# THE HIDDEN SURPLUS FROM RESEARCH JOINT VENTURES

## AN APPLICATION OF SYSTEMS RELIABILITY THEORY

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ABSTRACT

The paper's aim is two-fold: (a) to model some key features of the research process as a multi-component system, as understood by the mathematical theory of systems reliability; and (b) to apply the resulting model to Research Joint Ventures, showing that a potentially very large surplus can be realized purely by organizing research efficiently, i.e. even without any changes in R&D investment.

Keywords: optimal organization, systems reliability, Research Joint Ventures, parallel and series systems, majorization, organizational surplus.
JEL Codes: O32

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## INTRODUCTION

Consider two key features of the research process, namely, (i) that the successful completion of almost any research project involves the overcoming of more than one obstacle, and (ii) that typically there is more than one way to overcome any given research obstacle. This paper argues that there exists a well-developed theoretical apparatus ideally suited to analyze these two key features of research, namely *the theory of systems reliability* as pioneered by John von Neumann.[1]

To show the fruitfulness of this new approach, the paper addresses the problem of the optimal organization of Research Joint Ventures and highlights the existence of a hitherto neglected but potentially large *organizational surplus*, i.e., an unambiguous Pareto improvement that can be achieved with no change whatsoever in the R&D investment of the firms forming an RJV.

It should be understood from the outset that the technical results in the paper are well known in specialized fields of statistics, the main contribution of the paper being instead the novel application of this powerful tool to the organization of research in general and of RJVs in particular.

The paper is organized as follows: in Section 1, I argue that the theory of reliability of multi-component systems provides both the language and the techniques to model two key features of the process of research. In Section 2, I provide a very simple, but powerful application of the notion of research-as-a-reliable-system to the issue of the optimal organization of Research Joint Ventures and I show that a research-efficient RJV requires a radical re-structuring of R&D activities and not, as implicitly assumed in some of the literature, the mere coupling of previously separated R&D processes. The surprising result is that, even without any changes in the amount of R&D investment undertaken by RJV members, an optimally organized RJV generates a potentially large surplus. Section 3 concludes and hints at other applications of this analytical approach to

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[1] The seminal paper in this area is "Probabilistic logics" where John von Neumann (1956) showed how to combine unreliable devices (the so-called "Sheffer stroke") so that they can function as a system of higher reliability.

other fields in economic theory and industrial organization. All technical details are relegated to the Appendix.

## 1. RESEARCH AS A MULTI-COMPONENT SYSTEM

Suppose that each dot in Figure 1 represents a "switch", i.e., the most elementary example of a device that can either work or fail. In this particular instance, the five switches are connected in such a way that the whole structure can function (i.e., a connection from I to K can be established) even if not all five switches work. Indeed there are four different ways in which this structure can perform, namely , when any of the four "routes" $\{x_1,x_2\}$, $\{x_4,x_5\}$, $\{x_1,x_3,x_5\}$, $\{x_4,x_3,x_2\}$ is working.

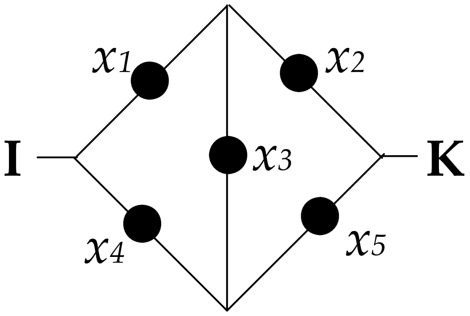

FIGURE 1

The very same structure can be interpreted in terms of a research problem, whereby in order to achieve a "discovery", i.e., to move from the state of Ignorance to the state of Knowledge, there is essentially a pair of two-obstacle search routes (namely $\{x_1,x_2\}$ and $\{x_4,x_5\}$) with an additional "bridging" route $(x_3)$.[2]

[2] To mention an example familiar to the reader, suppose that the aim is to write a publishable paper in mathematical economics where the first "obstacle" may be proving an existence theorem (which can be established either by topological means $(x_1)$ or constructively $(x_4)$ and the second obstacle may be proving a uniqueness theorem (which, again, can be done topologically $(x_2)$ or constructively$(x_5)$, with $(x_3)$ being a way of translating a topological story into a constructive argument.

The basic notion underlying this paper is that there exists a very close relationship between the way the process of research is structured and the way in which a multi-component system can be organized. With reference to the research problem described in Figure 1, each "dot" represents a "research unit" (laboratory, R&D personnel, etc.) assigned to a specific task. The structure of the problem to be solved is fully captured by the way in which the research units are connected and can be described as follows: there are two substantive research obstacles to be overcome, each of which can be tackled in two different ways. One obstacle can be surmounted by the successful accomplishment of either task $x_1$ or task $x_4$, while the removal of the other obstacle requires the completion of either task $x_2$ or task $x_5$, with task $x_3$ acting as a "translator", i.e. making the outcome of task $x_4$ (respectively, $x_1$) available to task $x_2$ (respectively, $x_5$).

This is hardly surprising: anyone engaged in research is only too painfully aware that typically the attainment of any research goal involves the overcoming of more than a single obstacle and that usually there is more than one way of tackling any given obstacle.

As this is, to the best of my knowledge, the first attempt to model research as a multi-component structure, I may be forgiven for concentrating on this aspect of the innovation process, happily ignoring other important features of research, e.g., its sequential nature[3]. This implies that the *order* in which components function (or fail) is immaterial. In order words, the results examined in the paper apply to any sort of *modular* research problem.

## 2. RELIABILITY AND THE OPTIMAL ORGANIZATION OF RESEARCH JOINT VENTURES

### 2.1 The simplest model of RJV organization

In order to see how the analytical apparatus of reliability theory can be brought to bear on the economics of the organization of R&D, I shall consider the simplest multi-component structure: a two-component series structure. This says that in order to complete the innovation process, two obstacles must be overcome. For

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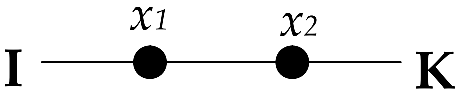
[3] As modelled, for example, in Weitzman (1979) and Weitzman and Roberts (1981).

example, one obstacle may be the development of a supersonic jet engine and the other the construction of an airframe capable of sustaining it.



$$\phi(\boldsymbol{x}) = x_1 x_2$$

FIGURE 2

In order to capture the inherent uncertainty of the research process, we can assume that success in each stage is stochastic. More specifically, we assume that success at stage *i* happens with probability $p_i$ and that successes are independent. In line with the statistical literature, we call the probability that the R&D unit *i* succeeds in task *i* as its "reliability". For the purpose of our model, let the reliability of each unit $p_i$ be either "high" or "low", i.e.; $1 \geq p_H > p_L > 0$, so that the reliability function (i.e., the probability that the whole project is completed successfully) is given by $h(\mathbf{p}) = p_j p_t,\ j,t = L,H$. Normalize the cost of a low-reliability component to unity, with $w$ ($>1$) being the cost of the high-reliability component.

