Another explanation for overbidding and another bias for underbidding in first-price auctions*

Oliver Kirchkamp[†] Philipp Reiss[‡]

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Abstract

First-price auction experiments find often substantial overbidding which is typically related to risk aversion. We introduce a model where some bidders use constrained linear bids. As with risk aversion this leads to overbidding if valuations are high, but in contrast to risk aversion the model predicts underbidding if valuations are low.

We test this model with the help of experiments, compare bidding in first-price and second-price auctions and study revenue under different treatments.

We conclude that at least part of the commonly observed overbidding is an artefact of experimental setups which rule out underbidding. Constrained linear bids seem to fit observations better.

Keywords: Auction, Experiment, Overbidding, Underbidding, Risk-Aversion.

(JEL C92, D44)

1 Introduction

First-price sealed bid auctions are a common institution and equilibrium bidding functions can often be derived in a straightforward way. Bidding behaviour has been studied in the lab and found to deviate systematically from equilibrium bids. Cox, Smith, and Walker (1983, 1985, 1988) are among the first to report on a large dataset with first-price auctions. Figure 1 shows data from one of their experiments. Participants repeatedly play a first-price auction with a fixed number of bidders. For each participant valuations are drawn

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[†]University of St Andrews, St Andrews, Fife KY16 9AL, Scotland (UK), oliver@kirchkamp.de

[‡]University of Magdeburg, Faculty of Economics and Management, reiss@ww.uni-magdeburg.de

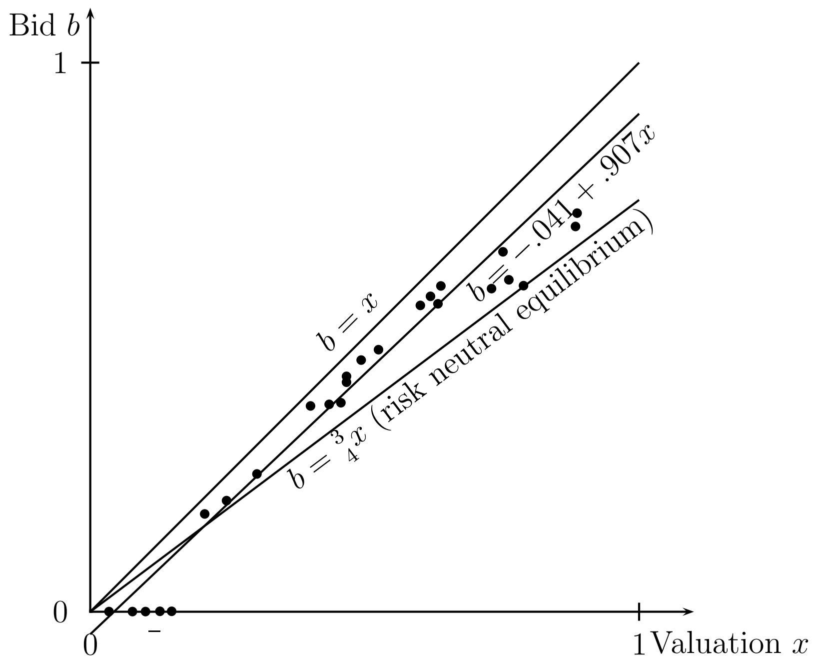


FIGURE 1: An example from an experiment by Cox, Smith, and Walker (1988) (page 83, figure 8, series 4, exp. 3, n=4, subject 2)

from a uniform distribution. The figure shows normalised valuations[1] that were drawn for a specific bidder on the horizontal axis. Bids are shown on the vertical axis.

The line $b = \frac{3}{4}x$ shows the equilibrium bidding function for this auction. We see that most bids are above the equilibrium bidding function. This is what we call overbidding and what, since then (see Cox, Smith, and Walker, 1983, 1985, 1988), has been found in many first-price auction experiments.

A standard explanation for overbidding in first-price auctions is risk aversion. One can assume specific functional forms for utility functions, e.g. constant relative risk aversion (CRRA). This implies a steeper equilibrium bidding function and, thus, a better approximation of actual bidding behaviour.

However, risk aversion does not explain all deviations from risk neutral equilibrium bidding. Cox, Smith, and Walker (1985) find overbidding even in experiments where subjects are payed in lottery tickets—i.e. where subjects with a von Neumann-Morgenstern utility would behave in a risk neutral way. Even worse, in experiments with third-price auctions, risk averse bidders should bid less and not more than the risk neutral equilibrium bid. Kagel and Levin (1993) find, nevertheless, overbidding in experiments with third price auctions.

Let us have a look at figure 1 again. For low valuations many bids are below, not above, the equilibrium bid. If bidders attach any utility to money this can not be an equilibrium. Still, Cox, Smith, and Walker (1988) find that when they approximate bids with the help of linear bidding functions the intercept is negative in some cases. Also

[1] In this experiment the smallest possible valuation was \$0 or \$0.10 and the largest possible valuation ranges from \$4.90-\$36.10. In the figure the valuations are normalised to $[0, 1]$. Bids that are shown in the figure are normalised correspondingly.

Ivanova-Stenzel and Sonsino (2004) report that 7.4% of the bids in their first-price auction are below the lowest possible valuation. However, underbidding does not find much attention in the experimental literature. One reason might be that underbidding is often ruled out implicitly or explicitly through the design of the experiment. Choosing valuations from an interval $[0, \bar{\omega}]$ may look like an innocent simplification but it may discourage bidders to enter bids which are smaller than the smallest possible valuation—which they otherwise might do. One aim of this study is methodological: we will compare different intervals for valuations and different restrictions on the lowest possible bid to find out whether these parameters have some impact on bidding behaviour.

A second aim in this paper is to find a simple explanation that is compatible with both the overbidding found in standard experiments but also compatible with the underbidding that we will find consistently once the auction design is changed slightly. In the next section we will introduce a model where all or some bidders are boundedly rational and use particularly constrained linear bids. As with risk aversion this leads to overbidding for high valuations, but in contrast to risk aversion constrained linear bids predict underbidding for low valuations. In section 4 we will test this model with the help of experiments. We will compare bidding in first-price and second-price auctions and study revenue under different treatments.

Anticipating our result we will find that at least part of the commonly observed overbidding seems to be an artefact of experimental setups which rule out underbidding. Constrained linear bids explain both the overbidding for high valuations as well as the underbidding for low valuations.

2 Model

In the following we will concentrate on a first-price sealed-bid auction with two bidders. Sometimes we will refer to the case of a second-price sealed-bid auction for comparison. In sections 2.1, 2.2, and 2.4 we will derive optimal bidding functions for different contexts.

