaen and Schachermayer (2004)), the absence of arbitrage opportunities is "essentially" equivalent to the existence of an equivalent martingale measure under which the discounted stock price process is a martingale. Let $\tilde{S}_t := e^{-\int_0^t r_u du} S_t$, for each $t \in \mathcal{T}$. Here, the martingale condition is given by considering an enlarged filtration as follows:

$$\tilde{S}_u = E^{\theta}[\tilde{S}_t | \mathcal{G}_u] \ , \quad \text{for any } t, u \in \mathcal{T} \text{ with } t \geq u \ , \tag{3.15}$$

where $E^{\theta}[\cdot]$ denotes an expectation with respect to $\mathcal{Q}_{\theta}$.

Proposition 3.2: Let

$$\tilde{\lambda}_i(\theta_i) := -r_i + (\theta_i + 1)\mu_i - \frac{1}{2}(\theta_i + 1)\sigma_i^2 + \frac{1}{2}(\theta_i + 1)^2 \sigma_i^2 \ , \ i = 1, 2, \ldots, N \ , \tag{3.16}$$

and $\tilde{\lambda}(\theta) := (\tilde{\lambda}_1(\theta_1), \tilde{\lambda}_2(\theta_2), \ldots, \tilde{\lambda}_N(\theta_N))$.

Then, the martingale condition is satisfied if and only if

$$\left\langle e^{(Q + diag(\tilde{\lambda}(\theta)))(t-u)} X_u, \mathbf{1}_N \right\rangle - \left\langle e^{(Q + diag(\lambda(\theta)))(t-u)} X_u, \mathbf{1}_N \right\rangle = 0 \ , \tag{3.17}$$

for all $X_u \in \mathcal{E}$ and for all $t, u \in \mathcal{T}$ with $t \geq u$.

Proof: By the Bayes' rule,

$$E^{\theta}[\tilde{S}_t | \mathcal{G}_u] = \frac{E[\Lambda_t \tilde{S}_t | \mathcal{G}_u]}{\Lambda_u} = \tilde{S}_u \frac{E[e^{-\int_u^t r_s ds} e^{((\theta+1)\cdot Y)_t} | \mathcal{G}_u]}{E[e^{(\theta \cdot Y)_t} | \mathcal{G}_u]} \tag{3.18}$$

The martingale condition is satisfied if and only if

$$\frac{E[e^{-\int_u^t r_s ds} e^{((\theta+1)\cdot Y)_t} | \mathcal{G}_u]}{E[e^{(\theta \cdot Y)_t} | \mathcal{G}_u]} = 1 \tag{3.19}$$

Note that

$$
\begin{aligned}
& E[e^{-\int_u^t r_s ds} e^{((\theta+1)\cdot Y)_t} | \mathcal{G}_u] \\
= \; & E\Big\{ \exp\Big[-\int_u^t r_s ds + \int_u^t (\theta_s + 1)\Big(\mu_s - \frac{1}{2}\sigma_s^2\Big) ds + \frac{1}{2}\int_u^t (\theta_s+1)^2 \sigma_s^2 ds \Big] \Big| \mathcal{G}_u \Big\} \\
= \; & E\Big[\exp\Big(\Big\langle \tilde{\lambda}(\theta), J(u,t) \Big\rangle \Big) \Big| \mathcal{G}_u \Big] \\
= \; & \Big\langle e^{(Q+diag(\tilde{\lambda}(\theta)))(t-u)} X_u, \mathbf{1}_N \Big\rangle \ ,
\end{aligned}
\tag{3.20}
$$

and that

$$
E[e^{(\theta\cdot Y)_t} | \mathcal{G}_u] = \langle e^{(Q+diag(\lambda(\theta)))(t-u)} X_u, \mathbf{1}_N \rangle \ . \tag{3.21}
$$

Hence, the result follows:

□

From Proposition 3.2, the risk-neutral Esscher parameters $(\theta_1, \theta_2, \ldots, \theta_N)$ satisfy the following system of N equations

$$
\begin{aligned}
& \Big\langle e^{(Q+diag(\tilde{\lambda}(\theta)))^T} e_i, \mathbf{1}_N \Big\rangle - \langle e^{(Q+diag(\lambda(\theta)))^T} e_i, \mathbf{1}_N \rangle = 0 \ , \\
& i = 1, 2, \ldots, N \ .
\end{aligned}
\tag{3.22}
$$

Note that for a $(N \times N)$ matrix M,

$$
\exp(M) = \sum_{n=0}^{\infty} \frac{M^n}{n!} \ , \tag{3.23}
$$

where $M^0 = \mathbf{I}$ (i.e. a $(N \times N)$ identity matrix) and $0! = 1$ by convention.

In practice, to solve the above system of N equations, we need to adopt a finite number of terms in the series expansion to $\exp(M)$ to approximate $\exp(M)$. We shall illustrate this and highlight some features of the solutions to the system of N equations through the following example in which we consider the case that there are two regimes and that $\exp(M)$ is approximated by the following second-order approximation:

$$\exp(M) \approx \mathbf{I} + M + \frac{1}{2!}M^2 \ . \tag{3.24}$$

Example 3.3: First, we suppose that $q_{11} = -q_{12} = -\eta$ and that $q_{21} = -q_{22} = \eta$. In this case, we need to solve the following pair of equations:

$$\begin{aligned} \left\langle e^{(Q+diag(\tilde{\lambda}(\theta)))T} e_1, \mathbf{1}_2 \right\rangle - \left\langle e^{(Q+diag(\lambda(\theta)))T} e_1, \mathbf{1}_2 \right\rangle = 0 \ , \\ \left\langle e^{(Q+diag(\tilde{\lambda}(\theta)))T} e_2, \mathbf{1}_2 \right\rangle - \left\langle e^{(Q+diag(\lambda(\theta)))T} e_2, \mathbf{1}_2 \right\rangle = 0 \ . \end{aligned} \tag{3.25}$$