Consider the following two-firm game: each firm chooses its R&D structure by assigning either a low- or a high-reliability unit to each of the two research tasks that have to be completed in order for the research project to e successful. It collects a reward equal to zero if the whole project fails, a reward $\pi_M$ if it alone succeeds, and a reward $\pi_D\ (< \frac{\pi_M}{2})$ if the other firm, too, is successful in completing the whole project, i.e.:

$$(1) \qquad \pi_i(h_i, h_j) = \begin{cases} 0 & \text{if} \quad h_i = 0 \\ \pi_M & \text{if} \quad h_i = 1 \quad \text{and} \quad h_j = 0 \\ \pi_D & \text{if} \quad h_i = 1 \quad \text{and} \quad h_j = 1 \end{cases}$$

Obviously, as each firm can opt for either a reliable (*HH*), or an intermediate (*HL*), or an unreliable (*LL*) structure, the payoff matrix for this game is as described in Table 1.

TABLE 1

| | | **FIRM 2** | | |
|---|---|---|---|---|
| | | *LL* | *HL* | *HH* |
| **FIRM 1** | *LL* | $p_L^2\pi_{LL}-2$,<br>$p_L^2\pi_{LL}-2$ | $p_L^2\pi_{HL}-2$,<br>$p_Hp_L\pi_{LL}-w-1$ | $p_L^2\pi_{HH}-2$,<br>$p_H^2\pi_{LL}-2w$ |
| | *HL* | $p_Hp_L\pi_{LL}-w-1$,<br>$p_L^2\pi_{HL}-2$ | $p_Hp_L\pi_{HL}-w-1$,<br>$p_Hp_L\pi_{HL}-w-1$ | $p_Hp_L\pi_{HH}-w-1$,<br>$p_H^2\pi_{HL}-2w$ |
| | *HH* | $p_H^2\pi_{LL}-2w$,<br>$p_L^2\pi_{HH}-2$ | $p_H^2\pi_{HL}-2w$,<br>$p_Hp_L\pi_{HH}-w-1$ | $p_H^2\pi_{HH}-2w$,<br>$p_H^2\pi_{HH}-2w$ |

where $\pi_{IJ}$ is the expected gross profits accruing to a *successful* firm whose *rival* has adopted a $p_Ip_J$ structure. $\pi_{IJ}$ is easily computed as $[\pi_M(1-p_Ip_J)+\pi_Dp_Ip_J]$.

The Nash equilibria in reliability structures of the two-stage game described in Table 1 obviously depend on the relationship between the additional cost of the high-quality component, $w-1$, and the expected increase in profits from using it, and specifically :

(1i) if $w-1>\dfrac{p_H^2-p_L^2}{2}\pi_{LL}$,

then the unique equilibrium is *(LL, LL);*

(1ii) if $w-1<\dfrac{p_H^2-p_L^2}{2}\pi_{HH}$

then the unique equilibrium is *(HH, HH);*

(1iii) if $\dfrac{p_H^2-p_L^2}{2}\pi_{LL}>w-1>\dfrac{p_H^2-p_L^2}{2}\pi_{HH}$,

then there are two asymmetric equilibria, *(HH, LL)* and *(LL, HH).*

It is interesting to note that an intermediate-reliability strategy is always dominated.

The main results of this paper apply to all possible equilibria. However, in order to get a better insight into the problem of which type of research units should be allocated to which research task, in what follows I shall assume that (1iii) holds, so that at a Nash equilibrium one firm pursues a high-reliability strategy and the other a low-reliability one. As I am interested in the relationship between multi-component R&D structures and research joint ventures, I shall ignore the equilibrium selection problem and simply assume that for some exogenous reason (e.g., historical accident) one firm will pursue the high-reliability strategy. This means that at an aggregate level there are two high- and two low-reliability units available to tackle a two-obstacle research project.

## 2.2 Efficient R&D arrangements

Now I can address the question of what is the most efficient arrangement of R&D resources when successful discovery requires the overcoming of two "obstacles" and given that the endowment of R&D resources comprises two high- and two low-reliability components.

I shall also point out how the answers provided generalize to $n$-component structures with $m$-obstacle research routes. This section relies on the notion of Schur-convexity and the techniques of majorization, which are very briefly summarized in the Appendix.[4]

The four possible arrangements that a four-component Research Joint Venture can take are best examined in pairs, beginning with:

### 2.2.1 Parallel-series arrangements

Consider the two following parallel-series arrangements:

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[4] For a full treatment, see the splendid book by W. Marshall and Ingram Olkin (1979).

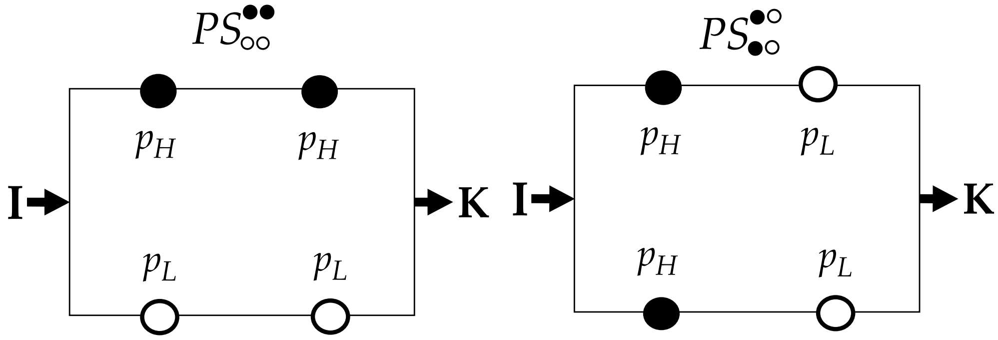

FIGURE 3

It is very simple to establish which parallel-series arrangement is more reliable:

$$
\begin{aligned}
h(PS^{\bullet\bullet}_{\circ\circ}) &\equiv p_H^2 + p_L^2 - p_H^2 p_L^2 > \\
&\quad 2p_H p_L - p_H^2 p_L^2 \equiv h(PS^{\bullet\circ}_{\bullet\circ})
\end{aligned}
\tag{2}
$$

This result generalizes to any parallel-series arrangement made up of *n* independent components:

> **THEOREM 1:** *The reliability of a parallel-series system is maximized when components ranked by reliability are allocated to ranked minimum path sets starting with the shortest.*

The proof of this remarkable theorem (first established in El-Neweihi *et al.* (1986)) provides a vivid example of the power of the techniques of majorization applied to reliability theory and is relegated to the Appendix. Notice that not all paths need have the same length (as in our simple example).

Theorem 1 states that when the route to discovery involves several obstacles and R&D is carried out by "teams" arranged in parallel, then all the "best" resources should be allocated to the most promising (i.e., shortest) research path, relegating the "worst" resources to the least promising.[5] Or, to put it figuratively, you should put all your (good) eggs in one basket.[6] This is an important result for any type of organization where competing activities are carried out by teams. It is especially significant in the context of the efficient organization of R&D in so far as it lends strong theoretical support to the popular policy of creating "centres of excellence".

