

Outside options: Another reason to choose the first-price auction*

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Abstract

In this paper we derive equilibrium bidding functions for first-price and second-price auctions with private values when bidders have outside options. We then study bidding behaviour with the help of experiments.

We find that bidders respond to outside options and to variations of common knowledge about competitors' outside options, though bidders in first-price auctions show more overbidding with outside options than without. In second-price auctions overbidding is not affected by outside options. As expected first-price auctions yield more revenue than second-price auctions. This revenue-premium is higher in the presence of outside options.

1 Introduction

During the last decade, auctions have increasingly attracted attention from academia and the wider public. A major part of this increased interest is due to growing popularity of using auctions as market institutions for C2C and B2C transactions, allocating public resources and procurement contracts. Cases in point are worldwide spectrum auctions, online auction platforms such as eBay and Ricardo and virtual B2B market places, e.g. Covisint for the automotive industry or Consip's AiR for Italian public procurement offers.

Often outside options are available to bidders in addition to the object offered in the particular auction. For the purpose of illustration, suppose somebody has the opportunity to buy a used watch either from a friend at some price or to participate in an online auction where a similar watch is offered. The bid in the auction might depend on the value from seizing the outside option, i.e. from buying the watch from the friend.

In this paper we augment the standard symmetric independent private value (SIPV) model to allow for public and private outside options. We derive equilibrium bidding functions and implement the auction in the laboratory.

To our knowledge, there is no literature on the effects of outside options on bidding behaviour. A model with symmetric independent private valuations and independent outside options can be reduced to a standard SIPV model.¹ One special case is studied by Holt (1980) who assumes

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¹See section 2.3.

that valuations are constant and the same for all bidders. A related case is studied by Weber (1983), Gale and Hausch (1994), and Reiß and Schöndube (2002) who study sequential auction models. A subsequent auction in such a sequential auction process can also be interpreted as a specific outside option. However, in these cases the value of the outside option is endogenous. In our paper we are focusing on the case of an exogenously given value of the outside option.

In our experiments we want to find out the following for the first-price and the second-price auction:

- Do outside options affect bids at all?
- Do bids in the laboratory deviate from equilibrium bids in the same way as they deviate in standard auctions without outside options?
- How are revenue and efficiency affected if outside options are present?

The plan of the paper is as follows: in section 2 we introduce outside options into the SIPV auction model and derive equilibrium bidding strategies for the first-price and second-price auctions, section 3 describes our experimental design, section 4 provides experimental results and section 5 concludes.

2 The Symmetric Independent Private Values auction model with outside options

There are n risk-neutral individuals with single-object demand. Each individual i has a valuation v_i for an object that is for sale in an auction. In addition to the auction offer each individual has access to a transaction alternative that can be substituted for the object offered in the auction. The value that an individual derives from executing the outside option instead of receiving the object auctioned off is denoted by w_i .² We assume here that receiving the auctioned object eliminates the value of the outside option entirely. Individuals may execute their outside options before, during or subsequently to the auction.

We distinguish between public and private outside options. With public outside options, each individual derives the same benefit from the outside option. This is common knowledge. In contrast, private outside options are individual-specific and private information.

We briefly report equilibrium bidding functions for first-price and second-price auction in the SIPV model in section 2.1. In section 2.2 we introduce public outside options. In section 2.3 we extend the SIPV model to allow for private outside options.

2.1 Bidding without outside options

Suppose that individual valuations v_i of the object that is offered in an auction are private information and independently and identically distributed according to a cumulative density function $F(v_i)$ where $v_i \in [\underline{v}, \bar{v}]$. Without outside options, the symmetric Bayes-Nash equilibrium bidding functions for the first- and second-price auctions are well-known (e.g. Riley and Samuelson, 1981, and Vickrey, 1961): for the first-price auction we have $b^{\text{fp}}(v) = v - \int_{\underline{v}}^v F^{n-1}(x)dx / F^{n-1}(v)$ and for the second-price auction $b^{\text{sp}}(v) = v$.

²Valuations of transaction alternatives are net of transaction costs. If, for instance, an alternative object is offered at a posted price, then w_i represents the value of the outside option net of its price. If there are many alternatives, then w_i corresponds to the best alternative net of prices.

2.2 Public outside options

This is the easy case. As in section 2.1 individual i has a valuation v_i for the auctioned object. Individual i could also execute the public outside option and obtain a value which is the same for all individuals and equals w . We assume $w \leq \underline{v}$. This ensures that every individual voluntarily participates in a standard auction.

We will use the payoff equivalence theorem to derive the equilibrium bidding functions for the public outside option case. This can be done in a straightforward way for standard auctions. From the bidder's perspective the auction with public outside options can be interpreted as a standard auction where bidders who fail to win the object receive a payment of w , thus, the application of the payoff equivalence theorem is possible.

Following Riley and Samuelson (1981), let $\Pi(x, v)$ be the expected payoff that a representative bidder with valuation v receives if mimicking valuation type x given that all competitors adhere to the common equilibrium bidding strategy $b^{\text{fp-pv}}(v)$. From payment equivalence without auction reserve price (cf. Riley and Samuelson, 1981, eqs. 7 and 8), we obtain immediately the expected equilibrium payment of the representative bidder with valuation v that depends on the expected equilibrium payoff to the lowest valuation type \underline{v} :

$$P(v) = vF^{n-1}(v) - \int_{\underline{v}}^v F^{n-1}(x)dx - \Pi(\underline{v}, v). \quad (1)$$

For the lowest valuation type to be indifferent between auction participation and not entering the auction, the bidder must receive at least the outside option w in the auction, leading to the condition $\Pi(\underline{v}, \underline{v}) = w$. For the first-price design with the modification that each bidder receives w if unsuccessful in the auction, the expected equilibrium payment is given by

$$P(v) = F^{n-1}(v) b^{\text{fp-pb}}(v) - [1 - F^{n-1}(v)] w. \quad (2)$$

Combining both expressions for expected equilibrium payment, (1) and (2), and solving for $b^{\text{fp-pb}}(v)$ leads to the intuitive result that the equilibrium bid under the first-price design with public outside option precisely matches that without public outside options reduced by the outside option's value:

$$b^{\text{fp-pb}}(v, w) = b^{\text{fp}}(v) - w \quad (3)$$

where $b^{\text{fp}}(v)$ is the equilibrium bidding function in the first-price auction without outside options defined in the preceding subsection.

The weakly dominant bidding strategy³ for the second-price auction is

$$b^{\text{sp-pb}}(v, w) = v - w. \quad (4)$$

Without outside options the above bidding functions always imply efficient allocations in equilibrium. Public outside options do not destroy this property.

2.3 Private outside options

The valuation of individual i 's is, again, v_i . The value from executing the outside option is w_i . Valuation pairs $(v, w) \in [\underline{v}, \bar{v}] \times [\underline{w}, \bar{w}]$ are independently distributed across individuals according to the probability density function $f(v, w)$ and are their private information. Again we assume that the lowest valuation is not lower than any outside option, i.e. $\underline{v} \geq \bar{w}$ ensuring that each individual submits a bid in the auction.