2.1 Unconstrained Equilibrium Bids

Deriving the Bayesian Nash Equilibrium for the first-price sealed bid auction is standard and is repeated here to make the reader familiar with the notation. Consider the case where valuations are distributed uniformly over $[0, 1]$ and utility is given by $u(x) = x^r$ where r is a parameter that measures attitude towards risk. A risk neutral individual would be described by $r = 1$, a risk averse individual would be characterised by $r < 1$. We will concentrate on the case $r \in (0, 1]$. To find a Bayesian Nash equilibrium let us assume that a symmetric increasing bidding function $\gamma(x)$ is invertible. In equilibrium all bidders bid according to γ. With two bidders we have to show that for each bidder it is a best reply to follow γ provided the other does so as well. Bidder 1 has a true valuation x and bids b. Since γ is invertible we can find a valuation z such that $b = \gamma(z)$. Bidder 1 wins the auction when the valuation of bidder 2 is smaller than z. The probability of this event is $G(z) \equiv z$. Bidder 1 chooses z to maximise $\text{EU} = G(z) \cdot u(x - \gamma(z))$ which

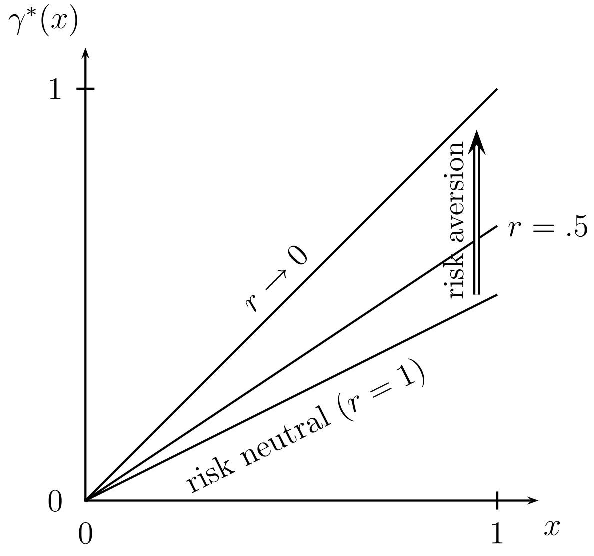


Figure 2: Bids in the Bayesian Nash Equilibrium

yields as a first order condition

$$(x - \gamma(z))^r - r \cdot z \cdot (x - \gamma(z))^{r-1} \gamma'(z) = 0 \tag{1}$$

In the symmetric equilibrium we have $z = x$, hence,

$$(x - \gamma(x))^{r-1} \cdot (\gamma(x) + x \cdot (r\gamma'(x) - 1)) = 0 \tag{2}$$

It is easy to see that in equilibrium $\gamma(0) = 0$ which yields the unique solution

$$\gamma^*(x) = \frac{x}{1 + r} \tag{3}$$

The second derivative $\partial^2 \mathrm{EU}/\partial z^2 = -(rx/(1+r))^{r-1}$ is negative, so we have indeed found a maximum. If valuations are not drawn from the interval $[0, 1]$ but from $[\underline{\omega}, \overline{\omega}]$ one finds similarly that the equilibrium bid is

$$\gamma^*(x) - \underline{\omega} = \frac{x - \underline{\omega}}{1 + r} \tag{4}$$

Figure 2 shows three examples for the case $x \in [0, 1]$. In a world where bidders are risk neutral $(r = 1)$ they will all bid according to $b = x/2$. When bidders are more risk averse $(r < 1)$ they will bid more than $x/2$. In the limit, if bidders are infinitely risk averse $(r \to 0)$, they will make bids equal to their valuation, $b = x$.

For second-price sealed-bid auctions a standard argument shows that it is always a weakly dominant strategy to make a bid equal to the own valuation.

$$\gamma^*(x) = x \tag{5}$$

This holds independently of the degree of risk aversion.

2.2 Constrained equilibrium bids

We have seen above that in the Bayesian Nash Equilibrium of the first-price auction bidders 'shade their bids', they make bids which are smaller than their valuation. When we looked at responses to unpaid computerised post-experimental questionaires of another first-price experiment[2] we saw some participants who said they would bid their valuation minus a constant amount. Shading by a constant amount can also be related to satisficing behaviour. A bidder who wants to gain a certain amount when winning the auction must bid the own valuation minus this amount. Finally, shading by a constant amount can also be interpreted as a simple rule given to a bidding agent. If first a principal has to define a bidding rule (before the individual valuation is revealed) and then the agent who follows this rule learns the valuation, then it might be simpler for the principal to require a fixed amount that the agent must gain from each trade.

Let us consider a population of bidders which are constrained to use a bidding function $\bar{\gamma}$ of the following structure

$$\bar{\gamma}_i(x) = \alpha_i + \beta x \tag{6}$$

with β exogenously given, positive[3], and the same for all bidders. We will specifically study the case $\beta = 1$. This is what we will call constrained linear bids in the following. In the Bayesian Nash Equilibrium in equation (3) we had $\beta = 1/(1+r)$.

Each bidder determines α_i individually. Given two valuations x_i and x_j of two bidders, bidder i wins if his opponent's valuation $x_j < x_i + (\alpha_i - \alpha_j)/\beta$. If $\alpha_i \leq \alpha_j$ the expected utility for a risk neutral bidder is

$$\mathrm{EU}_1 = \int_{\frac{\alpha_j-\alpha_i}{\beta}}^{1} \int_0^{x_i - \frac{\alpha_j-\alpha_i}{\beta}} x_i - (\alpha_i + \beta x_i) dx_j dx_i \tag{7}$$

The first order condition yields

$$\alpha_i^* = \frac{\alpha_j + \beta - 2\beta^2}{1+2\beta} \tag{8}$$

This condition is fulfilled for both players if

$$\alpha_i = \alpha_j = \frac{1}{2} - \beta \tag{9}$$

In the same way we can study the case $\alpha_i > \alpha_j$ which yields the same condition.

We should note that the risk neutral Bayesian Nash Equilibrium is a special case ($\alpha = 0$, $\beta = 1/2$) of equation (9).

Equation (7) can be generalised to also account for risk aversion:

$$\mathrm{EU}_2 = \int_{\frac{\alpha_j-\alpha_i}{\beta}}^{1} \int_0^{x_i - \frac{\alpha_j-\alpha_i}{\beta}} \left(x_i - (\alpha_i + \beta x_i)\right)^r dx_j dx_i \tag{10}$$

[2] **?**.

[3] One can show that the solution we obtain below in equation (9) extends to the case $\beta = 0$.

We can follow the same steps as above, however, for general β it is not possible to obtain a closed form solution for α_i as we could in equation (9). For the case $\beta = 1$, which we find particularly interesting, it is straightforward to find the equilibrium

$$\alpha_i = \alpha_j = -\frac{r}{2} \tag{11}$$

Risk averse bidders who are constrained to bid their valuation minus a constant amount will in equilibrium choose this constant to be $r/2$.

For the case of the second-price sealed-bid auction we see that the weakly dominant bidding strategy from equation (5) already has the structure of equation (6) with $\alpha_i = 0$ and $\beta_i = 1$. A bidder who is constrained to choose a bid that is the valuation shaded by a constant amount will, thus, not deviate from the weakly dominant bid.

2.3 Revenue comparison

With risk neutral bidders in the Bayesian Nash equilibrium the well known revenue equivalence theorem holds. For a wide range of auctions the expected revenue is the same. In particular the expected revenue in the first-price and in the second-price sealed bid auction do not differ.

It is easy to see that with risk averse bidders in the Bayesian Nash equilibrium the first-price auction generates a higher revenue. From equation (3) we see that bids increase with risk aversion in the first-price auction. If bidders follow the bidding strategy from equation (3) the expected revenue in the first-price auction is

$$R^{\mathrm{I}} = 2\int_0^1 \int_x^1 \frac{y}{1+r}\, dy\, dx = \frac{2}{3+3r} \tag{12}$$

In the second-price auction it is still a dominant strategy to bid the own valuation. Expected revenue in the second-price auction is, hence,

$$R^{\mathrm{II}} = 2\int_0^1 \int_x^1 x\, dy\, dx = \frac{1}{3} \tag{13}$$

Thus, $R^{\mathrm{II}} < R^{\mathrm{I}}$ for $r < 1$.