We need to evaluate $\left\langle e^{(Q+diag(\tilde{\lambda}(\theta)))T} e_i, \mathbf{1}_2 \right\rangle$ and $\left\langle e^{(Q+diag(\lambda(\theta)))T} e_i, \mathbf{1}_2 \right\rangle$, for each $i = 1, 2$. By employing the approximation in (3.24),

$$\left\langle e^{(Q+diag(\tilde{\lambda}(\theta)))T} e_1, \mathbf{1}_2 \right\rangle \approx 1 + \tilde{\lambda}_1(\theta_1)T + \frac{1}{2}\left[\tilde{\lambda}_1(\theta_1)^2 - \tilde{\lambda}_1(\theta_1)\eta + \tilde{\lambda}_2(\theta_2)\eta\right]T^2 \ . \tag{3.26}$$

Then,

$$\begin{aligned} & \left\langle e^{(Q+diag(\tilde{\lambda}(\theta)))T} e_1, \mathbf{1}_2 \right\rangle - \left\langle e^{(Q+diag(\lambda(\theta)))T} e_1, \mathbf{1}_2 \right\rangle \\ \approx \ & A_1\theta_1^3 + A_2\theta_1^2 + A_3\theta_1 + A_4 + A_5\theta_2 \ , \end{aligned} \tag{3.27}$$

where

$$\begin{aligned} A_1 &= \frac{1}{2}\sigma_1^4 T^2 \ , \\ A_2 &= \frac{1}{2}(3\mu_1 - r_1)T^2\sigma_1^2 \\ A_3 &= \sigma_1^2\Big(T - \frac{1}{2}\eta T^2\Big) + \frac{1}{2}(\mu_1 - r_1)(\sigma_1^2 + 2\mu_1)T^2 \\ A_4 &= \frac{1}{2}(\mu_1 - r_1)T^2 + (\mu_1 - r_1)\Big(T - \frac{1}{2}\eta T^2\Big) + \frac{1}{2}(\mu_2 - r_2)\eta T^2 \\ A_5 &= \frac{1}{2}\sigma_2^2\eta T^2 \ . \end{aligned}$$

Similarly,

$$\begin{aligned} &\left\langle e^{(Q+diag(\tilde{\lambda}(\theta)))T} e_2, \mathbf{1}_2 \right\rangle - \left\langle e^{(Q+diag(\lambda(\theta)))T} e_2, \mathbf{1}_2 \right\rangle \\ \approx\ & B_1\theta_2^3 + B_2\theta_2^2 + B_3\theta_2 + B_4 + B_5\theta_1 \ , \end{aligned} \quad (3.28)$$

where

$$\begin{aligned} B_1 &= \frac{1}{2}\sigma_2^4 T^2 \ , \\ B_2 &= \frac{1}{2}(3\mu_2 - r_2)T^2\sigma_2^2 \\ B_3 &= \sigma_2^2\Big(T - \frac{1}{2}\eta T^2\Big) + \frac{1}{2}(\mu_2 - r_2)(\sigma_2^2 + 2\mu_2)T^2 \\ B_4 &= \frac{1}{2}(\mu_2 - r_2)T^2 + (\mu_2 - r_2)\Big(T - \frac{1}{2}\eta T^2\Big) + \frac{1}{2}(\mu_1 - r_1)\eta T^2 \\ B_5 &= \frac{1}{2}\sigma_1^2\eta T^2 \ . \end{aligned}$$

Hence, the pair of equations (3.25) can be approximated by the following pair of equations

$$\begin{aligned} A_1\theta_1^3 + A_2\theta_1^2 + A_3\theta_1 + A_4 + A_5\theta_2 = 0 \ , \\ B_1\theta_1^3 + B_2\theta_2^2 + B_3\theta_2 + B_4 + B_5\theta_1 = 0 \ . \end{aligned} \quad (3.29)$$

In general, there are more than one set of the risk-neutral Esscher parameters $(\theta_1, \theta_2) \in \Re^2$ satisfying a pair of cubic polynomials with two variables (3.29). The number of pairs of solutions of (3.29) depends on the values of the coefficients A_i, B_i ($i = 1, 2, 3, 4, 5$).

□

Example 3.4: Consider the rate matrix in Example 3.3. However, we adopt the following first-order approximation to $\exp(M)$. That is,

$$\exp(M) \approx \mathbf{I} + M \ . \quad (3.30)$$

In this case, for $i = 1, 2$,

$$\left\langle e^{(Q+diag(\tilde{\lambda}(\theta)))T} e_i, \mathbf{1}_2 \right\rangle \approx 1 + \tilde{\theta}_i(\theta_i)T \ . \tag{3.31}$$

Then, for $i = 1, 2$,

$$\begin{aligned} & \left\langle e^{(Q+diag(\tilde{\lambda}(\theta)))T} e_1, \mathbf{1}_2 \right\rangle - \left\langle e^{(Q+diag(\lambda(\theta)))T} e_1, \mathbf{1}_2 \right\rangle \\ \approx \ & (\tilde{\lambda}_i(\theta_i) - \lambda_i(\theta_i))T \\ = \ & (\mu_i - r_i + \sigma_i^2 \theta_i)T \ . \end{aligned} \tag{3.32}$$

Hence, the pair of equations (3.25) can then be approximated by the following pair of equations:

$$\mu_i - r_i + \sigma_i^2 \theta_i = 0 \ , \quad i = 1, 2 \ . \tag{3.33}$$

This implies that

$$\theta_i = \frac{r_i - \mu_i}{\sigma_i^2} \ . \tag{3.34}$$

This coincides with the pair of the risk-neutral Esscher parameters in Elliott, Chan and Siu (2005) in which the regime-switching risk is not priced.