³The proof is identical to that for the equilibrium bidding strategy under the second-price auction with private outside options, see section 2.3.

Equilibrium bidding: first-price auction In order to derive the equilibrium bidding strategy in the first-price auction, we represent the bidding model such that we can solve it with standard procedures. Consider the utility maximisation problem of the representative risk-neutral individual i that submits bid b_i in the auction and faces the outside option w_i :

$$\max_{b_i} \Pr(b_i \text{ wins}) \cdot (v_i - b_i) + [1 - \Pr(b_i \text{ wins})] \cdot w_i \quad (5)$$

This program can be rearranged to

$$\max_{b_i} \Pr(b_i \text{ wins}) \cdot (v_i - w_i - b_i) + w_i \quad (6)$$

Since the outside option w_i is known to the individual and a constant, the argmax of that problem is the same as the one of the following problem where the new variable $x_i := v_i - w_i$ is introduced:

$$\max_{b_i} \Pr(b_i \text{ wins}) \cdot (x_i - b_i) \quad (7)$$

We interpret $x \in [v - \bar{w}, \bar{v} - w]$ as an individual's net valuation of the object. One way to interpret the transformation of the original maximisation problem into (7) is to suppose that the representative individual executes the outside option w_i before bidding in the auction, and, in case the auction is won (since we have single-object-demand) repays the outside option to the price of the auctioned object.

By this simple transformation we obtain a standard bidding problem in net valuations x . All there remains to do is to identify the probability density function that governs the distribution of net valuations. Note that infinitely many valuation pairs (v, w) lead to the identical net valuation \bar{x} . The probability density function of a given net valuation x is obtained by summing up all densities over the corresponding iso-net-valuation curve as follows

$$f_X(x) = \int_{\max\{w, v-x\}}^{\min\{\bar{w}, \bar{v}-x\}} f(x+w, w) dw. \quad (8)$$

Given a well-defined density function $f_X(x)$ with support $[v - \bar{w}, \bar{v} - w]$ and cumulative density function $F_X(x)$, we can invoke standard results to derive the equilibrium bidding function (e.g. Riley and Samuelson, 1981) assuming that there is no reserve price in the auction:

$$b^{\text{fp-pr}}(x) = x - \frac{\int_{v-\bar{w}}^x F_X^{n-1}(y) dy}{F_X^{n-1}(x)}. \quad (9)$$

By resubstitution, the equilibrium bidding function for our model is:

$$b^{\text{fp-pr}}(v, w) = v - w - \frac{\int_{v-\bar{w}}^{v-w} F_X^{n-1}(y) dy}{F_X^{n-1}(v-w)} \quad (10)$$

which is strictly increasing in v and strictly decreasing in w since $\partial b / \partial x > 0$ and $\partial x / \partial v = -\partial x / \partial w = 1$. We give an example for an equilibrium bidding function in appendix A.1.

Equilibrium bidding: second-price auction For the second-price auction without outside-options it can be shown with a standard argument that bidding ones own valuation in the auction is a weakly dominant strategy for each bidder. Following a similar argument one also finds that bidding ones own net-valuation $x_i = v_i - w_i$ is a weakly dominant strategy in the second-price auction with outside options.

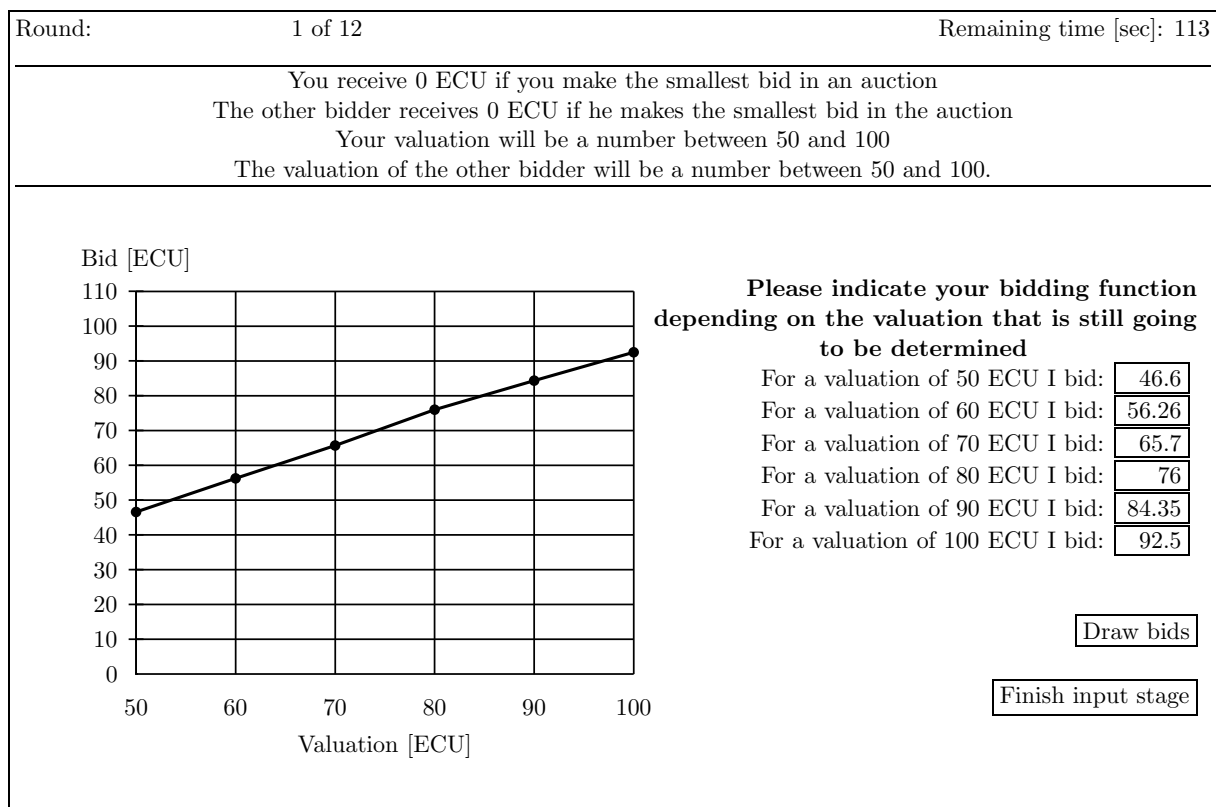


FIGURE 1: A typical input screen in the experiment (treatment A1, translated into English)

Efficiency with private outside options If all bidders follow the bidding function given by equation 9 then the object is allocated to the bidder with the highest net valuation. One can see easily that this leads to an efficient allocation.