If bidders in the first-price auction no longer follow the Bayesian Nash equilibrium but, instead, follow the linear bidding function from equation (6) then the expected revenue is

$$\bar{R} = 2\int_0^1 \int_x^1 \beta y + \alpha\, dy\, dx = \alpha + \frac{2\beta}{3} \tag{14}$$

We see that $R^{\mathrm{II}} < \bar{R}$ iff $\alpha > (1-2\beta)/3$. For the case of risk averse linear bidders that we study in equations (10) and (11) we find that $R^{\mathrm{II}} < \bar{R}$ iff $r < \frac{2}{3}$.

Thus, similar to the Bayesian Nash equilibrium more risk aversion increases the revenue obtained in the first-price auction. However, a higher degree of risk aversion is needed to outperform the second-price auction than in the Bayesian Nash equilibrium. In section 4.3 we will compare revenue of the first-price auction and the second-price auction in the experiment.

2.4 Optimising against a linear bidder

To complete the discussion of linear bidding functions let us consider the case of a rational bidder who knows that the opponent makes a bid $\bar{\gamma}(x_j)$ according to equation (6) where parameters α_j and β of the opponent are known but where the valuation x_j of the opponent is unknown. What is the best response $\hat{\gamma}$ in a first-price auction against such a linear bidder? Here we can not simplify equation (1) by assuming symmetry ($x = z$) as we did in the derivation of equation (2). Instead, we have to substitute equation (6) into (1). Solving the first order condition we obtain[4]

$$z = \frac{x - \alpha}{(1 + r)\beta} \tag{15}$$

which, using (6), yields

$$\hat{\gamma}(x) = \frac{x + r\alpha}{1 + r} \tag{16}$$

In a similar way we can study the problem of a bidder who knows that a fraction ρ of all bidders are rational and the remaining $1-\rho$ are linear bidders. Assume that a rational bidder makes a bid b, then call z the valuation of a linear bidder with exactly this bid (i.e. all linear bidders with a valuation smaller z will not win) and call y the valuation of a rational bidder with this bid. The first order condition becomes

$$\rho\cdot(x - \gamma(y))^{r-1} (x - \gamma(y) - ry\gamma'(y)) + (1-\rho)\cdot(x-\alpha-z\beta)^{r-1}(x-\alpha-z\beta-rz\beta) = 0\,. \tag{17}$$

If all rational bidders use the same symmetric bidding function we have $y = z$. Using $z = (\gamma(x) - \alpha)/\beta)$ we can solve for γ and find

$$\gamma(x) = \frac{x}{1 + r} + \alpha \cdot (1 - \rho) \cdot \frac{r}{1 + r \cdot (1 - \rho)} \tag{18}$$

which includes for $\rho = 1$ equation (3) and for $\rho = 0$ equation (16) as a special case.

We see that even if only a fraction of all bidders use linear bids with a negative intercept ($\alpha < 0$) then also rational bidders will behave as if they were constrained and use linear bids with a negative intercept.

3 Experimental setup

The purpose of the experiment to twofold: We want to find out how far constrained linear bids are consistent with actual behaviour and we want to check to what extent the existing experimental evidence on first-price auctions is an artefact of the design. To distinguish constrained linear bids from risk-averse bids we should be able to observe bids also for low valuations in a reliable way since particularly in this region predictions of risk-aversion and constrained linear bids differ. It has been argued that for low valuations bidders seldom win the auction and therefore have little chance to gain experience. To allow

[4] If $\beta = 1$ and α is chosen according to equation (9) we have always $\alpha < 0$ and $\beta > (1 - \alpha)/(1 + r)$. Then the inverse of $\bar{\gamma}$ is in $[0, 1]$ for all values of $\hat{\gamma}(x)$ with $x \in [0, 1]$ which allows us to move from equation (1) to (15). We actually do not need $\beta = 1$. E.g. if $r = 1$ then $\beta > 1/2$ is sufficient.

Treatment	$[\underline{\omega}, \overline{\omega}]$	restriction of bids	auction
−25	$[-25, 25]$	—	first-price
0	$[0, 50]$	—	first-price
0+	$[0, 50]$	only positive bids	first-price
25	$[25, 75]$	—	first-price
50	$[50, 100]$	—	first-price
50+	$[50, 100]$	only positive bids	first-price
50+II	$[50, 100]$	only positive bids	second-price

Table 1: Treatments

bidders to gain as much experience as possible we use a setup with two bidders only. We use the strategy method and play in each round five independent auctions which increases the chance to get feedback.

All that we have said above for bidders' valuations which are distributed uniformly over an interval $[0, 1]$ can be generalised to uniform distributions over an interval $[\underline{\omega}, \overline{\omega}]$. Table 1 lists the different intervals that we study in our experiments. Based on the theoretical considerations above we expect the following:

- In the treatments where underbidding is possible (i.e. the −25, 25, 50, and 50+ treatment) we should find underbidding for small and overbidding for large valuations which is consistent with constrained equilibrium bids but inconsistent with unconstrained equilibrium bids.

  We should find no underbidding in the 0+ treatment since there negative bids are not possible. The 0 treatment where negative bids are allowed is an intermediate case. Some participants might be tempted to assume that bids can not be smaller than zero, others might not.

- Underbidding for small and overbidding for large valuations might be particularly strong in the −25 treatment. While technically equivalent with the 25 and 50 treatment, this treatment involves negative and positive valuations at the same time which might be percived as more difficult and, thus, may give an additional incentive to use constrained linear bids.

- The comparison of the 50 and the 50+ treatment allows us to check whether the restriction to positive bids has any side effects even in an interval where the restriction should not matter.

- We introduce a treatment with a second-price auction (50+II) since there the equilibrium bid is a constrained linear bid, thus, in this treatment we expect neither over- nor underbidding.

Experiments were conducted between 12/2003 and 12/2004 in the experimental laboratory of the SFB 504 in Mannheim. 304 subjects participated in these experiments. A detailed list of the treatments is given in appendix A, instructions are shown in appendix B. The software we used was z-Tree (Fischbacher (1999)).