□

Note that in general, there are more than one $(\theta_1, \theta_2, \ldots, \theta_N)$ satisfying equation (3.22). In the second stage of our pricing method, we select a set of risk-neutral Esscher parameter $(\theta_1, \theta_2, \ldots, \theta_N)$ satisfying the martingale restrictions (3.22) that minimizes the maximum entropy between an equivalent martingale measure and the real-world probability measure over different states. First, we define the entropy between $\mathcal{Q}_\theta$ and $\mathcal{P}$ conditional on $X_0 \in \mathcal{E}$ as follows:

$$\begin{aligned} I(\mathcal{Q}_\theta, \mathcal{P}|X_0) \ &:= \ E\Big[\frac{d\mathcal{Q}_\theta}{d\mathcal{P}} \ln\Big(\frac{d\mathcal{Q}_\theta}{d\mathcal{P}}\Big)\Big|X_0\Big] \\ &= \ \frac{E[(\theta \cdot Y)_T e^{(\theta \cdot Y)_T}|X_0]}{E[e^{(\theta \cdot Y)_T}|X_0]} - \ln E[e^{(\theta \cdot Y)_T}|X_0] \ . \end{aligned} \tag{3.35}$$

Note that

$$\lambda(z\theta_i) := z\theta_i\mu_i - \frac{1}{2}z\theta_i\sigma_i^2 + \frac{1}{2}z^2\theta_i^2\sigma_i^2 \ . \tag{3.36}$$

The derivative of $\lambda(z\theta_i)$ with respect to z is:

$$\lambda_z(z\theta_i) = \theta_i\mu_i - \frac{1}{2}\theta_i\sigma_i^2 + z\theta_i^2\sigma_i^2 \ . \tag{3.37}$$

When $z = 1$,

$$\lambda_z(\theta_i) = \theta_i\mu_i - \frac{1}{2}\theta_i\sigma_i^2 + \theta_i^2\sigma_i^2 \ . \tag{3.38}$$

Consider a function $M^{X_0}(z)$ on z defined as below:

$$\begin{aligned} M^{X_0}(z) &:= E[e^{z(\theta\cdot Y)_T}|X_0] \\ &= \left\langle e^{(Q+diag(\lambda(z\theta)))T}X_0, \mathbf{1}_N \right\rangle \ . \end{aligned} \tag{3.39}$$

Then, the derivative of $M^{X_0}(z)$ with respect to z is:

$$M_z^{X_0}(z) = \left\langle e^{(Q+diag(\lambda_z(z\theta)))T}X_0, \mathbf{1}_N \right\rangle \ . \tag{3.40}$$

When $z = 1$,

$$M_z^{X_0}(1) = \left\langle e^{(Q+diag(\lambda_z(\theta)))T}X_0, \mathbf{1}_N \right\rangle \ . \tag{3.41}$$

Hence,

$$\begin{aligned} E[(\theta\cdot Y)_T e^{(\theta\cdot Y)_T}|X_0] &= M_z^{X_0}(1) \\ &= \left\langle e^{(Q+diag(\lambda_z(\theta)))T}X_0, \mathbf{1}_N \right\rangle \ . \end{aligned} \tag{3.42}$$

This implies that

$$\begin{aligned} I(\mathcal{Q}_\theta, \mathcal{P}|X_0) &:= E\Big[\frac{d\mathcal{Q}_\theta}{d\mathcal{P}}\ln\Big(\frac{d\mathcal{Q}_\theta}{d\mathcal{P}}\Big)\Big|X_0\Big] \\ &= \frac{\left\langle e^{(Q+diag(\lambda_z(\theta)))T}X_0, \mathbf{1}_N \right\rangle}{\left\langle e^{(Q+diag(\lambda(\theta)))T}X_0, \mathbf{1}_N \right\rangle} - \ln\left\langle e^{(Q+diag(\lambda(\theta)))T}X_0, \mathbf{1}_N \right\rangle \ . \end{aligned} \tag{3.43}$$

Define

$$\Theta := \{\theta \in \Re^N | \theta \text{ satisfies (3.22)}\} \ .$$

Let $I(\mathcal{Q}_\theta, \mathcal{P})$ denote the maximum entropy between $\mathcal{Q}_\theta$ and $\mathcal{P}$ over different values of X_0. That is,

$$I(\mathcal{Q}_\theta, \mathcal{P}) := \max_{i=1,2,\ldots,N} I(\mathcal{Q}_\theta, \mathcal{P} | X_0 = e_i) \ . \tag{3.44}$$

Then, we select a set of the risk-neutral Esscher parameters $\theta := (\theta_1, \theta_2, \ldots, \theta_N) \in \Theta$ such that $I(\mathcal{Q}_\theta, \mathcal{P})$ is minimized.

Note that the elements in Θ are determined by approximating $\exp(M)$ with a polynomial with a finite order. For example, a second-order polynomial is used to approximate $\exp(M)$ in Example 3.3. In this case, Θ contains a finite number of elements, say $\Theta := (\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(N)})$. Then, the risk-neutral Esscher parameter can be determined by solving the following minimization problem:

$$\min_{j=1,2,\ldots,N} I(\mathcal{Q}_{\theta^{(j)}}, \mathcal{P}) \ . \tag{3.45}$$

The maximum entropy $I(\mathcal{Q}_\theta, \mathcal{P})$ also involves the term $\exp(M)$. To evaluate $I(\mathcal{Q}_\theta, \mathcal{P})$, we can also approximate $\exp(M)$ by a polynomial with a finite order. In order to be consistent, the order of polynomial for approximating $\exp(M)$ in the evaluation of $I(\mathcal{Q}_\theta, \mathcal{P})$ can be chosen to be the same as that in solving the martingale condition.