3 Experimental design and procedures

To test the theoretical implications of public and private outside options in the SIPV model, we use five treatments in a between-subjects design. We varied the type of outside options (none/public/private) and the auction design (first-price/second-price). Appendix A.2 summarises the treatment parameters used for each of the 19 experimental sessions. In total there were 340 subjects. Each session required less than 90 minutes. The experiment was programmed and conducted with the software *z-Tree* (Fischbacher, 1999).

Treatment types: We distinguish between three types of treatments, A, B, and C. In the baseline treatment, A, we ran standard auction games without outside options under the independent private values assumption with two bidders where the highest bid wins. Valuations were randomly drawn from a uniform distribution with support $[50,100]$ during the experiment and rounded to the second decimal place. The baseline treatment A used either a first-price sealed-bid auction design (treatment A1) or a second-price sealed-bid auction (treatment A2). There were neither minimum bids nor entry fees. Bids and valuations were denominated in experimental currency units (ECU). Each experiment session had twelve auction rounds in the strangers-matching design such that no subject was matched with the same subject in two consecutive rounds.

Strategy method: In standard auction experiments bidders first learn their payoff-relevant valuation before they submit a bid. In our experiment we used the strategy method to elicit a subject’s continuous bidding function. Figure 1 shows a typical input screen. We asked for bids for six hypothetical valuations 50,60,70,80,90 and 100. Bids for intermediate valuations were determined by linear interpolation. The interpolated bidding function was graphically displayed at all times. Subjects were free to adjust their set of entered bids and thereby their specified bid functions as often as they wished. Bids were entered via keyboard and had to be nonnegative, not larger than 200, and had to have not more than two decimal places.

Related implementations of the strategy method have been used in a few other experiments. Selten and Buchta (1999) introduced the strategy method for auction experiments. There subjects could specify a piecewise linear bidding functions with up to 10 000 segments either using a graphical input mode or via keyboard (valuations ranged between 0.00 and 100.00). Their implementation suffered from the problem that 46% of observed bidding functions were non-monotonic—subjects might have been drawing interesting landscapes rather than bidding functions (p. 81). Pezanis-Christou and Sadrieh (2004) used a simplified version of the Selten and Buchta (1999) implementation where subjects could specify two segments which allowed for a single kink in the bidding schedule. In their study, approximately 15% of bidding functions are non-monotonic in their asymmetric auction treatments and approximately 5% in their symmetric auction treatments.⁴ In the experimental studies of Güth et al. (2002, 2003) the set of possible valuations was restricted to eleven values. For each of these eleven valuations, subjects had to enter a corresponding bid. In our experiment about 10.15% of all bids are not monotonic.⁵ A characteristic common to all implementations including ours is that subjects were required to submit their bidding schedules before their valuations were drawn.

Multiple feedback: A distinctive feature of our design is that pairs of matched subjects participated in five unrelated auctions after specification of their bidding schedules instead of one single auction. For each of these five auctions and for each of the subjects, a valuation was independently drawn from a uniform distribution with support [50,100]. In each of the five auctions, subjects’ bids were determined according to their specified bidding functions and their valuations. This design feature was included to decrease the random component in income determination and to increase its strategic component. This supports the use of theoretical predictions based on risk-neutrality. In addition, it allows to better motivate the elicitation of entire bidding functions for subjects.

Subjects were informed about their five valuations, their submitted bids, whether they won the object, the price of the object, and their income in all five auctions. They were also informed about their income for each round which was the sum of the income in the five auctions. No information about competitor’s valuations and incomes was revealed. Since in second-price auctions the competitor’s bid determines the price this bid was revealed in these auctions.

Outside options: Treatments B and C constitute modifications of treatment A which allow for an outside option. The outside option was implemented as an exogenous income for the bidder who did not win the auction. The values of these outside options were drawn from a uniform distribution with support [0, 50] and held constant for four consecutive auction rounds. The value of the outside option was announced to each individual bidder before they specified their bidding functions. Treatments B and C differed in the amount of information that subjects had about their competitors’ outside options. In treatment B the outside option was public information and the same for both bidders. In treatments C outside options were drawn independently

⁴The percentages are inferred from bar charts in Pezanis-Christou and Sadrieh (2004), figures 3 and 5.

⁵See appendix B.

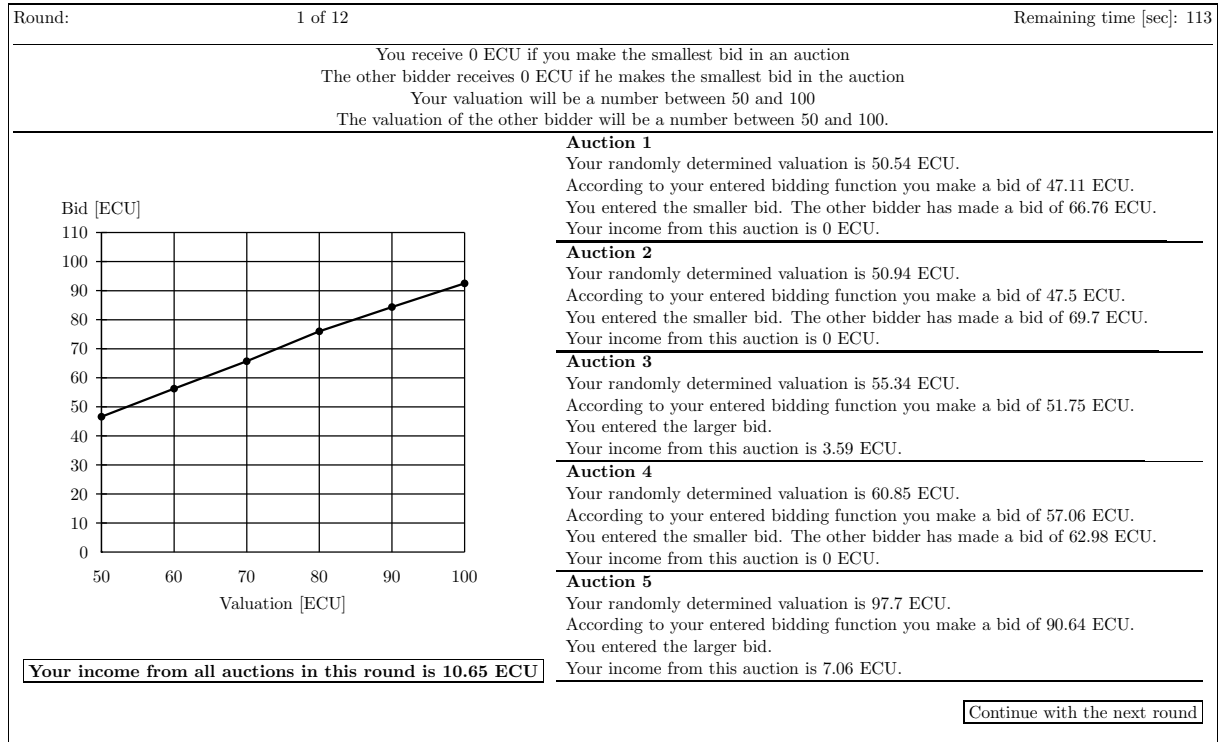


FIGURE 2: A typical feedback screen in the experiment (treatment A1, translated into English)

TABLE 1: Number of independent observations and subjects

type of outside options	first-price auction	second-price auction
none	A1: 8 (86 subjects)	A2: 6 (58 subjects)
public	B1: 6 (52 subjects)	–
private	C1: 8 (72 subjects)	C2: 8 (72 subjects)

for each bidder from a uniform distribution with support $[0, 50]$. This procedure was common knowledge, though the individual values were private information.