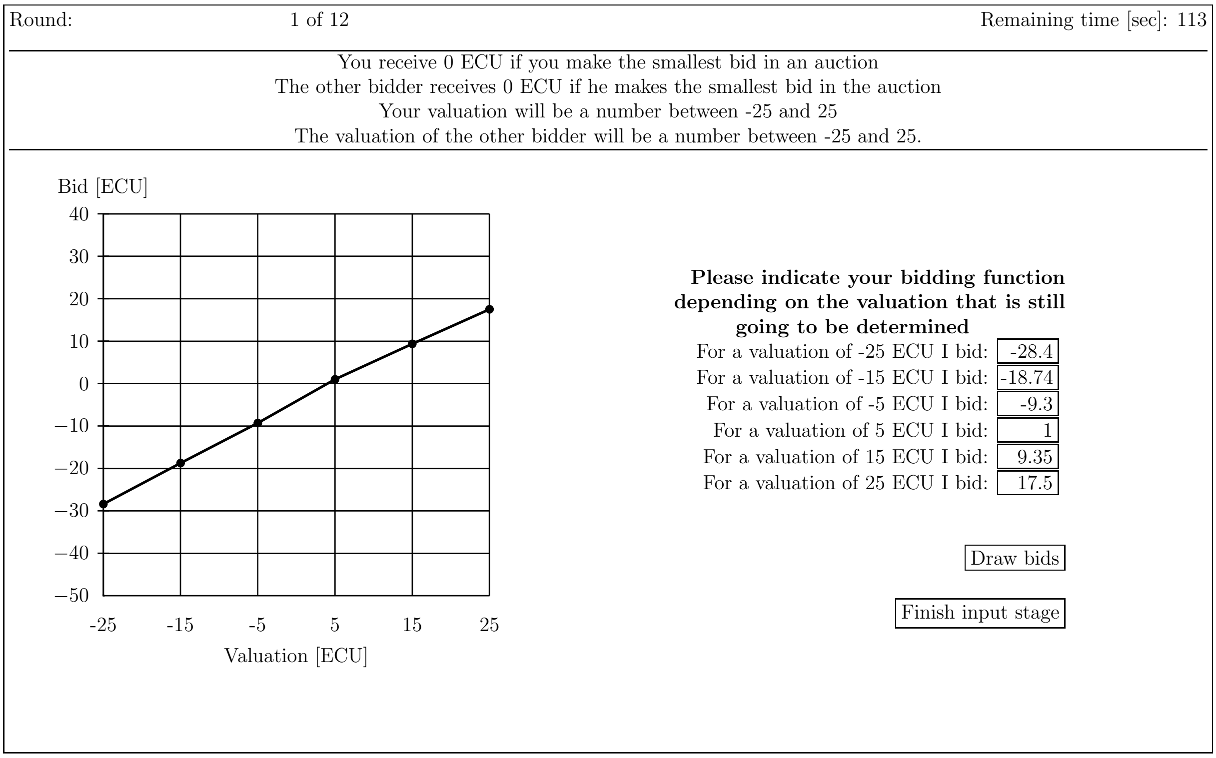


FIGURE 3: A typical input screen in the experiment (translated into English)

A typical screen during the experiment is shown in figure 3 (translated into English). In each round participants enter bids for six valuations which are equally spaced between $\underline{\omega}$ and $\overline{\omega}$. Bids for all other valuations are interpolated linearly. When all participants have determined their bidding functions we draw five random and independent valuations for each participant. Each of these five random draws corresponds to an auction for which the winner is determined and the gain of each player is calculated. The sum of the gain of these five auctions determines the total gain from this round. A typical feedback screen is shown in figure 4. Participants play 12 rounds, each consisting of a bid input stage and a feedback stage. After that participants complete a small questionnaire and are payed in cash according to their gains in the experiment.

4 Results

4.1 Convergence of bidding behaviour

Before we look at details of bidding behaviour we have to check whether behaviour stabilises over the course of the experiment. To do this we count how often participants change support points of their bidding function. In each period and for each participant this can be a number between zero and six. Zero if the participant continues to use the bidding function from the last period, and six if all hypothetical bids are changed. The development of the median of this distribution is reported in the left graph of figure 5. By definition all six support points are new in the first period, thus, period 1 must start with

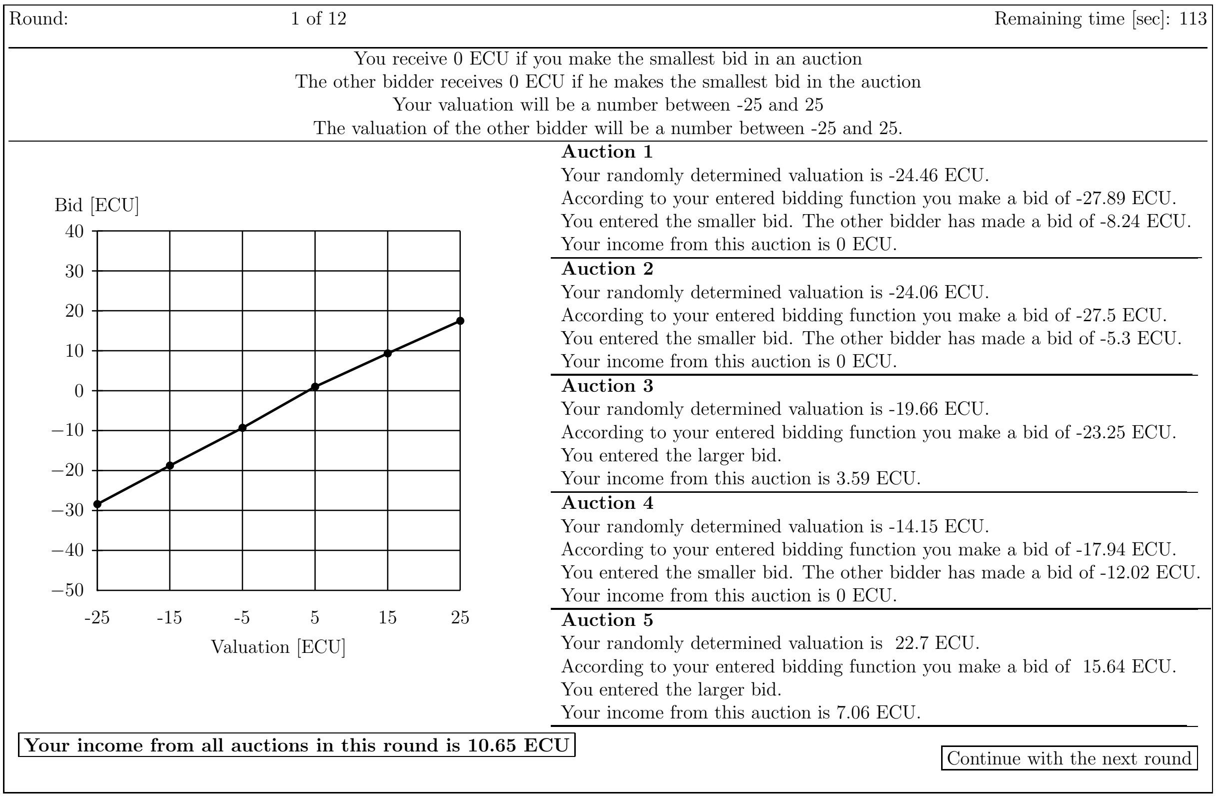
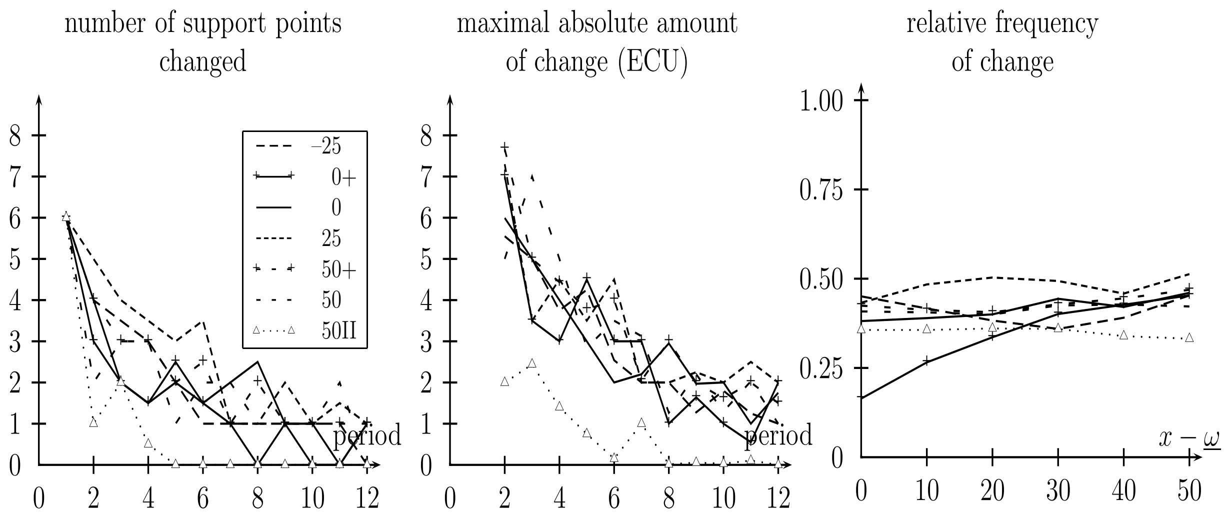