Now, we consider a European option with payoff $V(S_T)$ at maturity T. Given $\mathcal{G}_t$, the conditional price of the option is:

$$V_t := E^\theta\Big[\exp\Big(-\int_t^T r_u du\Big) V(S_T) \Big| \mathcal{G}_t\Big] \ . \tag{3.46}$$

Note that (S, X) is a two-dimensional Markov process with respect to $\mathcal{G}$. Then, given $S_t = s$ and $X_t = x$, the price of the option at time t is:

$$V(t, s, x) = E^{\theta}\Big[\exp\Big(-\int_t^T r_u du\Big)V(S_T)\Big|S_t = s, X_t = x\Big] \ . \qquad (3.47)$$

The price can be computed by Monte Carlo simulation. We shall discuss this in some detail in the next section.

§4. Numerical Experiments

In this section, we shall conduct numerical experiments to illustrate the effect of pricing regime-switching risk. We shall consider two models, namely, Model I and Model II. Model I refers to the pricing model in which the regime-switching risk is priced. We assume that in Model I, the term $\exp(M)$ is approximated by the second-order approximation in (3.24) as in Example 3.3. Model II refers to the pricing model in which the regime-switching risk is not priced. We can see from Example 3.4 that this corresponds to the case that $\exp(M)$ is approximated by a first-order approximation in (3.30). We shall compare the prices of standard European-style call options obtained from Model I and those from Model II. The option prices from Model I and Model II are computed by Monte Carlo simulation and the simulation procedure is described in the sequel.

Suppose we wish to evaluate the price of a standard European call option at the current time $t = 0$ with maturity T and strike price K. First, we note that the call option $C(0, S_0, X_0)$ can be evaluated as follows:

$$\begin{aligned} C(0, X_0, S_0) &= E^{\theta}\Big[\exp\Big(-\int_0^t r_u du\Big)(S_T - K)^+\Big|S_0, X_0\Big] \\ &= E\Big[\frac{d\mathcal{Q}_{\theta}}{d\mathcal{P}}\exp\Big(-\int_0^t r_u du\Big)(S_T - K)^+\Big|S_0, X_0\Big] \\ &= \frac{E[e^{(\theta\cdot Y)_T}e^{-\int_0^T r_u du}(S_T - K)^+|S_0, X_0]}{E[e^{(\theta\cdot Y)_T}|X_0]} \ . \end{aligned} \qquad (4.1)$$

The key idea of the simulation procedure is to first approximate $E[e^{(\theta\cdot Y)_T}e^{-\int_0^T r_u du}(S_T-K)^+|S_0,X_0]$ and $E[e^{(\theta\cdot Y)_T}|X_0=x]$ separately by Monte Carlo simulation and then use the ratios of the two approximations to approximate $C(0,X_0,S_0)$.

Since the Markov chain X and the log return process Y are continuous-time processes, we need to consider their discrete versions for the simulation. We divide the time horizon $[0,T]$ into N subintervals $[t_j,t_{j+1}]$ ($j=0,1,\ldots,J-1$) of equal length $\Delta=\frac{T}{J}$, where $t_0=0$ and $t_J=T$. For the discrete-time version of the Markov chain X, we suppose that the transition probability matrix in a subinterval is $\mathbf{I}+Q\Delta$ given X_0. Given the simulated path of X, the sample paths of the processes $\{\mu_{t_j}\}_{j=1}^J$, $\{\sigma_{t_j}\}_{j=1}^J$, $\{\theta_{t_j}\}_{j=1}^J$ and $\{r_{t_j}\}_{j=1}^J$ are identified. Then, we adopt the Euler foward discretization scheme to discrete the process Y as follows:

$$Y_{t_{j+1}}=Y_{t_j}+\Big(\mu_{t_j}-\frac{1}{2}\sigma_{t_j}^2\Big)\Delta+\sigma_{t_j}\xi_{t_{j+1}}\ , \tag{4.2}$$

where $\{\xi_{t_{j+1}}\}_{j=0,1,\ldots,J-1}$ and $\xi_{t_{j+1}}\sim N(0,\Delta)$.

Given $\{X_{t_j}\}_{j=1}^J$ and $Y_0=0$, we then sample $\{Y_{t_j}\}_{j=1}^J$ using (4.2) recursively.

The simulation procedure is summarized as follows:

Step I: For each $l=1,2,\ldots,L$, simulate the discrete-time version of the Markov chain X and obtain $\{X_{t_j}^{(l)}\}_{j=1}^J$

Step II: Given $\{X_{t_j}^{(l)}\}_{j=1}^J$, identify the sample paths of the processes $\{\mu_{t_j}^{(l)}\}_{j=1}^J$, $\{\sigma_{t_j}^{(l)}\}_{j=1}^J$, $\{\theta_{t_j}^{(l)}\}_{j=1}^J$ and $\{r_{t_j}^{(l)}\}_{j=1}^J$, for each $l=1,2,\ldots,L$

Step III: For each $l=1,2,\ldots,L$, simulate the discrete-time version of the log return process Y and obtain $\{Y_{t_j}^{(l)}\}_{j=1}^J$

Step IV: Approximate the call price by:

$$C(0,S_0,X_0)\ \approx\ \frac{\sum_{l=1}^L e^{\sum_{j=1}^J\theta_{t_j}^{(l)}(Y_{t_j}^{(l)}-Y_{t_{j-1}}^{(l)})}e^{-\sum_{j=1}^J r_{t_j}^{(l)}}(S_0e^{Y_T^{(l)}}-K)^+}{\sum_{l=1}^L e^{\sum_{j=1}^J\theta_{t_j}^{(l)}(Y_{t_j}^{(l)}-Y_{t_{j-1}}^{(l)})}}. \tag{4.3}$$

Note that the above simulation procedure is applicable for both Model I and Model II. The main difference between Model I and Model II is on the set of risk-neutral Esscher parameters.