Treatments B1 and C1 use the first-price auction, treatment C2 uses the second-price auction. Table 1 summarises the number of independent observations and the number of subjects by treatment.

Procedures: At the beginning of each experimental session, subjects read written instructions, then they took a brief treatment-specific computerised quiz to ensure their familiarity with the instructions and the experiment. Then subjects went through twelve rounds of the actual experiment. At the end of the last auction round, subjects completed a brief computerised questionnaire and received their earned income in cash. To compensate for the unavailable outside option in treatments A1 and A2 subjects received in these treatments an additional payment of 3 Euro.

Equilibrium predictions: Table 3 summarises the equilibrium bidding strategies for the different treatments. Figures 3 and 4 below illustrate equilibrium bids.

treatment	Bayes-Nash equilibrium prediction	bid range
A1	$b^{A1}(v) = 25 + \frac{v}{2}$	$b^{A1} \in [50, 75]$
B1	$b^{B1}(v, w) = 25 + \frac{v}{2} - w$	$b^{B1} \in [0, 75]$
C1	$b^{C1}(x) = \begin{cases} \frac{2}{3}x & \text{if } x \in [0, 50] \\ \frac{100x^2 - 2/3x^3 - 250000/3}{200x - x^2 - 5000} & \text{if } x \in [50, 100] \end{cases}$ where $x \equiv v - w$	$b^{C1} \in [0, 50]$
A2	$b^{A2}(v) = v$	$b^{A2} \in [50, 100]$
C2	$b^{C2}(v, w) = x$, where $x \equiv v - w$	$b^{C2} \in [0, 100]$

TABLE 2: Equilibrium bidding predictions

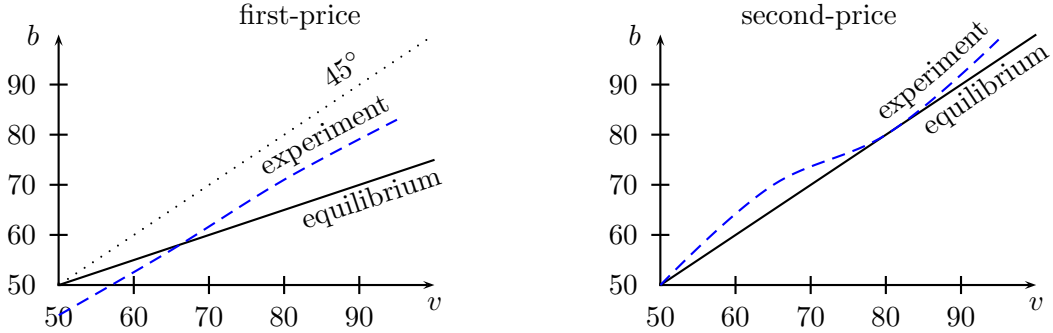


FIGURE 3: Median bids in treatments without outside options (A1 and A2)

4 Experimental results

4.1 Bids

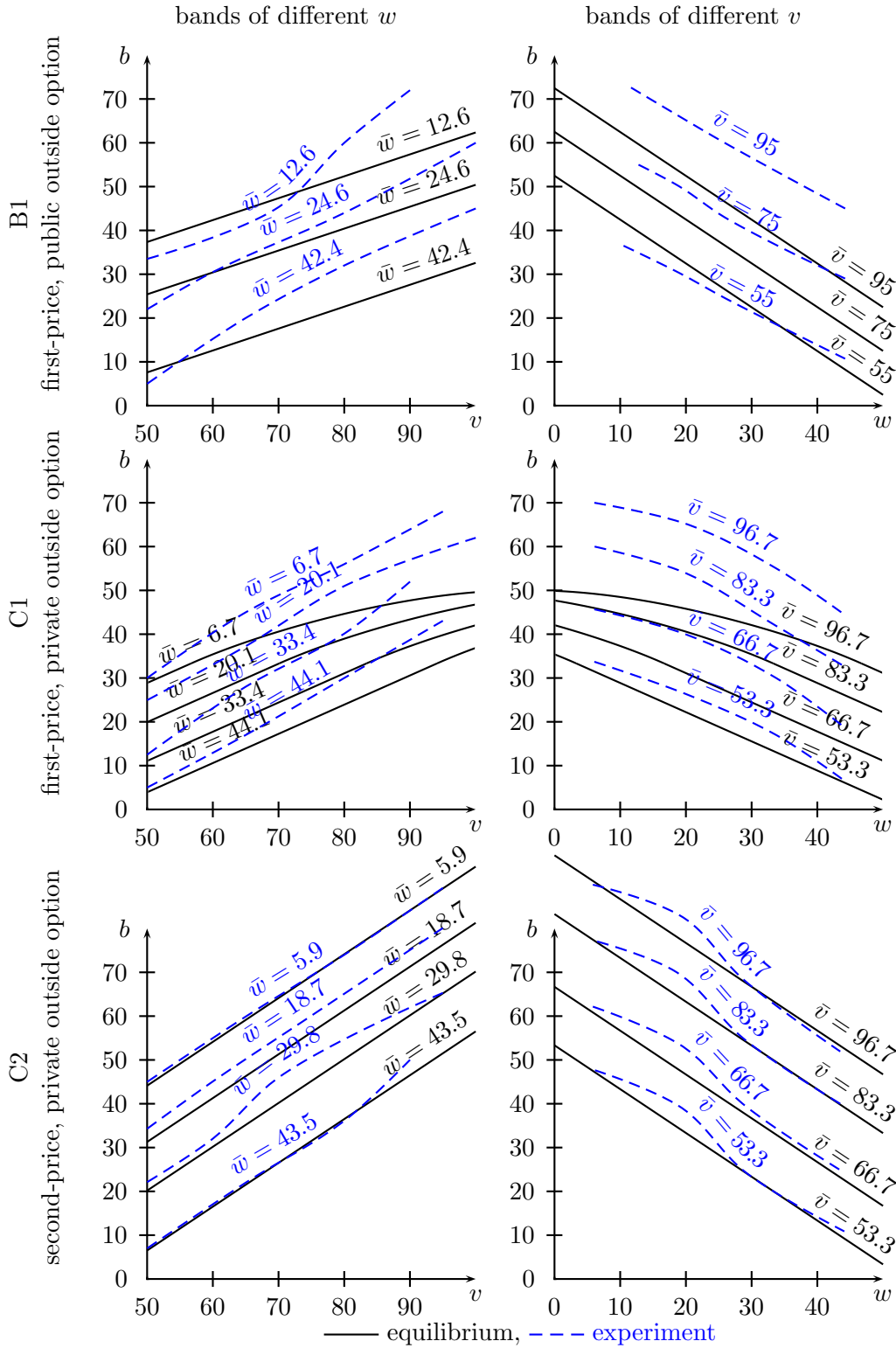
Median bids from the treatments without outside options (A1 and A2) are shown in figure 3. The equilibrium bid is shown as a solid line, median bids in the experiment are shown as dashed curves in the figure. We make the standard observation: There is a substantial amount of overbidding in the first-price auction and a smaller amount of overbidding in the second-price auction.