I now turn to the remaining pair of possible RJV arrangements, namely:

### 2.2.2 Series-parallel arrangements

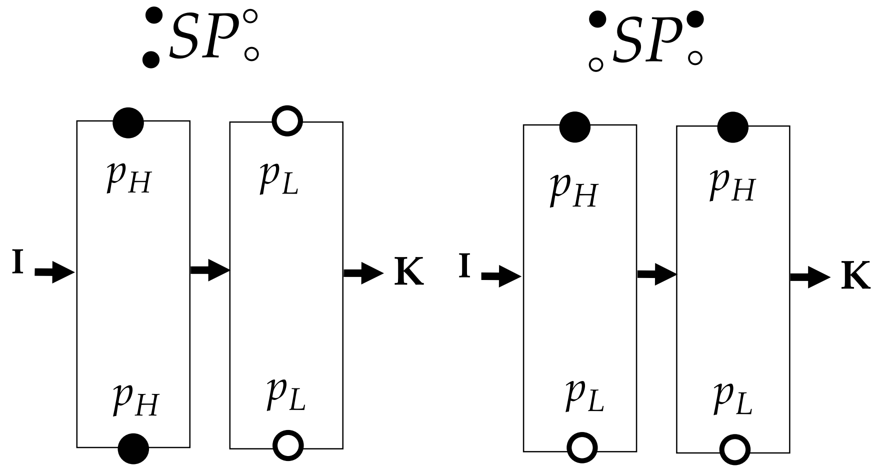

FIGURE 4

Again, it is trivial to establish that ${}^{\bullet}_{\circ}SP^{\bullet}_{\circ}$ is the more reliable arrangement:

$$h({}^{\bullet}_{\bullet}SP^{\circ}_{\circ}) \equiv (2p_H - p_H^2)(2p_L - p_L^2) < (p_H + p_L - p_H p_L)^2 \equiv h({}^{\bullet}_{\circ}SP^{\bullet}_{\circ}) \tag{3}$$

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[5] The Research Assessment Exercise (the ambitious attempt to rank every department in every University in the UK) is implicitly aimed at concentrating resources in higher-ranked departments and hence is implicitly based on the assumption that University research is a parallel-series process (see La Manna (2001) for a "reliability-based" analysis of the RAE).

[6] I owe this analogy to John Sutton.

This result (that generalizes to any $n$-component series-parallel system, see Appendix) is the dual to Theorem 1 and implies the reverse of the "centres of excellence" policy, i.e., talent should be *dispersed* across the various stages of the research process.

### 2.2.3 Comparison between parallel-series and series-parallel structures

Which is the best *overall* R&D arrangement for the four-component two-path structures analyzed in this section? The answer is of some consequence as far as the suitability of the approach taken in this paper for the analysis of RJVs is concerned. Suppose that the parallel-series structure $PS_{\circ\circ}^{\bullet\bullet}$ were to turn out to be the superior arrangement. The resulting research joint venture would be the mere "coupling" of the individual firms' previous research strategies, implying simply a full sharing of information. On the other hand, if the alternative ${}_{\circ}^{\bullet}SP_{\circ}^{\bullet}$ were preferable, the resulting research joint venture would imply a complete re-organization of research activities with units from the two firms combining at *each stage* of the R&D process. As it turns out, not only is the ${}_{\circ}^{\bullet}SP_{\circ}^{\bullet}$ structure always more reliable than the $PS_{\circ\circ}^{\bullet\bullet}$ structure, but also a far more general result can be established, demonstrating that for research joint ventures to be *organizationally efficient* they have to involve more than a simple coupling of individual research routes.

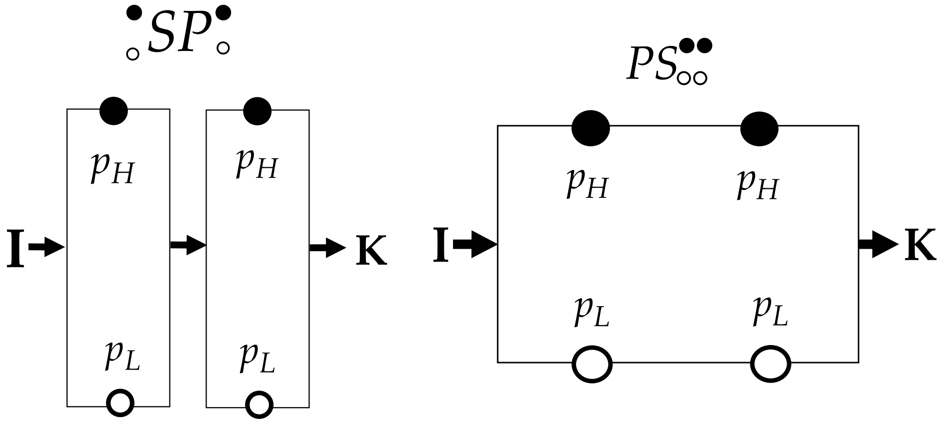

FIGURE 5

In the Appendix we prove

> **THEOREM 2**: *for any n-component coherent system, the series-parallel structure is always more reliable than the parallel-series structure.*

For the two-path case described above the proof is trivial:

$$\begin{aligned} h({}^{\bullet}_{\circ}SP^{\bullet}_{\circ}) - h(PS^{\bullet\bullet}_{\circ\circ}) = {} & [1-(1-p_H)(1-p_L)]^2 - \\ & [1-(1-p_H^2)(1-p_L^2)] = \\ & 2(1-p_H)(1-p_L)p_H p_L > 0 \\ & \forall p_H, p_L \quad 0 < p_H, p_L < 1 \end{aligned} \tag{4}$$

## 2.3 The simple welfare analysis of multi-component RJVs

Traditional analyses of the RJVs cannot ask the question of which is the more efficient form of R&D organization, for they lack the analytical apparatus required to address this issue. Those models that do attempt to examine organizational matters do so either from a principal-agent perspective[7] or indirectly by endogenizing spillover effects.[8] Here we can apply Theorem 2 directly to the two organizational forms open to an RJV in the two-path case and obtain quite straightforward and revealing results:

> **THEOREM 3**: *If R&D is not organized efficiently (in a reliability sense), then an RJV can be formed only if firms collude in the final-product market.*

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[7] See, for example, Aghion, and Tirole (1994, 1997).

[8] For example, Katsoulacos and Ulph (1998) examine the issue of "research design" in RJVs by extending D'Aspremont and Jacquemin's (1988) RJV-cum- spillover model in that the spillover coefficient is determined by the RJV members endogenously in the first stage, with R&D levels being determined in the second stage.