FIGURE 4: A typical feedback screen in the experiment (translated into English)



The figure shows how many of the six support points (hypothetical bids) of a bid function a median bidder changes and the maximal absolute amount of this change.

FIGURE 5: Convergence of bids over time

6 changes for all treatments. We see that participants quickly adjust and that during the second half of the experiment the median bidder changes no more than one or two support points in each period. The graph in the middle of figure 5 shows how the absolute amount of these changes changes over time. For each participant and each period we determine the largest absolute change in the six hypothetical bids from one period to the next. The median of this distribution is shown in the graph. We see that these changes are small compared to the range of the valuation. The right part of the figure shows that with the exception of the 0+ treatment changes are distributed evenly over valuations. Below we will see that 0+ is indeed a special treatment.

4.2 Bids in the experiment

Overbidding The median amount of overbidding $b(x) - \gamma^*(x)$ as a function of the valuation x, is shown in figure 6. Let us look at the $50 + II$ and the 0+ treatment first.

The $50 + II$ treatment is a second-price treatment where bidders have a weakly dominant bidding strategy. We should not expect much overbidding in this treatment and, indeed, our median bidders nicely follow the equilibrium prediction for this treatment: The triangles which denote the median difference between actual and equilibrium bids are just on the horizontal axis.

The 0+ treatment is the traditional first-price treatment. The lowest possible valuation is 0, and bids are constrained to be positive. As to be expected in a first-price auction there is overbidding and the amount of overbidding increases with the valuation. This finding is consistent with risk-aversion and confirms findings from several previous experiments, starting with Cox, Roberson, and Smith (1982).

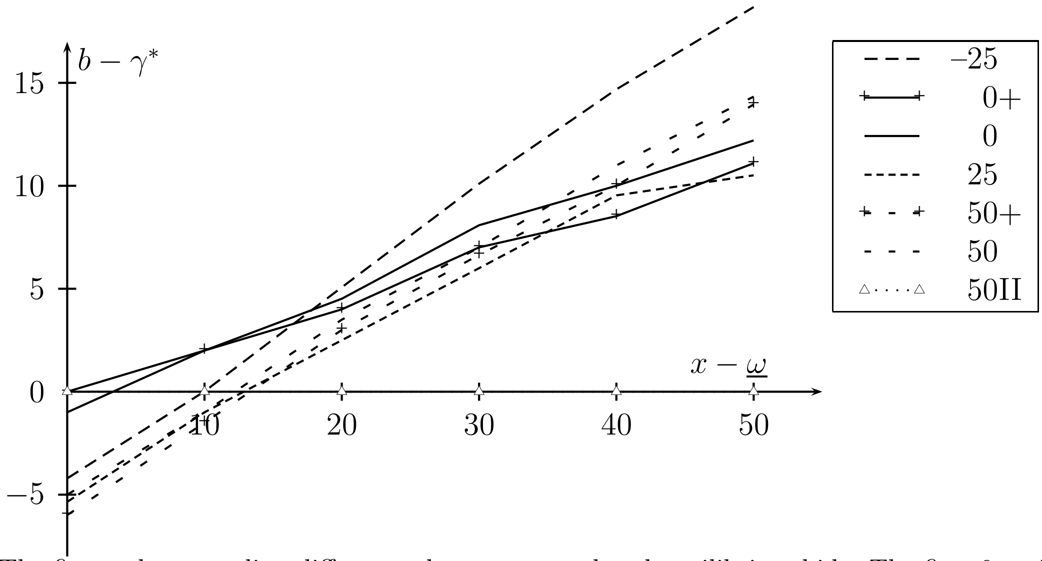
The other five treatments allow for underbidding and, indeed, show underbidding if the valuation is small. This is not consistent with unconstrained equilibrium behaviour with or without risk-aversion but it is consistent with the constrained linear bids presented in section 2.

All this is tested formally in table 2 which shows mean overbidding for the highest and lowest valuation, $\underline{\omega}$ and $\overline{\omega}$. The table compares overbidding for the lowest and highest valuation, $\underline{\omega}$ and $\overline{\omega}$. For each treatment n is the number of independent observations. We test whether $b(\underline{\omega}) < \gamma^*(\underline{\omega})$ (underbidding for low valuation) and whether $b(\overline{\omega}) > \gamma^*(\overline{\omega})$ (overbidding for high valuation). Mean bids are shown together with results of a parametric t-test ($P_{>t}$) and a non-parametric binomial test (P_{bin}). As in many other experiments with first-price auctions, we find a significant amount of overbidding for the highest possible valuation $\overline{\omega}$ in all treatments. We also find underbidding for the smallest possible valuation $\underline{\omega}$ in all first-price treatments where underbidding is possible, i.e. always, except in the 0+ treatment.

Individual bids Figure 7 shows the result of estimating equation (19) for each bidder individually.

$$b(x) = \underline{\omega} + \alpha + \beta \cdot (x - \underline{\omega}) + u \tag{19}$$

The first six periods of the experiment are discarded. Outlayers have been eliminated using Hadi's method. For the second price auction 20 bidders can not be distinguished



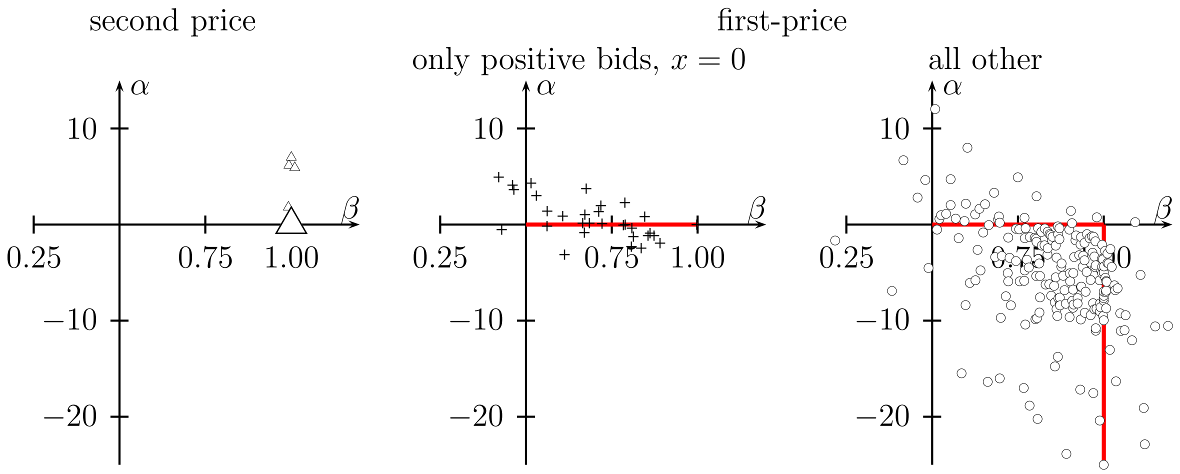
The figure shows median differences between actual and equilibrium bids. The first 6 periods from each session are discarded.