Figure 4 shows median bids from the treatments with outside options (B1, C1, and C2).⁶ Since in these treatments bids depend on two parameters, the valuation v and the outside option w , we use a different way to represent bids than in figure 3: The left graphs show v on the horizontal axis and use different bands of w to show bidding. In the right graphs we show w on the horizontal axis and use different bands of v to show bidding.⁷ Solid lines show the equilibrium bid for the mean of the parameter, dashed lines show median splines through bids in the lab for all parameter values in the corresponding band.

We see that for all three treatments shown in the figure the data is qualitatively in line with the equilibrium bidding functions. Bids increase with the value of the valuation v and bids decrease with the value of the outside option w .

⁶Since A1 is a special case of B1 (with the outside option $w = 0$) we include the A1 treatments in the graphs for B1.

⁷The bands are constructed in the following way: The empirical distribution of the parameter is divided into quantiles of equal size. We use three quantiles in the B1 treatment (since the number of realisations of w is small in the B1 treatment, $w \in \{0.00, 10.51, 12.79, 14.59, 23.49, 25.82, 25.96, 37.28, 44.24, 46.07\}$), and we use four quantiles in the C1 and C2 treatment. The mean value of the parameter is shown next to each curve.



To show how bids depend on v and w we show two diagrams for each treatment. In each diagram one parameter is shown on the horizontal axis and the other parameter is shown through different bands. In the left diagram the parameter on the axis is v , in the right diagram this is w . Different values of the other parameter (w on the left, v on the right) are shown as curves for different bands (see footnote 7). Means of this parameter value are shown next to the curve. Since A1 is a special case of B1 the A1 data is included in the B1 diagram.

FIGURE 4: Median bids in treatments with outside options

treatment	equil. value	expl. var.	coeff. $\hat{\beta}$	robust σ_{β}	t	$P > t $
A1	0	constant	-0.116	1.801	-0.06	0.951
	$\frac{1}{2}$	v	0.871	0.021	41.64	0.000
B1	0	constant	1.872	1.142	1.64	0.162
	$\frac{1}{2}$	v	0.797	0.012	64.67	0.000
	-1	w	-0.767	0.025	-30.14	0.000
A1+B1	0	constant	1.631	1.385	1.18	0.260
	$\frac{1}{2}$	v	0.843	0.017	49.37	0.000
	-1	w	-0.867	0.027	-31.97	0.000

TABLE 3: Estimation of equation (11) for the A1 and B1 case.

Treatment B1: public outside options in the first-price auction: To gain a better understanding of bidding behaviour we regress bids b on valuations v and outside options w following equation (11) for treatments A1 and B1 separately and jointly.

$$b = \beta_v v + \beta_w w + \beta_0 + u \quad (11)$$

Since $w = 0$ in treatment A1 we do not estimate β_w in this treatment. Calculations of standard errors take into account that observations might be correlated within matching groups but not across matching groups; Rogers, 1993). Regression results are summarised in table 4.1.

We find that the estimated coefficient of the public outside option value w is significantly different from zero (two-sided t -test, $p = 0.000$), but with $\hat{\beta}_w = -0.77$ significantly smaller than the equilibrium prediction $\beta_w = -1$ (two-sided t -test, $p = 0.000$). Also the coefficient of the valuation v is, as found in several other studies, larger in the experiment than in equilibrium ($p = 0.000$).

The fact that bidders do not fully exploit their outside option is interesting in light of the debate on the declining price anomaly.⁸ If this exploitation failure arises with endogenous outside options, too, then one might expect a series of falling prices as observed in the field since bids in the early auctions would be too high.

Treatment C1: Private outside options in first-price auctions In treatment C1 the equilibrium bidding function is not linear. We, thus, estimate equation (12)

$$b = \beta_v v + \beta_w w + \beta_{\Delta} \Delta + \beta_0 + u \quad (12)$$

where Δ is defined as

$$\Delta \equiv b^{C1}(v - w) - \frac{2}{3}(v - w). \quad (13)$$

Thus, in the linear part of the bidding function Δ is zero. For values of v and w where the equilibrium bidding function from table 3 becomes nonlinear the value of Δ captures the difference. Table 4 summarises the regression results. We see that estimates for β_v and β_w are larger in absolute terms than they should in equilibrium. β_v is actually significantly larger ($F_{1,7} = 15.47$, $p = 0.0057$), while the deviation of β_w is not significant ($F_{1,7} = 2.77$, $p = 0.1402$). The coefficient β_{Δ} has the correct sign, i.e. participants in the experiment may pick up the nonlinearity. However, β_{Δ} is significantly smaller than its equilibrium value ($F_{1,7} = 45.53$, $p = 0.0003$). β_v is larger in absolute terms than β_w , though not significantly so ($F_{1,7} = 2.82$, $p = 0.1370$).

⁸See Ashenfelter and Graddy (2003) and the references therein. For the theoretical reference solution, see Weber (1983); for an experimental study that reproduces this phenomenon in the laboratory, see Keser and Olson (1996).

treatment	equil. value	expl. var.	coeff. $\hat{\beta}$	robust σ_β	t	$P > t $
C1	2/3	v	.8055877	.0353191	22.81	0.000
	-2/3	w	-.7201958	.0321798	-22.38	0.000
	1	Δ	.2213714	.1153977	1.92	0.097
	0	constant	-1.658815	2.963631	-0.56	0.593

TABLE 4: Estimation of equation (12) for the C1 case.

treatment	equil. value	expl. var.	coeff. $\hat{\beta}$	robust σ_β	t	$P > t $
A2	1	v	.9703532	.0152585	63.59	0.000
	0	constant	4.732475	1.285897	3.68	0.014
C2	1	v	.92333	.0284033	32.51	0.000
	-1	w	-.970783	.0574175	-16.91	0.000
	0	constant	8.84315	2.635614	3.36	0.012

TABLE 5: Estimation of equation (11) for the C2 case.