The proof is extremely simple: if the two firms simply combine (i.e., *join in parallel*) their pre-RJV R&D projects and hence the resulting organizational form is the $PS_{\circ\circ}^{\bullet\bullet}$ case, the RJV's gross profit[9] are given by:

$$\pi^{RJV} \equiv (p_L^2 + p_H^2 - p_L^2 p_H^2) 2\pi_D \tag{5}$$

whereas the sum of individual firm' profits prior to the formation of the RJV is given by:

$$\begin{aligned} \pi^0 &\equiv p_L^2 \pi_{HH} + p_H^2 \pi_{LL} \\ & p_L^2 [(1 - p_H^2)\pi_M + p_H^2 \pi_D] + p_H^2 [(1 - p_L^2)\pi_M + p_L^2 \pi_D] \end{aligned} \tag{6}$$

i.e.,

$$\pi^0 - \pi^{RJV} = (p_L^2 + p_H^2 - 2p_L^2 p_H^2)(\pi_M - 2\pi_D) > 0 . \tag{7}$$ [10]

In other words, traditional models of Research Joint Ventures do not allow RJV members to optimize over organization forms, in spite of the fact that the benefits to an RJV derived from the efficient re-organization of R&D can be substantial, as shown in the following:

> Remark: For a wide range of $p_L, p_H$ pairs, an efficient RJV generates an organizational surplus that makes it profitable even if its members do not collude in the final-product market and do not adjust their R&D investment.

In comparing the pre- and post-RJV rewards in the case of the efficient RJV, we need examine only gross profits, as we are deliberately keeping R&D investment constant, so that the costs of the four R&D components are the same across regimes. Therefore, if any surplus is generated, this is entirely due to the

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[9] We only need examine gross profit as the RJV is assumed to utilise the same four "components" its members we using before forming the RJV. This assumption is immaterial, as the same results obtained in the text apply for any permutation of R&D resources (e.g., in the cases where all four components are of high (low) reliability, only two high (low) reliability components are used, etc.).

[10] To confirm that our results apply also to the symmetric-equilibrium cases (1i) and (1ii), it is simple to confirm that if both firms joining the RJV employ research units of reliability $p$ (where $p = p_H, p_L$), then (7) simplifies to $\pi^0 - \pi^{RJV} = 2p^2(1 - p^2)(\pi_M - 2\pi_D) > 0$ .

increased efficiency of the *organization* of R&D, not to a change in R&D expenditure (as assumed in most of the literature on RJVs).

Define the profit gain from a reliability-efficient RJV as:

$$g(p_H, p_L) \equiv (p_H + p_L - p_H p_L)^2 2\pi_D - \left[ p_H^2 \pi_{LL} + p_L^2 \pi_{HH} \right] \tag{8}$$

Using the definitions of $\pi_{LL}$ and $\pi_{HH}$, we can write (8) as

$$\begin{aligned} g(p_H, p_L) \equiv {} & (p_H + p_L - p_H p_L)^2 2\pi_D - \\ & p_H^2 \left[ (1 - p_L^2)\pi_M + p_L^2 \pi_D \right] + \\ & p_L^2 \left[ (1 - p_H^2)\pi_M + p_H^2 \pi_D \right] \end{aligned} \tag{9}$$

As a benchmark, we can consider the linear Cournot case[11] where $\frac{\pi_M}{\pi_D} = \frac{9}{4}$. Using this and setting $g(p_H, p_L) = 0$ we can solve the resulting quadratic equation for $p_H$ and obtain:

$$\tilde{p}_H(p_L) = \frac{8p_L(1 - p_L) + p_L\sqrt{63 - 144p_L + 82p_L^2}}{1 + 16p_L - 18p_L^2} \tag{10}$$

Thus $g(p_H, p_L)$ is positive for $\forall p_L < p_H < \tilde{p}_H$, i.e., for all pairs $(p_H, p_L)$ in the shaded area in Figure 6.

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[11] i.e., inverse market demand is given by $min\{0, A - Q\}$ and the marginal cost of production is constant.

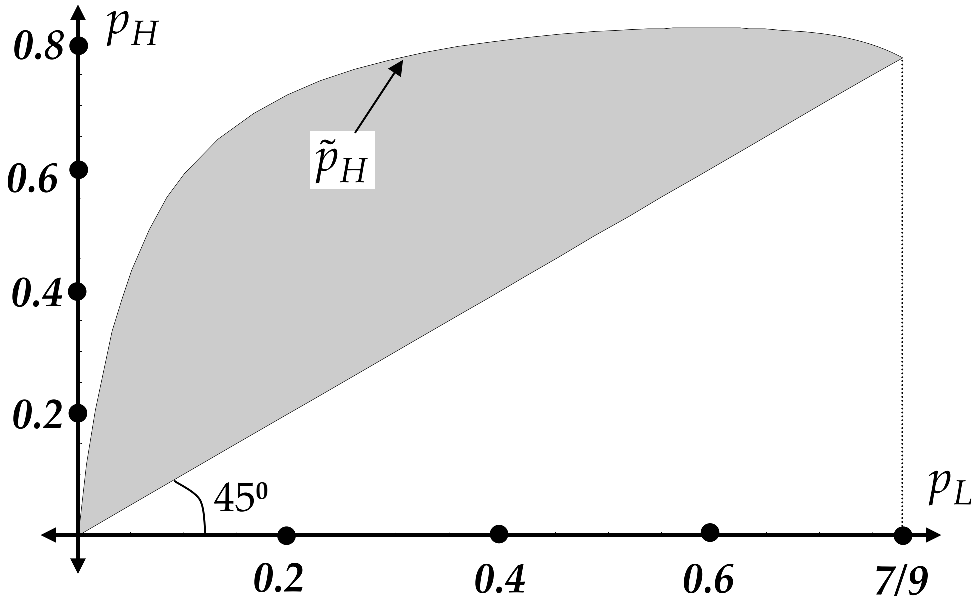

FIGURE 6

To underline the fact that the surplus enjoyed by RJV members is due entirely to the re-organization of research and not to any adjustment in R&D investment or quality, notice that if all four R&D components have the *same* reliability, *p*, then the RJV generates a surplus for $\forall p < \frac{7}{9}$. To obtain a feel for the relative magnitude of this "hidden" organizational surplus, define $rg(p)$ as the relative gain of the organizationally-optimal RJV compared to the organizational form implicitly assumed in standard information-sharing RJVs, namely:

$$(11) \qquad rg(p) \equiv \frac{(2p-p^2)\pi_D}{p^2\left[(1-p^2)\pi_M + p^2\pi_D\right]} = \frac{(2-p)^2}{2.25-1.25p^2}$$

As we can see from Figure 7, the relative gain can be fairly substantial, especially for low values of *p,* e.g., if R&D units have a probability of success of *0.25*, an organization-optimized RJV yields a *41%* higher surplus than a conventional RJV.