Now, we shall consider some specimen values for the parameters in both Model I and Model II. Here, we suppose that there are two states in the economy $(N = 2)$. State 1 represents a "Good" economy while State 2 represents a "Bad" economy. The transition probabilities of the two-state Markov chain are $q_{11} = -q_{12} = -\eta$ and $q_{21} = -q_{22} = \eta$. Then, we suppose that the model parameters are given by:

$$\begin{aligned} r_1 &= 0.05 \ ; \\ r_2 &= 0.01 \ ; \\ (\mu_1, \sigma_1) &= (0.35, 0.1) \ ; \\ (\mu_2, \sigma_2) &= (0.05, 0.2) \ ; \\ \eta &= 0.5 \ . \end{aligned}$$

Before presenting the pricing results, we consider an example to illustrate the implementation of the two-stage pricing method.

Example 4.1: Consider the above specimen values of the model parameters and assume that $T = 0.5$ (i.e. a half-year option). Here, we consider Model II. In this case, we obtain the following two equations from the martingale condition:

$$\begin{aligned} 0.0000125\theta_1^3 + 0.00125\theta_1^2 + 0.031\theta_1 + 0.145 + 0.000625\theta_2 &= 0 \\ 0.0002\theta_2^3 + 0.0007\theta_2^2 + 0.0182\theta_2 + 0.03645 + 0.0025\theta_1 &= 0 \qquad (4.4) \end{aligned}$$

Figure 1 presents the graphs of the above system of polynomials.

[Figures 1 about here]

From Figure 1, we see that there are three possible pairs of solutions. These pairs of solutions and their corresponding maximum entropies are presented as follows:

$$
\begin{array}{ll}
(\hat{\theta}_1, \hat{\theta}_2) & I(\mathcal{Q}_{\hat{\theta}}, \mathcal{P}) \\
(-6.03, -1.21) & 1.48 \\
(-29.64, 1.86) & 3.22 \\
(-64.29, 4.77) & 0.43
\end{array}
\tag{4.5}
$$

We then pick the pair of solution $(\hat{\theta}_1, \hat{\theta}_2) = (-64.29, 4.77)$ since it minimizes the maximum entropy.

To visualize the effect of pricing regime-switching risk on the risk-neutral Esscher parameters over time, we plot $\hat{\theta}_1$ and $\hat{\theta}_2$ against time for both Model I and Model II in Figure 2.

[Figure 2 about here]

From Figure 2, we see that the impact of pricing regime-switching risk on the risk-neutral Esscher parameters is significant.

□

Then, we consider the computation of the call prices. We assume that $X_0 = e_1$, $S_0 = 100$, $J = 100$ and $M = 50,000$ Figure 3 displays the plots of the call price vensus the strike price K for different maturities.

[Figure 3 about here]

From Figure 3, we see that the impact of pricing regime-switching risk on the call prices is significant. In particular, the call prices from Model I are substantially lower than those from Model II for in-the-money options and nearly-at-the-money options when the maturities are 3 months, 6 months and 9 months. For longer maturity options (i.e. the maturity is 12 months), the call prices from Model I are higher than those from Model II for different values of the strike price.

Figure 4 presents the plot of the call price against the time to maturity.

[Figure 4 about here]

From Figure 4, we see that the effect of pricing regime-switching risk on the qualitative behavior of the call price against the maturity is significant for in-the-money options and at-the-money options.

Figure 5 displays the three-dimensional plot of the call price against the strike price and the time to maturity.

[Figure 5 about here]

§4. Conclusion

We developed a two-stage pricing model for valuing an option when the price dynamics of the underlying asset is governed by a Markov-modulated geometric Brownian motion. The key feature of the model is that we are able to price both the diffusion risk and the regime-switching risk. The two-stage procedure involves the use of the Esscher transform to determine the martingale condition in the first stage and a min-max entropy problem in the second state. We conducted numerical experiments to investigate the effect of pricing regime-switching risk. We found that the impact of pricing regime-switching risk on the option prices is significant.

References

1) Avellaneda, M. 1998. “Minimum Relative-Entropy Calibration of Asset-pricing Models,” *International Journal of Theoretical and Applied Finance*, 1: 447-472.
2) Black, F. and Scholes, M. 1973. “The Pricing of Options and Corporate Liabilities,” *Journal*

of Political Economy 81: 637-659.

3) Buchen, P.W. and Kelly, M. 1996. “The Maximum Entropy Distribution of an Asset Inferred from Option Prices,” *Journal of Financial and Quantitative Analysis*, 31:143-159.

4) Buffington, J. and Elliott, R.J. 2002a. “Regime Switching and European Options,” *In Stochastic Theory and Control, Proceedings of a Workshop, Lawrence, K.S.*, October, Springer Verlag, 73-81.

5) Buffington, J. and Elliott, R.J. 2002b. “American options with regime switching,” *International Journal of Theoretical and Applied Finance* 5: 497-514.

6) Bühlmann, H., Delbaen, F., Embrechts, P. and Shiryaev, A.N. 1996. “No-Arbitrage, Change of Measure and Conditional Esscher Transforms,” *CWI Quarterly* 9(4): 291-317.