The regression results confirm what we see in the middle section of figure 4. In the first-price auction with private outside options bidders react slightly too sensitively to their own valuation v . This is to be expected from other first-price auctions without outside options and this leads to some overbidding. Bidders also reduce their bids according to their outside options almost as they should in equilibrium. In addition to the so far ‘standard’ overbidding, bidders also forget to shade their bids when v is large and w is small. The nonlinearity in the equilibrium bidding function which lowers equilibrium bids is not reflected in the experimental bidding function.

Treatment A2 and C2: Second-price auctions For the second-price auction we can again estimate equation (11). Results are reported in table 5. The β_v coefficients in both treatments are not significantly different from each other ($F_{1,13} = 2.28$, $p = 0.1549$). Also, coefficients are close to equilibrium values, though in the C2 treatment β_v is significantly smaller than β_w ($F_{1,7} = 7.29$, $p = 0.0307$).

Most importantly, the estimated coefficient for the outside option value β_w does not significantly differ from the equilibrium value ($F_{1,7} = 0.26$, $p = 0.6265$). It appears that subjects on average fully exploit their outside option by decreasing their bid by precisely the outside option value in second-price auctions, although they failed to do so in first-price auctions. Recall that in treatment B1, subjects decreased bids only by 0.77 per unit of outside option value instead of 1.00.

Outside options increase overbidding in first-price auctions To assess the impact of outside options on overbidding we estimate the following equation for the first-price treatments:

$$b = \beta_A b_A^{eq} + \beta_B b_B^{eq} + \beta_C \frac{2}{3}(v - w) + \beta_\Delta \Delta + u \quad (14)$$

b_A^{eq} is the equilibrium bid in the first-price auction without outside options and zero otherwise. b_B^{eq} is the equilibrium bid in the first-price auction with public outside options and zero otherwise. The term $2/3(v - w)$ is the linear part of the equilibrium bid in the first-price auction with private outside options and zero in the case without outside options. Δ is the nonlinear part of the bidding function with private outside options as defined in equation (13) and zero in the case without outside options. Results are given in table 6.

equil. value	expl. var.	coeff. $\hat{\beta}$	robust σ_β	t	$P > t $
1	β_A	1.056127	.0093481	112.98	0.000
1	β_B	1.129079	.0319034	35.39	0.000
1	β_C	1.231377	.0232246	53.02	0.000
1	β_Δ	.4675264	.1490796	3.14	0.005

TABLE 6: Estimating equation (14)

equil. value	expl. var.	coeff. $\hat{\beta}$	robust σ_β	t	$P > t $
1	β_A	1.030342	.0066221	155.59	0.000
1	β_C	1.053819	.0201858	52.21	0.000

TABLE 7: Estimating equation (15)

As expected we see overbidding in the standard case without outside options. The coefficient β_A is larger than one and significantly so.⁹ But once public outside options are introduced we observe more overbidding: The coefficient β_B is significantly larger than β_A .¹⁰ Introducing private outside options leads to even more overbidding than with public outside options: The coefficient β_C is significantly larger than β_B .¹¹ Not only bidders in the case with private outside options bid more than without outside options, furthermore they fail to correct for the concavity of the bidding function: The coefficient β_Δ is significantly smaller than one.¹² Since Δ is always negative a too small β_Δ means more overbidding. It appears that the subtle difference in common knowledge stemming from different types of outside options affects bidding behaviour.¹³

When we repeat this exercise for second-price auctions we find that these are less affected by outside options. Table 7 shows estimates of equation (15) for the second-price treatments.

$$b = \beta_A b_A^{eq} + \beta_C b_C^{eq} + u \quad (15)$$

We see a small but significant amount of overbidding also in the second-price auction.¹⁴ Introducing outside options in the second-price auction does, however, not further increase overbidding.¹⁵

Outside options boost revenue-dominance of first-price auction Based on the revenue equivalence theorem which also applies to the case of outside options the expected revenue for a first-price auction equals that for a second-price auction. Since the experimental study of Cox et al. (1982) it is well known that the first-price design generates larger revenues than the second-price auction in the absence of outside options. Table 8 shows for the different treatments the difference between the expected revenue given the bidding functions in the lab and the expected revenue given equilibrium bidding functions.¹⁶ Indeed, first-price auctions obtain a higher revenue than the second-price auctions in our experiments. The difference is

⁹ $F_{1,21} = 36.05, p = 0.0000.$

¹⁰ $F_{1,21} = 4.82, p = 0.0396.$

¹¹ $F_{1,21} = 6.72, p = 0.0170.$

¹² $F_{1,21} = 12.76, p = 0.0018.$

¹³Güth and Ivanova-Stenzel (2003) report that the manipulation of common knowledge in asymmetric private value auctions (competitor's valuation distribution is know/unknown) "changes behaviour only slightly and hardly ever in significant ways." (p. 198f.)

¹⁴The coefficient β_A is significantly larger than 1 ($F_{1,13} = 20.99, p = 0.0005$).

¹⁵The coefficient β_B is not significantly different from β_A ($F_{1,13} = 1.22, p = 0.2892$).

¹⁶We determine the expected revenue by evaluating the bidding function for each participant and each period 100 times for a random valuation and matching the bidder as in the experiment.

	treatment		excess revenue	robust $\hat{\sigma}$	t	$P > t $
A2	second-price	no outside option	1.519887	.3717887	4.09	0.000
C2	second-price	private outside option	1.31426	.9255003	1.42	0.164
A1	first-price	no outside option	7.185551	.5245006	13.70	0.000
B1	first-price	public outside option	10.82654	.6744602	16.05	0.000
C1	first-price	private outside option	12.36949	.4828499	25.62	0.000

The table shows average excess revenue (the difference between expected revenue in the lab and the expected revenue with equilibrium bids).

TABLE 8: Average excess revenue

	treatment		relative frequency of efficient allocations	robust $\hat{\sigma}$	95% confidence interval	
A2	second-price	no outside option	.875546	.0175144	.8399898	.9111022
C2	second-price	private outside option	.8252546	.0205445	.7835471	.8669622
A1	first-price	no outside option	.855562	.0071832	.8409793	.8701448
B1	first-price	public outside option	.8065385	.0111459	.7839112	.8291658
C1	first-price	private outside option	.8360185	.0153102	.8049371	.8671

TABLE 9: Relative frequency of efficient allocations

significant in the no-outside-option treatments A1 and A2 ¹⁷ and also significant in the private-outside-option treatments C1 and C2 ¹⁸. Most importantly, the difference in revenue between the first-price and the second-price auction increases when outside options are introduced.¹⁹

Efficiency In our auction we call an allocation efficient if the object is obtained by the bidder with the highest net valuation. Since in all treatments bidding functions are monotonic and the same for all bidders in the net valuation we have always an efficient allocation in equilibrium. Before we did our experiment we suspected that with outside options the situation is more complex, hence, we would find more inefficient allocations. This, however, does not seem to be the case. Table 9 shows relative frequencies of efficient allocations for the different treatments. We see that differences in efficiency are small and also not significant.