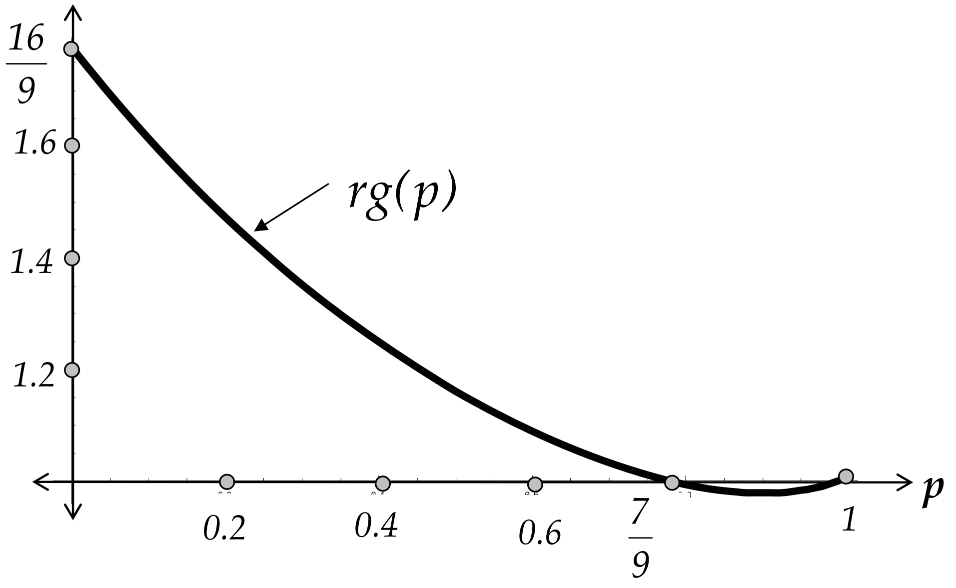

FIGURE 7

It may be suggested that the re-organization of research required to form an organizationally efficient RJV may involve a cost that more than offsets any organizational surplus. This is unlikely for two reasons: (i) as shown above, gross surplus is quite substantial and hence only an equally large cost of re-organization can yield a negative net surplus, but, more importantly (ii) the logic of re-organizing the research activity of RJVs applies also to the activities of individual firms, as shown in the following simplified example.

Consider a firm engaged in a two-obstacle research project (as described in Figure 2) and assume that it assigns to each task a single research unit, costing $w$ and succeeding with probability $p$. Without loss of generality, normalize so that the net expected profit is zero, i.e.:

(12) $$p^2\pi - 2w = 0$$

Now ask the following question: would the increased probability of completing the two-stage research project deriving from deploying optimally two additional research units more than offset the additional cost? i.e, assuming that the reliability and cost of the additional units are $p\beta$ and $w\beta$, respectively, would the firm benefit from switching from the structure at the top of Figure 8 to the one at the bottom?

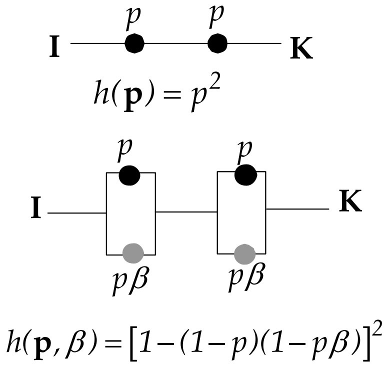

FIGURE 8

We can write the increase in net expected profit as:

(13) $[1-(1-p)(1-p\beta)]^2 \pi - 2w - 2w\beta$, i.e., using (12) and simplifying:

(14) $2w[(1+\beta-p\beta)^2 - (1+\beta)]$

Bearing in mind the constraint $p\beta \le 1$, we can see that next expected profit is positive for all combinations of $p$ and $\beta$ in the shaded area in Figure 9.

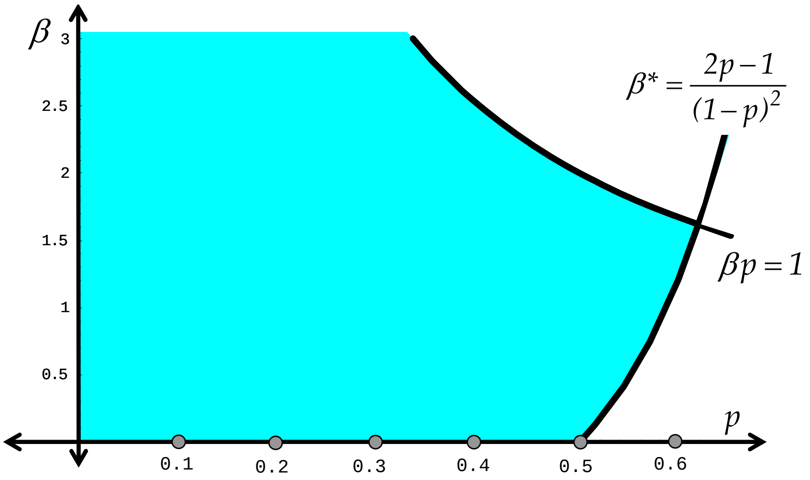

FIGURE 9

The implications of Figure 9 are quite interesting: it suggests that if the probability of success of the "initial" units is less than half, then coupling each of them in parallel with another unit is **always** profitable, irrespective of whether the "additional" units are less than, more than or as reliable as the initial ones. Or, to put it another way, provided the probability of component reliability is less than 0.5, one unit per task is **never** profit maximizing.

This means that if firms optimize over the organizational structure of their own research process before forming an RJV, they are likely (at least for "risky" projects) to be deploying already R&D units working in parallel and therefore forming an organization-efficient RJV (where members join in parallel their R&D resources for each research task) is unlikely to involve a significant cost of re-organization.

## 3. EXTENSIONS AND CONCLUSION

I hope to have convinced the reader that the theory of reliability of multi-component systems can provide valuable insights into the economics of research. Indeed, I should like to put forward the conjecture that reliability theory can be equally useful in analyzing *any* type of organization where the achievement of alternative goals requires the successful execution of multiple tasks. Some of the targets for the application of systems reliability theory are obvious, e.g., the growing literatures on networks and on decentralized information processing, to name but two. Other potential applications are less apparent, but none the less quite fruitful. These include Michael Kremer's influential O-ring theory of development (Kremer 1993) and the literature on "systems competition", as captured by Farrell *el al*. (1998), to which I turn briefly now.

### 3.1 The O-ring theory of economic development and reliability

> Many production processes consist of a *series* of tasks, mistakes in any of which can dramatically reduce the product's value. The space shuttle Challenger had thousand of components: it exploded because if was launched at a temperature that caused *one* of those components, the O-rings, to malfunction.

These are the opening sentences in Kremer's "O-ring" article. The first sentence envisages a multi-stage system organized in such a way that failure at any one stage effectively brings about the failure of the whole system. The second sentence considers a dramatic example of such a system, where the failure of one component, the O-rings, brought about the destruction of the entire space shuttle.[12]

[12] It is instructive to consider in slight more detail the tale of the ill-fated Challenger. The relevant subsystem turned out to be the two solid rocket motors that propel the shuttle into orbit. Each solid rocket motor contains three joints, for a total of six joints for the entire Shuttle. It is crucial that the small gap between each part of the motor be perfectly sealed; that's why O-rings are used. All six O-rings must operate if the propellant is not to escape from the solid rocket motor. In reliability terms, the O-rings form a six-component series system, as described in Figure 8.