7) Delbaen, F. and Schachermayer, W. 2004. “What is a Free Lunch?,” *Notices of the American Mathematical Society*, 51(5), 526-528.

8) Elliott, R.J. 1982. *Stochastic Calculus and Applications*. Springer: Berlin-Heidelberg-New York.

9) Elliott. R.J. 1993. “New Finite Dimensional Filters and Smoothers for Noisily Observed Markov Chains,” *I.E.E.E. Transactions on Information Theory* 39: 265-271.

10) Elliott, R.J., Aggoun, L. and Moore, J.B. 1994. *Hidden Markov Models: Estimation and Control*. Springer-Verlag: Berlin-Heidelberg-New York.

11) Elliott, R.J. and J. van der Hoek. 1997. “An application of hidden Markov models to asset allocation problems,” *Finance and Stochastics* 3: 229-238.

12) Elliott, R.J., W.C. Hunter and B.M. Jamieson. 2001. ”Financial signal processing,” *International Journal of Theoretical and Applied Finance* 4: 567-584.

13) Elliott, R.J. and J. Hinz. 2002. “Portfolio analysis, hidden Markov models and chart analysis

by PF-diagrams," *International Journal of Theoretical and Applied Finance* 5 :385-399.

14) Elliott, R.J., W.P. Malcolm and A.H. Tsoi. 2003. "Robust parameter estimation for asset price models with Markov modulated volatilities," *Journal of Economics Dynamics and Control* 27(8): 1391-1409.

15) Elliott, R.J., Chan, L. and Siu, T.K. 2005. "Option Pricing and Esscher Transform Under Regime Switching," *Annals of Finance* 1(4): 423-432.

16) Elliott, R.J. and Osakwe, C-J.U. 2006. "Option Pricing for Pure Jump Processes with Markov Switching Compensators", *Finance and Stochastics* 10(2): 250 - 275.

17) Frittelli, M. 2000. "The Minimal Entropy Martingale Measure and the Valuation Problem in Incomplete Markets," *Mathematical Finance* 10(1): 39-52.

18) Gerber, H.U. and Shiu, E.S.W. 1994. "Option Pricing by Esscher Transforms (with discussions)," *Transactions of the Society of Actuaries* 46: 99-191.

19) Guo, X. 2001. "Information and Option Pricings," *Quantitative Finance* 1: 38-44.

20) Hamilton, J. D. 1989. "A new approach to the economic analysis of nonstationary time series and the business cycle," *Econometrica* 57: 357–384.

21) Harrison, J. M. and Kreps, D.M. 1979. "Martingales and Arbitrage in Multiperiod Securities Markets," *Journal of Economic Theory* 20: 381-408.

22) Harrison, J.M., and Pliska, S.R. 1981. "Martingales and Stochastic Integrals in the Theory of Continuous Trading," *Stochastic Processes and Their Applications* 11: 215-280.

23) Harrison, J.M., and Pliska, S.R. 1983. "A Stochastic Calculus Model of Continuous Trading: Complete Markets," *Stochastic Processes and Their Applications* 15: 313-316.

24) Hull, J. and White, A. 1987. "The Pricing of Options on Assets with Stochastic Volatility," *Journal of Finance* 42: 281-330.

25) McLeish, D.L., and Reesor, R.M. 2003. “Risk, Entropy, and the Transformation of Distributions,” *North American Actuarial Journal* 7(2): 128-144.

26) Merton, R.C. 1973. “The Theory of Rational Option Pricing,” *Bell Journal of Economics and Management Science* 4: 141-183.

27) Merton, R.C. 1976. “Option Pricing When Underlying Stock Returns are Discontinuous,” *Journal of Financial Economics* 3: 125-144.

28) Miyahara, Y. 2001. “Geometric Lévy Process and MEMM: Pricing Model and Related Estimation Problems,” *Asia-Pacific Financial Markets* 8: 45-60.

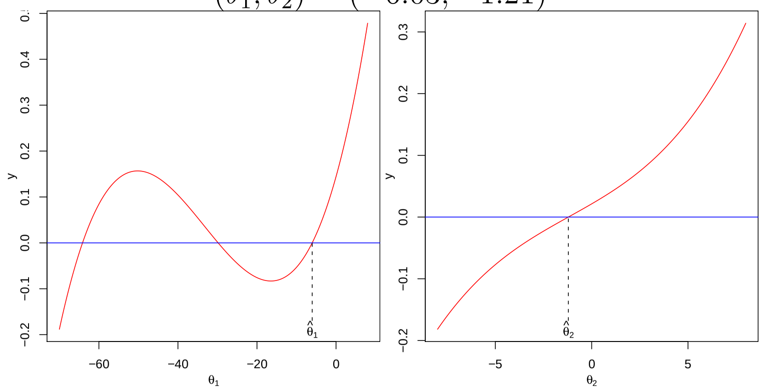
29) Naik, V. 1993. “Option Valuation and Hedging Strategies With Jumps in the Volatility of Asset Returns,” *Journal of Finance* 48: 19691984.

30) Pliska, S. 1997. *Introduction to Mathematical Finance: Discrete Time Models*. Blackwell Publishers: United States.