5 Conclusion

We have introduced a bidding model that allows for public and private outside option and experimentally tested its properties. A key feature of our experimental design is that we collected entire bidding functions. We find that, in line with the theoretical prediction, higher-valued outside options lead to less aggressive bidding (ie. lower bids) than in the first-price and second-price auction model without outside options.

In contrast to theoretical revenue equivalence of first-price and second-price auction, our laboratory analysis shows that the first-price auction generates larger revenue than the second-price auction. Importantly, outside options magnify significantly the revenue-premium of the

¹⁷ $F_{1,35} = 77.66, p = 0.0000.$

¹⁸ $F_{1,35} = 112.16, p = 0.0000.$

¹⁹In the no-outside option treatment the revenue premium of the first-price auction is $7.19 - 1.52 = 5.67$ ECU while the introduction of private outside options strongly increase it to $12.37 - 1.31 = 11.06$ ECU. The difference of these revenue premia is significant, $F_{1,35} = 19.33, p = 0.0001.$

first-price auction since overbidding in first-price auctions is more prominent with outside options than without. There is no such effect for second-price auctions. In the private outside option case overbidding is further increased since bidders do not take fully into account the concavity of the bidding function.

Taken together, our analysis suggests that outside options crucially influence bidding behaviour in a way that is qualitatively predicted by theory and that the particular nature of outside options matters. However, actual bidding behaviour seems to deviate from the predictions in important ways.

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A Appendix

A.1 Private outside options, first-price bidding function

In this section we give an example for the equilibrium bidding function for the first-price auction in the case with private outside options. Suppose $(v, w) \in [50, 100] \times [0, 50]$ with $f(v, w) = 1/2500$ and $n = 2$. It follows that net valuations $x \in [0, 100]$. This is the parameterisation that we use in our experiment. The cumulative density function of $X \equiv V - W$ is given by

$$F_X(x) = \begin{cases} \frac{x^2}{5000} & \text{if } x \in [0, 50] \\ \frac{200x - x^2 - 5000}{5000} & \text{if } x \in [50, 100]. \end{cases} \quad (16)$$

As a result, the symmetric Bayes-Nash equilibrium bidding function is given by

$$b^{\text{fp-pr}}(x) = \begin{cases} \frac{2}{3}x & \text{if } x \in [0, 50] \\ \frac{300x^2 - 2x^3 - 250000}{600x - 3x^2 - 15000} & \text{if } x \in [50, 100]. \end{cases} \quad (17)$$

The bidding function $b^{\text{fp-pr}}(x)$ is continuous at $x = 50$. The bidding function $b^{\text{fp-pr}}(v, w)$ can be obtained by resubstitution:

$$b^{\text{fp-pr}}(v, w) = \begin{cases} \frac{2}{3}(v-w) & \text{if } (v-w) \in [0, 50] \\ \frac{300(v-w)^2 - 2(v-w)^3 - 250\,000}{600(v-w) - 3(v-w)^2 - 15\,000} & \text{if } (v-w) \in [50, 100]. \end{cases} \quad (18)$$

A.2 List of experimental sessions

Seventeen sessions were conducted at the experimental laboratory at the SFB 504 at the University of Mannheim in 2003 and 2004; two sessions were conducted at the MaXLab at the University of Magdeburg in April 2005.

Date	Treatment	outside option	auction	ECU/Euro	participants
20031211-18:23	A1	none	first price	25	14
20031212-10:45	A1	none	first price	25	14
20040519-15:53	A1	none	first price	35	18
20050414-08:55	A1	none	first price	35	20
20050414-13:17	A1	none	first price	35	20
20031212-14:23	B1	public	first price	120	18
20031212-15:53	B1	public	first price	120	16
20031212-17:33	B1	public	first price	120	18
20031211-10:19	C1	private	first price	150	16
20031211-12:31	C1	private	first price	150	18
20031211-14:25	C1	private	first price	150	20
20031211-16:17	C1	private	first price	150	18
20041130-17:41	A2	none	second price	35	20
20041201-14:09	A2	none	second price	35	20
20041201-15:57	A2	none	second price	35	18
20041129-10:27	C2	private	second price	150	20
20041129-15:29	C2	private	second price	175	16
20041130-10:43	C2	private	second price	175	18
20041130-15:59	C2	private	second price	175	18

B Monotonicity of observed bidding functions

	A1	B1	C1	A2	C2	all
strictly decreasing	0.0%	0.0%	0.1%	0.0%	0.0%	0.02%
weakly decreasing	0.0%	0.1%	0.5%	0.0%	0.1%	0.15%
constant	0.2%	3.1%	2.7%	0.4%	1.0%	1.37%
weakly increasing	6.7%	3.9%	6.4%	4.2%	3.9%	5.17%
strictly increasing	82.7%	84.1%	80.9%	86.8%	82.3%	83.14%
nonmonotonic	10.5%	8.8%	10.0%	8.6%	12.6%	10.15%
total number	1 032	624	864	696	864	4 080

Every bidding function is classified only once. A bidding function is classified as "weakly increasing" if it had at least one horizontal segment but less than five.

C Conducting the experiment and instructions

Participants were recruited by email and could register for the experiment on the internet. At the beginning of the experiment participants drew balls from an urn to determine their allocation to seats. Being seated participants then obtained written instructions in German. These

instructions vary slightly depending on the treatment. In the following we give a translation of the instructions.

After answering control questions on the screen subjects entered the treatment described in the instructions. After completing the treatment they answered a short questionnaire on the screen and where then payed in cash. The experiment was done with the help of z-Tree Fischbacher (1999).

C.1 General information

You are participating in a scientific experiment that is sponsored by the Deutsche Forschungsgemeinschaft (German Research Foundation). If you read the following instructions carefully then you can — depending on your decision — gain a considerable amount of money. It is, hence, very important that you read the instructions carefully.

The instructions that you have received are only for your private information. **During the experiment no communication is permitted.** Whenever you have questions, please raise your hand. We will then answer your question at your seat. Not following this rule leads to exclusion from the the experiment and all payments.

During the experiment we are not talking about Euro, but about ECU (Experimental Currency Unit). Your entire income will first be determined in ECU. The total amount of ECU that you have obtained during the experiment will be converted into Euro at the end and payed to you in **cash**. The conversion rate will be shown on your screen at the beginning of the experiment.

C.2 Information regarding the experiment.

Today you are participating in an experiment on auctions. The experiment is divided into separate rounds. We will conduct **12 rounds**. In the following we explain what happens in each round.