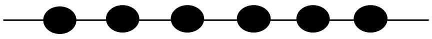

FIGURE 8: SIX O-RINGS IN SERIES

This is why Kremer used the failure of just one of the Shuttle's O-rings to illustrate his special production function where effectively various tasks are arranged in series.

However, this is not the end of the story: as one would expect, NASA was well aware that a series arrangement is very risky, in so far as a single component failure results in catastrophic system failure. To obviate this problem, each O-ring was coupled with a redundant (also called secondary, or backup) O-ring that would take over in case of failure of the primary component. Thus the *actual* arrangement in the Challenger was as in Figure 9, i.e., it was a **series** of **parallel** subsystems.

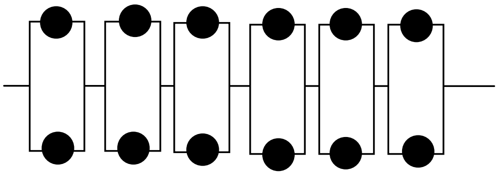

FIGURE 9: O-RINGS, THE TRUE STORY

The ultimate cause of the disaster was not the low temperature at launch, but flawed statistical analysis of the correlation between temperature at launch and O-ring failures in previous Shuttle missions. Had the correct analysis been undertaken, the solid rocket motor manufacturers would have realized that low launch-temperature could prevent each of the pairs of O-rings to function *independently*, thereby depriving each secondary O-ring of its safety value and thus reducing substantially the reliability of the entire system. The tragedy of Challenger is a dramatic example of the value of **parallel** arrangements.

What are the implications of the full story of the Challenger's disaster for the O-ring theory of economic development? Just to give a flavour of the argument, consider the dual of the production process envisaged by Kremer, i.e., instead of a series arrangement where $n$ tasks have to be carried out, take a *parallel* arrangement where a task can be carried out in $n$ alternative ways.

Using Kremer's assumptions and notation, the *series* arrangement yields a profit function of the form:

$$(15) \qquad \pi[\{q_i\}] = (\prod_{i=1}^{n} q_i)nB - \sum_{i=1}^{n} w(q_i); \qquad \frac{d^2y}{dq_i d(\prod_{j\neq i} q_i)} = nB > 0$$

where $q_i$ is effectively the reliability (i.e. probability of carrying out the task successfully) of worker $i$, $B$ is output per worker when all tasks are performed successfully and $w(q_i)$ is the wage rate. A crucial result in Kremer's analysis is that:

> the search for equilibria can be restricted to those allocations of workers to firms in which all workers employed by any single firm have the same $q$. This is because the derivative of the marginal product of skill for the ith worker with respect to the skill of the other workers is positive: $\frac{d^2y}{dq_i d(\prod_{j\neq i} q_i)} = nB > 0$

This means that, given a distribution of workers' reliabilities, firms will be heterogeneous, but workers within each firm will not be, as a firm with high-quality workers in all but one tasks will outbid all other firms for high-quality workers for that task.

Contrast this with a *parallel* arrangement, which yields the following profit function:

$$(16) \qquad \pi[\{q_i\}] = \left[1 - \prod_{i=1}^{n}(1-q_i)\right]nB - \sum_{i=1}^{n} w(q_i)$$

In this case, the cross derivative described above is *negative* ($\frac{d^2y}{dq_i d(\prod_{j\neq i}(1-q_j)} = -nB < 0$) and therefore the opposite result obtains: firms will be homogenous, each employing a heterogeneous workforce.

## 3.2 The vertical organization of industry and reliability

My final example of the potential for applying reliability theory refers to the literature on standards compatibility and particularly to the important paper by Farrell *et al.* (1998) on the vertical organization of industry. In a separate paper I extend their analysis by taking full advantage of the insights provided by reliability theory. Here I shall confine myself to pointing out the extraordinarily close analogy between their model and a multi-component structure, as formalized in this paper.

> Getting a final good to a consumer often involves more than one stage or component of production, and complementary activities must be coordinated and combined ... we examine ... two very different ways in which an industry can be organized ... competing on final products only (systems), or competing at intermediate stages (components). (Farrell *et al.* (1998), p. 144)

Farrell *et al.* call the former type of organization *closed*, as only the units within the organization can contribute intermediate inputs, and the latter *open* (for the reverse reason). It is not difficult to see that the closed (respectively, open) organization corresponds perfectly to a parallel-series (respectively, series-parallel) system and thus it is not surprising that each of the main propositions in their article has a precise counterpart in terms of reliability theory. Consider, for example, what they call "weak link principle": the quality of a system is the *minimum* of the qualities of its components. Not altogether surprisingly (from a reliability-theory perspective) they obtain the result that the open organization is more efficient than the closed regime, i.e., in reliability terms, a series-parallel structure is always more efficient than a parallel-series structure – precisely what Theorem 2 above has shown within a more general setting.

## 3.3 Concluding remarks

I hope that the glimpses of the potential of reliability theory supplied in the paper will be sufficient to convince economists that systems reliability theory may provide the language and the techniques (i) to address some interesting but hitherto neglected economic problems (such as the optimal organization of research), and (ii) to organize in a more general framework some recent and future developments in the economics of organizing.

## APPENDIX

In this Appendix it is assumed that the reader is familiar with the basic notions of systems reliability, as can be found, for example, in the classic text by Barlow and Proschan (1975). The emphasis here is rather on re-interpreting reliability theory in terms of organization of research.

### Some key reliability concepts

Let $x_i$ be a binary indicator variable assigned to component *i:*

(A1) $$x_i = \begin{cases} 1 & \text{if component is working} \\ 0 & \text{if component fails} \end{cases} \quad ; \quad \boldsymbol{x} \equiv (x_1, \ldots, x_n)$$

The starting point of *system* reliability is that the state of a *n*-component system (whether it functions or not) depends on the state of its components, i.e.

(A2) $$\phi(\boldsymbol{x}) = \begin{cases} 1 & \text{if system is working} \\ 0 & \text{if system fails} \end{cases}$$

It is natural to restrict attention to structure functions that have been "cleared" of useless components, i.e., components the performance of which does not affect the overall performance of the system, i.e.:

(A3) $$\begin{aligned} &\phi(\boldsymbol{1}) = 1, \ where \ \ \boldsymbol{1} = (1,1,\ldots,1) \\ &\phi(\boldsymbol{0}) = 0, \ where \ \ \boldsymbol{0} = (0,0,\ldots,0) \\ &\nexists \ x_i \ such \ that \ \phi(x_{-i}, 0_i) = \phi(x_{-i}, 1_i) \quad \forall x_{-i} \end{aligned}$$

For example, the system represented by the structure function $\phi(x_1, x_2) = 1 - (1 - x_1)(1 - x_1 x_2)$ is obviously dominated by its substructure $x_1$, in so far as the functioning of component $x_i$ is necessary and sufficient. In the context of the research process, I regard the requirement (A3) as the neatest statement of Occam's razor. Structures satisfying (A3) are called *monotonic* (or *coherent*).