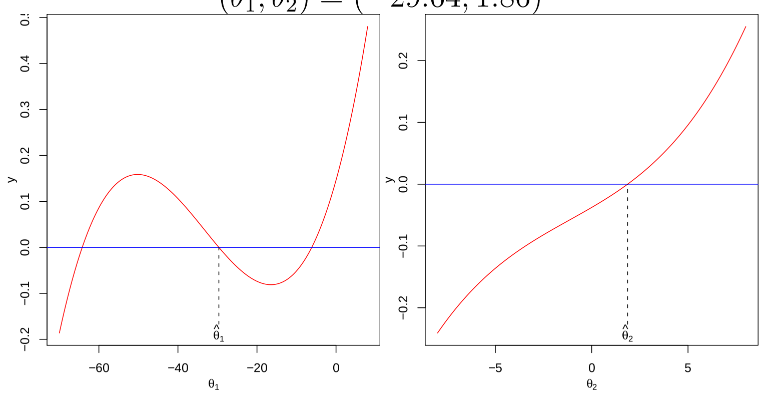
31) Wiggins J. 1987. “Options Values Under Stochastic Volatility: Theory and Empirical Evidence,” *Journal of Financial Economics* 19: 351-372.

Figure 1

$(\hat{\theta}_1, \hat{\theta}_2) = (-6.03, -1.21)$



$(\hat{\theta}_1, \hat{\theta}_2) = (-29.64, 1.86)$



$(\hat{\theta}_1, \hat{\theta}_2) = (-64.29, 4.77)$

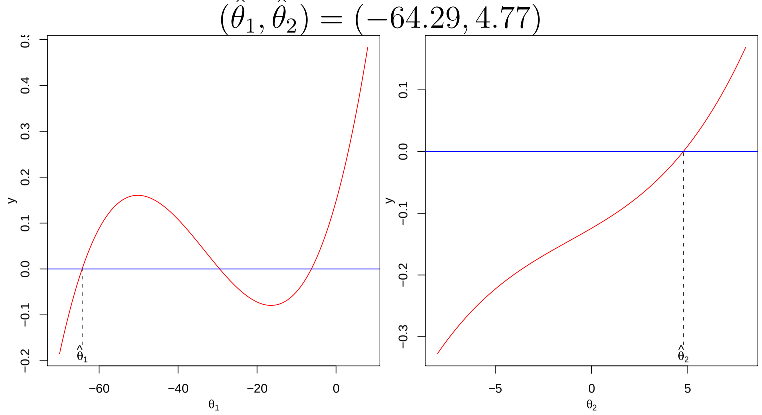


Figure 2

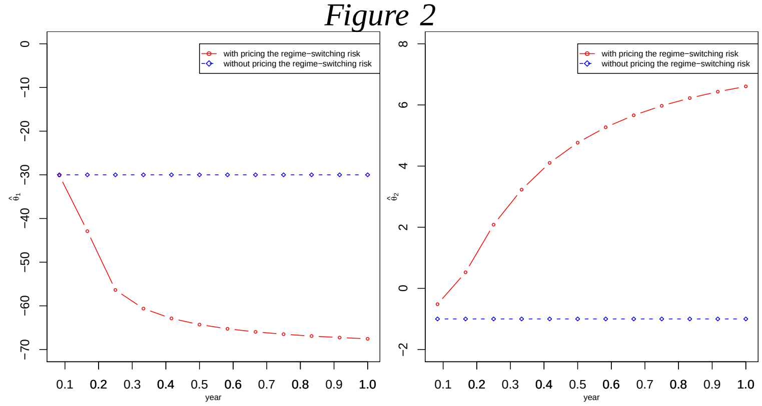

with pricing the regime−switching risk
without pricing the regime−switching risk
year
year
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
0 −10 −20 −30 −40 −50 −60 −70
8 6 4 2 0 −2