Figure 6: Overbidding for different treatments

			$b(\underline{\omega}) - \gamma^*(\underline{\omega})$				$b(\overline{\omega}) - \gamma^*(\overline{\omega})$			
treatment		n	mean	t	$P_{>t}$	P_{bin}	mean	t	$P_{>t}$	P_{bin}
-25	−	4	-4.653	-7.703	.0023	.0625	16.229	20.753	.0001	.0625
0	−	6	-3.453	-2.003	.0508	.0156	10.586	5.798	.0011	.0156
0	+	4	.841	2.009	.9309	1	9.397	14.154	.0004	.0625
25	−	3	-4.953	-5.574	.0154	.125	8.461	4.176	.0264	.125
50	−	4	-7.53	-4.643	.0094	.0625	6.639	2.806	.0338	.0625
50	+	14	-2.597	-1.704	.0561	.212	17.272	7.169	.0000	.0001
all firstprice		29	-4.856	-7.466	.0000	.0000	10.389	13.882	.0000	.0000
secondprice		6	3.549	3.98	.9947	.9844	27.485	36.084	.0000	.0156

The table compares overbidding for the lowest and highest valuation, $\underline{\omega}$ and $\overline{\omega}$. For each treatment n is the number of independent observations. We test whether $b(\underline{\omega}) < \gamma^*(\underline{\omega})$ (underbidding for low valuation) and whether $b(\overline{\omega}) > \gamma^*(\overline{\omega})$ (overbidding for high valuation). Mean deviations from the risk neutral Bayesian Nash equilibrium bid are shown together with results of a parametric t-test ($P_{>t}$) and a non-parametric binomial test (P_{bin}). As in many other experiments we find a significant amount of overbidding for the highest possible valuation $\overline{\omega}$ in all treatments. However, we also find underbidding for the smallest possible valuation $\underline{\omega}$ in all first-price treatments where underbidding is possible, i.e. always, except in the 0+ treatment and in the second-price auction.

Table 2: Overbidding for different treatments



Each dot represents the estimation of equation (19) for a single bidder with the first six periods of the experiment discarded. For the second price auction 20 bidders can not be distinguished visually from $\alpha = 0$ and $\beta = 1$ (big triangle). Outlayers have been eliminated using Hadi's method.

FIGURE 7: Individual bidding

visually from $\alpha = 0$ and $\beta = 1$, i.e. the equilibrium value. This is shown as a big triangle in the figure.

The graph in the middle shows estimates for the 0+ treatment. We find that the absolute value of the intercept α is small for most bidders. There is no visible underbidding.

The right graph shows all the other treatments. Consistent with the aggregate observations we see that the possibility to make bids smaller than the smallest possible valuation has two effects:

- For many bidders the coefficient β is now larger and closer to one. We have more bidders who bid similar to constrained linear bids.
- Most bidding functions have now a negative intercept—there is underbidding.

4.3 Revenue in the experiment

In section 4.2 we have found overbidding as well as underbidding as long as the design permits small bids. In the current section we investigate the net effect on revenue. Do the effects of overbidding and underbidding cancel out if small bids are possible or is overbidding still the dominating effect? Will first-price auctions still generate more revenue than second-price auctions? In table 3 we look at the excess revenue, i.e. the average difference between the actual revenue R and the equilibrium value R^*. For each treatment n is the number of independent observations. We test whether $R > R^*$ (revenue in the experiment is higher than revenue in equilibrium). This is always the case and in most cases it is significant for the individual treatment. Most importantly, average revenue for all first-price auctions is significantly higher than equilibrium revenue both with parametric

treatment	n	$R - R^*$ mean	t	$P_{>t}$	P_{bin}
−25	4	10.708	33.693	.0001	.125
0	6	7.918	7.308	.0008	.0312
0 +	4	7.227	47.702	0	.125
25	3	6.071	3.985	.0576	.25
50	4	4.544	3.575	.0374	.125
50 +	8	7.188	11.512	0	.0078
all first-price	29	7.329	16.739	0	0
second-price	6	2.483	5.219	.0034	.0312

	n	$R^I - R^{II}$ (for $\underline{\omega} = 50$ only) mean	t	$P_{>t}$	$z, P_{>z}$
	18	4.021	5.334	.0001	$z = 2.81$, $P_{>z} = .005$

The table compares excess revenue $R - R^*$ for different treatments. n is the number of independent observations. Mean excess revenue is shown together with results of a parametric t-test ($P_{>t}$) and a non-parametric binomial test (P_{bin}). Data from the first 6 periods of each session is discarded.

Table 3: Excess revenue for different treatments

and with non-parametric tests. Also in the second-price auction average revenue is significantly higher than in equilibrium. In the lower part of table 3 we compare revenue in the first-price and in the second-price auction. We find that average revenue is significantly higher in the first-price auction.[5]

5 Concluding remarks

In this paper we have presented a series of experiments to better understand systematic deviations from equilibrium bids in first-price auctions.

Many first-price auction experiments find that subjects bid more than the risk neutral equilibrium bid. The lack of risk-neutrality is a tempting explanation for overbidding behaviour, though it was soon realised that risk aversion does not seem to explain everything.

The idea we are proposing here, namely that some bidders use constrained linear bids, is independent of the representation of payoffs as lottery tickets or as money and consistent with the traditional experimental evidence. We have seen in section 2 that optimal constrained linear bids imply underbidding for small valuations and that the presence of a small proportion of bidders with constrained linear bids is sufficient to make rational bidders behave as if they were constrained in a similar way. Our experiment has confirmed the presence of underbidding for a large variety of scenarios. Constrained linear bids seem to fit better with what we observe in the laboratory than risk-aversion.

[5]The non-parametric test used here is a Wilcoxon rank-sum test.

References

Cox, J. C., B. Roberson, and V. L. Smith, 1982, Theory and behavior of single object auctions, in *Research in experimental economics*, ed. by V. L. Smith. JAI Press, Greenwich, Conn.

Cox, J. C., V. L. Smith, and J. M. Walker, 1983, Test of a Heterogeneous Bidder's Theory of First Price Auctions, *Economic Letters*, 12(3-4), 207–212.

——— , 1985, Experimental Development of Sealed-Bid Auction Theory: Calibrating Controls for Risk Aversion, *American Economic Review*, 75(2), 160–165.