The key concept is that the functioning of a system/organization depends on the way in which its components/units are connected. It turns out that a structure function is fully identified by the way in which the shortest ways to success (or alternatively, to failure) are arranged. In other words, we can define as *path vector* a vector **x** such that $\phi(\mathbf{x}) = 1$ and a corresponding *path set* $C_1(\mathbf{x}) \equiv \{ i \,|\, x_i = 1 \}$. A *minimal* path vector is a path vector **x** such that $\mathbf{y} < \mathbf{x} \Rightarrow \phi(\mathbf{y}) = 0$, i.e., a minimal path set is the minimal set of components whose functioning ensures the functioning of the whole system. Consider any structure function

$\phi(\mathbf{x})$ with minimal path sets $P_1,\ldots,P_r$. To each $P_j\ (j=1,\ldots,r)$ we now associate a binary function

(A4) $\rho_j(\mathbf{x}) \equiv \prod_{i\in P_j} x_i$ , i.e.,

$\rho_j(\mathbf{x})$ is the structure function of a *series* system made up of the components in $P_j$. The final step is to recall that any system $\phi(\mathbf{x})$ works iff at least one of its *r* minimal path series structures works:

(A5) $$\phi(\mathbf{x}) = \coprod_{j=1}^{r} \rho_j(\mathbf{x}) \equiv 1 - \prod_{j=1}^{r}(1-\rho_j(\mathbf{x})) \equiv \coprod_{j=1}^{r} \prod_{i\in P_j} x_i$$

i.e., our structure can be interpreted as the *parallel* arrangement of its path *series* structures. This representation is particularly convenient in the context of research, for it allows us to visualize the multi-obstacle search for a "discovery" as the list of all the shortest alternative routes to success.[13]

So far we have assumed that the state of components is deterministic. As a first step towards realism we can now consider the case in which the states of components are stochastic and *statistically independen*t. Let the state $x_i$ of component *i* be random where the probability that *i* works, $P[x_i=1]=p_i=Ex_i$, is called the *reliability* of *i.* Similarly, the reliability of the whole system is given by: $P[\phi(\mathbf{x})=1]=h=E\phi(\mathbf{x})$. Because of the independence assumption, system reliability is a function of component reliabilities: $h=h(\mathbf{p})$, where *h* is the *reliability function* of structure $\phi$ When applying this apparatus to the analysis of organization, we can regard $p_i$ as any relevant index of unit *i*'s contribution

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[13] In an analogous way, define as a cut vector a vector x such that $\phi(\mathbf{x})=0$ and the corresponding cut set is $C_0(\mathbf{x})$, where $C_0(\mathbf{x}) \equiv \{i \mid x_i = 0\}$. A minimal cut vector is a cut vector x such that $\mathbf{y} > \mathbf{x} \Rightarrow \phi(\mathbf{y}) = 1$. Similarly with the jth minimal cut set $K_j$ of a monotonic structure we can associate a binary function $\kappa_j(\mathbf{x}) = \coprod_{i\in K_j} x_i$ , which takes the value 0 if all components in the jth minimal cut set fail and 1 otherwise (*j=1,…,k,* where *k* is the number of minimal cut sets). As the underlying structure fails iff at least one of its minimal cut structures fails, we can write the identity $\phi(\mathbf{x}) \equiv \prod_{j=1}^{k} \kappa_j(\mathbf{x})$, i.e. the structure function can be regarded as the series arrangement of its minimal cut parallel structures.

to the functioning of the organization, not necessarily in terms of reliability (e.g., $p_i$ can be interpreted as the *quality* of unit *i*).

Recalling that a structure function is fully determined in terms of minimal paths and cuts, we can compute system reliability by taking the expectation of the structure function:

$$\text{(A6)} \quad h(\mathbf{p}) \equiv E\phi(\mathbf{x}) \equiv E\coprod_{j=1}^{r} \prod_{i\in P_j} x_i \equiv E\prod_{j=1}^{k} \coprod_{i\in K_j} x_i$$

In this section I hope to have indicated how the theory of reliability of multi-component structures can deployed to assess the efficiency of research processes. Indeed, more generally, reliability theory provides useful insights into the efficient arrangement of *any* structure where multiple tasks can be carried out in alternative ways.

**Majorization**

DEFINITION: $for\ \mathbf{x},\mathbf{y} \in \Re^n,\ \mathbf{x} \prec \mathbf{y}\ iff \begin{cases} \sum_{i=1}^{k} x_{[i]} \le \sum_{i=1}^{k} y_{[i]},\ k=1,\ldots,n-1 \\ \sum_{i=1}^{n} x_{[i]} = \sum_{i=1}^{n} y_{[i]} \end{cases}$

where $x_{[1]} \ge x_{[2]} \ge \ldots \ge x_{[n]}$ denotes the decreasing arrangement of **x**. When $\mathbf{x} \prec \mathbf{y}$, **x** is said to *be majorized* by **y**, or **y** *majorizises* **x**. See Figure A1.

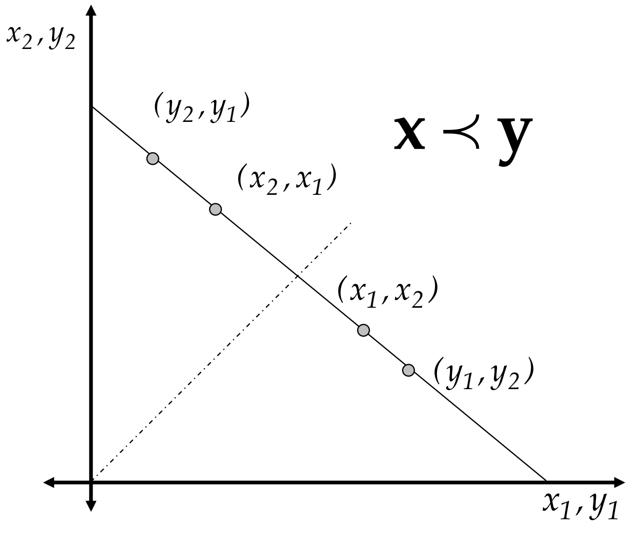

**FIGURE A4: y** majorizes **x**

DEFINITION: A real-valued function $f$ defined over a set $A \subset \Re^n$ is *Schur-convex* on $A \subset \Re^n$ if $\mathbf{x} \prec \mathbf{y}$ *on* $A \Rightarrow f(\mathbf{x}) \le f(\mathbf{y})$.

DEFINITION: A real-valued function $f$ defined over a set $A \subset \Re^n$ is *Schur-concave* on $A \subset \Re^n$ if $\mathbf{x} \prec \mathbf{y}$ *on* $A \Rightarrow f(\mathbf{x}) \ge f(\mathbf{y})$.