Figure 3: Call price vs Strike Price

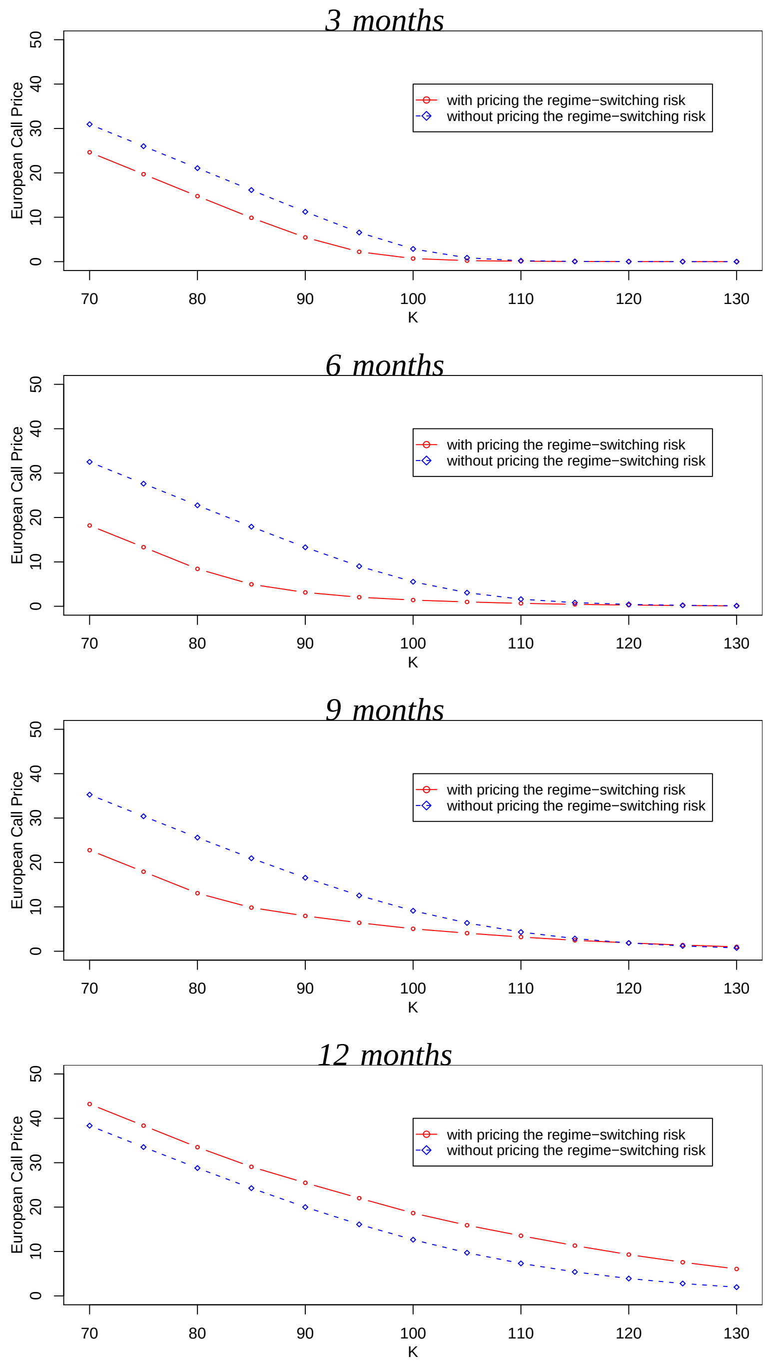


Figure 4: Call price vs Time to Maturity

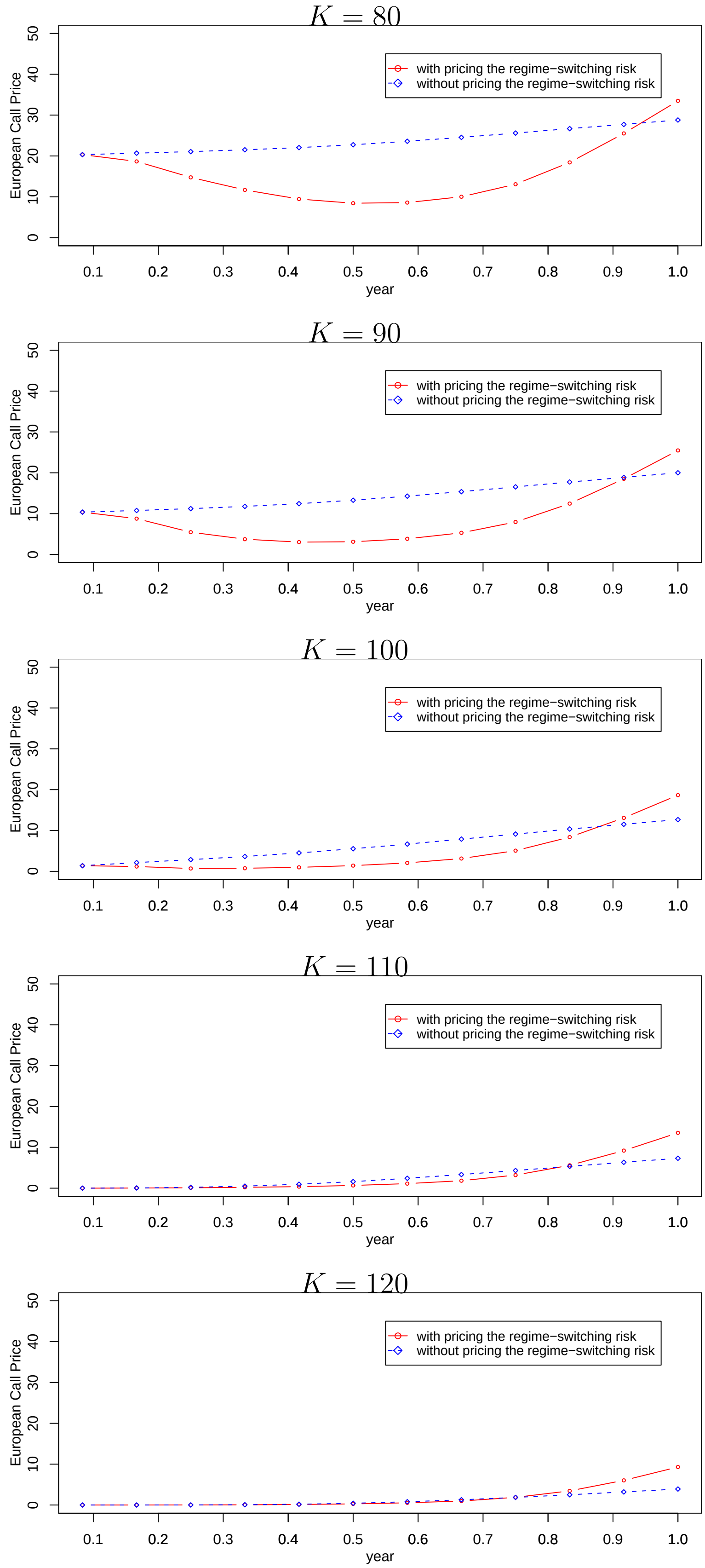
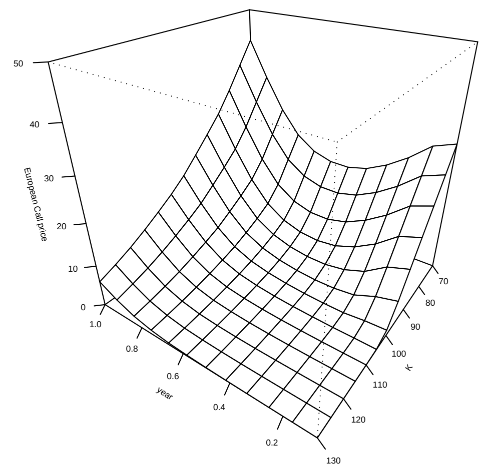


Figure 5: Call price

with pricing the regime-switching risk



without pricing the regime-switching risk