Cox, J. C., V. L. Smith, and J. M. Walker, 1988, Theory and Individual Behavior of First-Price Auctions, *Journal of Risk and Uncertainty*, 1, 61–99.

Fischbacher, U., 1999, z-Tree. Toolbox for Readymade Economic Experiments, Working paper 21, IEW, University of Zurich.

Ivanova-Stenzel, R., and D. Sonsino, 2004, Comparative Study of One Bid versus Two Bid Auctions, *Journal of Economic Behavior and Organization*, to appear.

Kagel, J. H., and D. Levin, 1993, Independent private value auctions: Bidder behavior in first-, second- and third-price auctions with varying numbers of bidders., *Economic Journal*, 103, 868–879.

A List of independent observations

date	group	$\underline{\omega}$	$\underline{b}$	second-price	participants
20040518-17:41	0	-25	-125	0	8
20040518-17:41	1	-25	-125	0	8
20040519-10:37	0	-25	-125	0	8
20040519-10:37	1	-25	-125	0	8
20040518-10:19	0	0	-100	0	8
20040518-10:19	1	0	-100	0	6
20040518-12:15	0	0	-100	0	6
20040518-12:15	1	0	-100	0	6
20040519-17:39	0	0	-100	0	8
20040519-17:39	1	0	-100	0	6
20040518-15:55	0	0	0	0	8
20040518-15:55	1	0	0	0	8
20040519-12:33	0	0	0	0	8
20040519-12:33	1	0	0	0	8
20040602-14:11	0	25	-75	0	8

continued on next page

date	group	$\underline{\omega}$	$\underline{b}$	second-price	participants
20040602-14:11	1	25	-75	0	8
20040602-16:03	0	25	-75	0	10
20040517-12:21	0	50	-50	0	8
20040517-12:21	1	50	-50	0	6
20040517-17:17	0	50	-50	0	8
20040517-17:17	1	50	-50	0	8
20031211-18:23	0	50	0	0	14
20031212-10:45	0	50	0	0	14
20040519-15:53	0	50	0	0	8
20040519-15:53	1	50	0	0	10
20050414-08:55	0	50	0	0	10
20050414-08:55	1	50	0	0	10
20050414-13:17	0	50	0	0	10
20050414-13:17	1	50	0	0	10
20041130-17:41	0	50	0	1	10
20041130-17:41	1	50	0	1	10
20041201-14:09	0	50	0	1	10
20041201-14:09	1	50	0	1	10
20041201-15:57	0	50	0	1	10
20041201-15:57	1	50	0	1	8

The parameter $\underline{b}$ is the smallest possible bid. In the +treatments $\underline{b} = 0$, otherwise $\underline{b} = \underline{\omega} - 100$. The highest bid that participants could enter was always $\overline{\omega} + 100$.

B Conducting the experiment and instructions

Participants were recruited by email and could register for the experiment on the internet. At the beginning of the experiment participants drew balls from an urn to determine their allocation to seats. Being seated participants then obtained written instructions in German. These instructions very slightly depending on the treatment. In the following we give a translation of the instructions.

After answering control questions on the screen subjects entered the treatment described in the instructions. After completing the treatment they answered a short questionnaire on the screen and where then payed in cash. The experiment was done with the help of z-Tree (Fischbacher (1999)).

B.1 General information

You are participating in a scientific experiment that is sponsored by the Deutsche Forschungsgemeinschaft (German Research Foundation). If you read the following instructions carefully then you can—depending on your decision—gain a considerable amount of money. It is, hence, very important that you read the instructions carefully.

The instructions that you have received are only for your private information. **During the experiment no communication is permitted.** Whenever you have questions, please raise your hand. We will then answer your question at your seat. Not following this rule leads to exclusion from the the experiment and all payments.

During the experiment we are not talking about Euro, but about ECU (Experimental Currency Unit). Your entire income will first be determined in ECU. The total amount of ECU that you have obtained during the experiment will be converted into Euro at the end and payed to you in **cash**. The conversion rate will be shown on your screen at the beginning of the experiment.

B.2 Information regarding the experiment

Today you are participating in an experiment on auctions. The experiment is divided into separate rounds. We will conduct **12 rounds**. In the following we explain what happens in each round.

In each round you bid for an object that is being auctioned. Together with you another participant is also bidding for the same object. Hence, in each round, there are **two bidders**. In each round you will be allocated randomly to another participant for the auction. *Your co-bidder in the auction changes in every round.* The bidder with the highest bid has obtained the object. If bids are the same the object will be allocated randomly.

For the auctioned object you have a valuation in ECU. This valuation lies between x and $x + 50$ ECU[6] and is determined randomly in each round. The range from x to $x + 50$ will be shown to you at the beginning of the experiment on the screen and is the same in each round.[7] **From this range you will obtain in each round new and random valuations for the object.** The other bidder in the auction also has a valuation for the object. The valuation that the other bidder attributes to the object is determined by the same rules as your valuation and changes in each round, too. All possible valuations of the other bidder are also in the interval from x to $x + 50$ from which also your valuations are drawn. All valuations between x and $x+50$ are equally probable. Your valuations and those of the other player are determined independently. You will be told your valuation in each round. *You will not know the valuation of the other bidder.*

[6] In the 0+ and 50+ treatments the valuation would be announced precisely: “This valuation lies between 0 and 50 ECU” in the 0+ treatment and “This valuation lies between 50 and 100 ECU” in the 50+ treatment. Whenever x is mentioned in the remainder of the instruction the same comment applies: In the 0+ and 50+ treatments the valuation is always announced precisely.

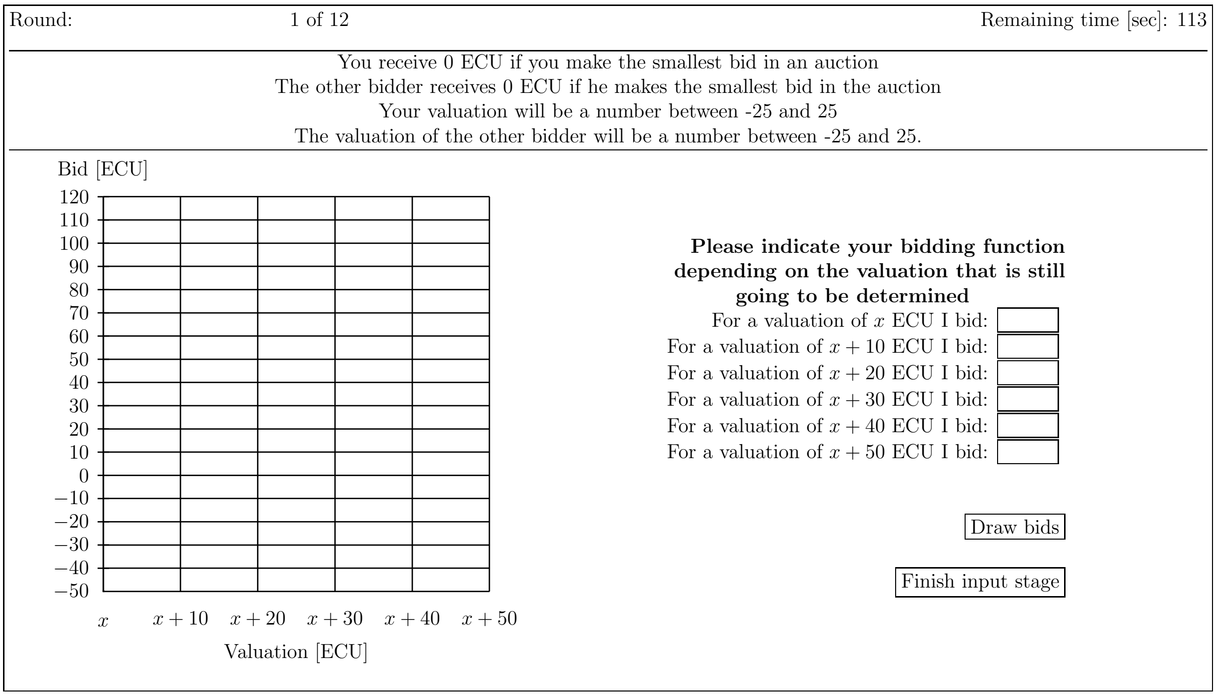
[7] This sentence was not shown in the 0+ and 50+ treatments, though in all treatments the range was shown on the screen.