Necessary and sufficient conditions for $f$ to be Schur-concave on $\mathbf{I}^n$ are:

*(i)* $f$ *is symmetric on* $\mathbf{I}^n$

(A7) *(ii)* $(z_1 - z_2)\left(\frac{\partial f(z)}{\partial z_1} - \frac{\partial f(z)}{\partial z_2}\right) \le 0, \forall z \in \mathbf{I}^n$

if the last inequality is reversed, $f$ is Schur-convex.

We now have the necessary apparatus to prove Theorems 1 and 2.

*Proof of Theorem 1.* Let $P_1, \dots, P_r$ be the minimal path set of a parallel-series structure (as all components are assumed to be independent, these sets are disjoint) and w.l.o.g. rank their sizes as $n_1 \le \dots \le n_r$. The problem to be solved is how to allocate $n = \sum_{j=1}^{r} n_j$ independent components of reliability $p_i$ to these minimal path sets. We can show that reliability is maximized when the $n_1$ most reliable components are allocated to the shortest minimal path $P_1$, the next $n_2$ most reliable components to $P_2$, and so on. Using (A6) we can write:

$$h(\mathbf{p}) = 1 - \prod_{j=1}^{r}\left(1 - \prod_{i \in P_j} p_i\right)$$

If we let $\alpha_i$ be the hazard of component $i$ with $\alpha_i = -\log p_i$ and let $A_j \equiv \sum_{i \in P_j} \alpha_i$, $\mathbf{A} = A_1, \dots, A_r$ we can write the system reliability function as:

$$h(\mathbf{A}) = 1 - \prod_{j=1}^{r}\left(1 - e^{-A_j}\right)$$

By applying the Schur condition (A7), it is easy to check that $h(\mathbf{A})$ is Schur-convex. It follows that if $\mathbf{A}^* \succ \mathbf{A} \Rightarrow h(\mathbf{A}^*) \ge h(\mathbf{A})$. The theorem follows by noticing that the allocation of $n_r$ components with the highest hazards to $P_r$, of the $n_{r-1}$ components with the second highest hazards to $P_{r-1}$, etc. indeed majorizes any other allocation.

*Proof of (3) for any n-component series-parallel system.*

Let $C_1,\ldots,C_k$ be the minimal cut sets of an *n*-components series-parallel structure. W.l.o.g. let the sizes of these sets be $n_1 \le \ldots \le n_k$, with $n = \sum_{i=1}^{k} n_i$. Again, the problem is how to allocate *n* independent components with individual reliabilities $p_1,\ldots,p_n$.

System reliability is *minimized* when the best $n_k$ components are allocated to $C_k$, the next $n_{k-1}$ to $C_{k-1}$ and so on.

Let $y_i = -log(1-p_i)$ be the hazard of component *i* and let $Y_i \equiv \sum_{i \in C_j} y_i$ be the hazard of the cut set $C_j,\ j=1,\ldots,k.$ $\underset{=}{.}$ Define $\mathbf{Y} \equiv (Y_1,\ldots,Y_k)$. So we can write the reliability function as:

$$h(\mathbf{Y}) = \prod_{j=1}^{k}\left(1 - \prod_{i \in C_j}(1-p_i)\right) = \prod_{j=1}^{k}\left(1-e^{-Y_j}\right).$$

By applying condition (A7) we can verify that *h*(**Y**) is Schur-concave and thus $\mathbf{Y^*} \succ \mathbf{Y} \Rightarrow h(\mathbf{Y^*}) \le h(\mathbf{Y})$.

Conversely, an allocation that minimizes **Y** in the sense of majorization maximizes system reliability (there may be more than one).

*Proof of Theorem 2*: Suppose that successful discovery necessitates the overcoming of *m* "obstacles" (or stages) and that each stage can be tackled in *t* different ways. The general structures of the resulting series-parallel and parallel-series arrangements are respectively:

$$\phi^{SP}(\mathbf{x}) = \prod_{i=1}^{m}\left(1 - \prod_{j=1}^{t}(1-x_i^j)\right)$$

$$\phi^{PS}(\mathbf{x}) = 1 - \prod_{j=1}^{t}\left(1 - \prod_{i=1}^{m} x_i^j\right)$$

If we let the states of component $x_i^j, \left\{X_i^j\right\}_{i=1,\ldots,m}^{j=1,\ldots,t}$ $x_i^j$, be independent binary random variables with $1 > p_i^j = P\left\{X_i^j = 1\right\} > 0$, we can show that the reliability functions associated with $\phi^{SP}$ and $\phi^{PS}$, respectively, satisfy the following inequality:

$$P\left\{\phi^{SP}(\mathbf{X})=1\right\}=h\left(\mathbf{1}-\prod_{j=1}^{t}(1-\mathbf{p}^{j})\right)\geq 1-\prod_{j=1}^{t}\left(1-h(\mathbf{p}^{j})\right)=P\left\{\phi^{PS}(\mathbf{X})=1\right\}$$

where $\mathbf{p}^{j}=(p_1^{j},\ldots,p_m^{j})$.

The probability that the *i*th parallel arrangement of components in a series-parallel functions, i.e., $P\left\{max\left[X_i^1,\ldots,X_i^t\right]=1\right\}$ is, of course, equal to $1-(1-p_i^1)\cdots(1-p_i^t)$.

Thus the probability that the whole SP structure functions, i.e., $E\left\{\phi\left[\mathbf{X}^1,\ldots\mathbf{X}^t\right]\right\}$ equals $h\left(\mathbf{1}-(\mathbf{1}-\mathbf{p}^1)\cdots(\mathbf{1}-\mathbf{p}^t)\right)$.

As $\phi$ is a coherent structure, then $\phi\left(max\left[\mathbf{X}^1,\ldots,\mathbf{X}^t\right]\right)>max\left[\phi(\mathbf{X}^1),\ldots,\phi(\mathbf{X}^t)\right]$. Bearing in mind that $max\left[\phi(\mathbf{X}^1),\ldots,\phi(\mathbf{X}^t)\right]$ is a Bernoulli random variable and hence its expectation equals the probability of its being equal to 1, finally we can write:

$$\begin{aligned}P\left\{\phi^{SP}(\mathbf{x})=1\right\}&=h\left(\mathbf{1}-(\mathbf{1}-\mathbf{p}^1)\cdots(\mathbf{1}-\mathbf{p}^t)\right)=E\left\{\phi\left(max\left[\mathbf{X}^1,\ldots,\mathbf{X}^t\right]\right)\right\}>\\&E\left\{max\left[\phi(\mathbf{X}^1),\ldots,\phi(\mathbf{X}^t)\right]\right\}=P\left\{max\left[\phi(\mathbf{X}^1),\ldots,\phi(\mathbf{X}^t)\right]=1\right\}=\\&1-P\left\{\phi(\mathbf{X}^1)=0,\ldots,\phi(\mathbf{X}^t)=0\right\}=\\&1-\left[1-h(\mathbf{p}^1)\right]\cdots\left[1-h(\mathbf{p}^t)\right]\equiv P\left\{\phi^{PS}(\mathbf{x})=1\right\}.\end{aligned}$$

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