B.2.1 Experimental procedure

The experimental procedure is the same in each round and will be described in the following. Each round in the experiment has two stages.

1. Stage

In the first stage of the experiment you see the following screen:[8]

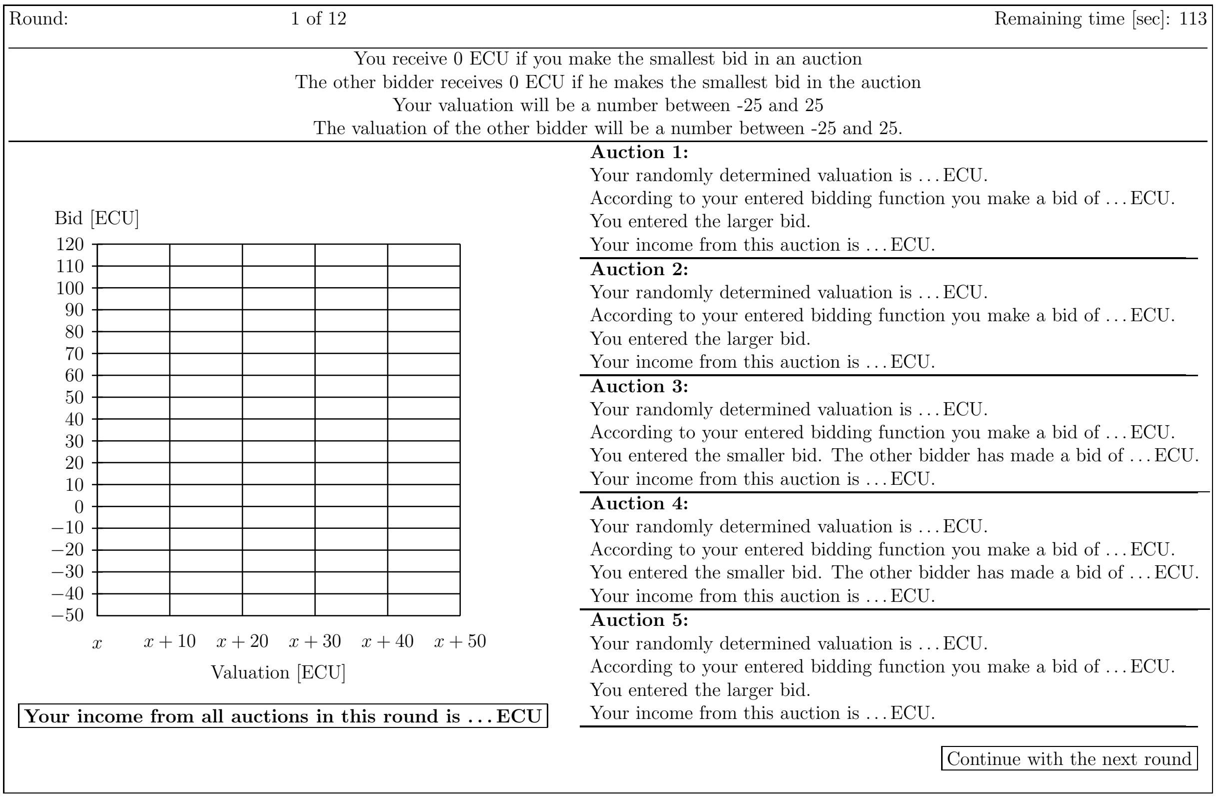


At that stage **you do not know your own valuation for the object in this round.** On the right side of the screen you are asked to enter a bid for **six hypothetical valuations** that you might have for the object. These six hypothetical valuations are x, $x + 10$, $x + 20$, $x + 30$, $x + 40$, and $x + 50$ ECU. Your input into this table will be shown in the graph on the left side of the screen when you click on "draw bids". In the graph the hypothetical valuation is shown on the horizontal axis, the bids are shown on the vertical axis. Your input in the table is shown as six points in the diagram. **Neighbouring points are connected with a line automatically.** These lines determine your bid for all valuations *between* the six points for those you have made an input. For the other bidder the screen in the first stage looks the same and there are as well bids for six hypothetical valuations. The other bidder can not see your input.

[8]In the 0+ and 50+ treatments the interval was already shown exactly in the instructions and consistently also in the figures in the instructions. In the other treatments the interval x to $x + 50$ was, as you see in the figure, described as x to $x + 50$. From the first round of the experiment on the current numbers were given.

2. Stage

The actual auction takes place in the second stage of each round. In each round we will play not only a single auction but **five auctions**. This is done as follows: **Five times a random valuation is determined** that you have for the object. Similarly for the other bidder five random valuations are determined. You see the following screen:[9]



For each of your five valuations the computer determines your bid according to the graph from stage 1. If a valuation is precisely at x, $x + 10$, $x + 20$, $x + 30$, $x + 40$, or $x + 50$ the computer takes the bid that you gave for this valuation. If a valuation is between these points your bid is determined according to the joining line. In the same way the bids of the other bidder are determined for his five valuations. Your bid is compared with the one of the other bidder. The bidder with the higher bid has obtained the object.

Your income from the auction:

For each of the five auctions the following holds:

- The bidder with the higher bid obtains the valuation he had for the object in this auction added to his account minus his bid for the object.

[9] In the instructions the following figure was shown. This figure does not show the bidding function in the graph and the specific bids, gains and losses that would be shown during the experiment.

- If the bidder with the higher bid has a negative valuation for the object, the ECU account is reduced by this amount.[10]
- If the bid of bidder with the higher is a negative number, the amount is added to his ECU account.[11]
- The bidder with the smaller bid obtains **no income** from this auction.

You total income in a round is **the sum of the ECU income from those auctions in this round where you have made the higher bid.**

This ends one round of the experiment and you see in the next round again the input screen from stage 1.

At the end of the experiment your total ECU income from all rounds will be converted into Euro and payed to you in cash together with your Show-Up Fee of 3.00 Euro.

Please raise your hand if you have questions.

[10] This item is not shown in the 0+ and 50+ treatments.
Note that, in order to be able to use same instructions for all treatments we mention the possibility of negative valuations in all, except the 0+ and 50+ treatments, even if subjects learn later that their valuation is drawn from an interval that contains only positive numbers.

[11] This item not shown in the 0+ and 50+ treatments.