In each round you bid for an object that is being auctioned. Together with you another participant is also bidding for the same object. Hence, in each round, there are **two bidders**. In each round you will be allocated randomly to another participant for the auction. *Your co-bidder in the auction changes in every round.* {The following sentence was only included in the instructions for A1, B1, and C1: The bidder with the highest bid obtains the object. If bids are the same the object will be allocated randomly.}

{The following paragraph was only included in the instructions for A2 and C2:

In each round you submit your maximum bid for an object that is auctioned. The maximum bid is the largest amount that you want to pay for the object. Your co-bidder submits his maximum bid for the object at the same time as you do. In the auction, the price for the object will be increased in steps of 0.01 ECU. As soon as the price matches one of both maximum bids (either yours or that of your co-bidder), the corresponding player stops bidding in the auction. The bidder who remains alone in the auction obtains the object. **Thus, the bidder with the higher maximum bid obtains the object. The price is equal to the lower maximum bid.** In case you or your co-bidder stop bidding at the same time, the bidder who obtains the object is randomly determined.}

For the auctioned object you have a valuation in ECU. This valuation lies between 50 and 100 ECU and is randomly determined in each round. **From this range you will obtain in each round new and random valuations for the object.** The other bidder in the auction also has a valuation for the object. The valuation that the other bidder attributes to the object is determined by the same rules as your valuation and changes in each round, too. All possible valuations of the other bidder are also in the interval from 50 to 100 from which also your valuations are drawn. All valuations between 50 and 100 are equally probable. Your valuations and those of the other player are determined independently. You will be told your valuation in each round. *You will not know the valuation of the other bidder.*

[The following three paragraphs were only inserted in the instructions for treatments B1, C1, and C2:

The auction income is calculated as follows:

- {B1 and C1: The bidder with the higher bid obtains the valuation he had for the object in this auction added to his account minus his bid for the object.} {C2: The bidder who remains alone in the auction obtains the valuation he had for the object in this auction added to his account minus the price of the object. The price is given by the smaller one of both maximum bids, ie. the price at which one of the bidders stops bidding.}
- {B1: The bidder with the smaller bid obtains a payment that both bidders are} {C1: The bidder with the smaller bid obtains a randomly determined payment that he is} {C2: The bidder that first stops bidding in the auction obtains the randomly determined payment that he is} informed about before. The determination of this payment is explained further below.

At the beginning of each round, you are informed about the payment {B1: that is obtained by the bidder who does} {C1,C2: that you obtain if you do} not receive the object in that round. The value of this payment will be randomly determined and remains constant for four rounds. **Thus, you are assigned a new randomly determined value for the payment after four rounds.** {B1: **This payment is identical for you and your co-bidder.** For the value of this payment, all values in the range of 0 and 50 ECU are equally probable.} {C1,C2: This randomly determined payment can be any value in the range of 0 and 50 ECU with equal probability. Also the other bidder is assigned such a payment. That will be determined according to the same rules as your payment. **You are not informed about the payment of the other bidder.** Your payment and the payment of the other bidder are independent of one another.}]

C.2.1 Experimental procedure

The experimental procedure is the same in each round and will be described in the following. Each round in the experiment has two stages.

1. Stage

In the first stage of the experiment you see the following screen:²⁰

²⁰This figure does not show the bidding function in the graph and the specific bids that would be shown during the experiment. Figures are slightly treatment dependent.

Round:	1 of 12	Remaining time [sec]: 113																									
You receive 0 ECU if you make the smallest bid in an auction The other bidder receives 0 ECU if he makes the smallest bid in the auction Your valuation will be a number between 50 and 100 The valuation of the other bidder will be a number between 50 and 100.																											
<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="text-align: right;">Bid [ECU]</td></tr> <tr><td style="text-align: right;">170</td></tr> <tr><td style="text-align: right;">160</td></tr> <tr><td style="text-align: right;">150</td></tr> <tr><td style="text-align: right;">140</td></tr> <tr><td style="text-align: right;">130</td></tr> <tr><td style="text-align: right;">120</td></tr> <tr><td style="text-align: right;">110</td></tr> <tr><td style="text-align: right;">100</td></tr> <tr><td style="text-align: right;">90</td></tr> <tr><td style="text-align: right;">80</td></tr> <tr><td style="text-align: right;">70</td></tr> <tr><td style="text-align: right;">60</td></tr> <tr><td style="text-align: right;">50</td></tr> <tr><td style="text-align: right;">40</td></tr> <tr><td style="text-align: right;">30</td></tr> <tr><td style="text-align: right;">20</td></tr> <tr><td style="text-align: right;">10</td></tr> <tr><td style="text-align: right;">0</td></tr> <tr><td style="text-align: right;">50</td></tr> <tr><td style="text-align: right;">60</td></tr> <tr><td style="text-align: right;">70</td></tr> <tr><td style="text-align: right;">80</td></tr> <tr><td style="text-align: right;">90</td></tr> <tr><td style="text-align: right;">100</td></tr> <tr><td style="text-align: center;">Valuation [ECU]</td></tr> </table>	Bid [ECU]	170	160	150	140	130	120	110	100	90	80	70	60	50	40	30	20	10	0	50	60	70	80	90	100	Valuation [ECU]	<p style="text-align: center;">Please indicate your bidding function depending on the valuation that is still going to be determined</p> <p>For a valuation of 50 ECU I bid: <input style="width: 40px;" type="text"/></p> <p>For a valuation of 60 ECU I bid: <input style="width: 40px;" type="text"/></p> <p>For a valuation of 70 ECU I bid: <input style="width: 40px;" type="text"/></p> <p>For a valuation of 80 ECU I bid: <input style="width: 40px;" type="text"/></p> <p>For a valuation of 90 ECU I bid: <input style="width: 40px;" type="text"/></p> <p>For a valuation of 100 ECU I bid: <input style="width: 40px;" type="text"/></p> <p style="text-align: right; margin-top: 10px;"><input type="button" value="Draw bids"/></p> <p style="text-align: right;"><input type="button" value="Finish input stage"/></p>
Bid [ECU]																											
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Valuation [ECU]																											

At that stage **you do not know your own valuation for the object in this round.** {B1: The payment that the bidder with the smaller bid obtains} {C1,C2: The payment that you obtain if you do not receive the object} {B1,C1,C2: is displayed on the screen.} On the right side of the screen you are asked to enter a {A1,B1,C1: bid} {A2,C2: maximum bid} for **six hypothetical valuations** that you might have for the object. These six hypothetical valuations are 50, 60, 70, 80, 90, and 100 ECU. Your input into this table will be shown in the graph on the left side of the screen when you click on {A1,B1,C1: “draw bids”} {A2,C2: “draw maximum bids”}. In the graph the hypothetical valuation is shown on the horizontal axis, the {A1,B1,C1: bids} {A2,C2: maximum bids} are shown on the vertical axis. Your input in the table is shown as six points in the diagram. **Neighbouring points are connected with a line automatically.** These lines determine your {A1,B1,C1: bid} {A2,C2: maximum bid} for all valuations *between* the six points for those you have made an input. For the other bidder the screen in the first stage looks the same and there are as well {A1,B1,C1: bids} {A2,C2: maximum bids} for six hypothetical valuations. The other bidder can not see your input.

2. Stage

The actual auction takes place in the second stage of each round. In each round we will play not only a single auction but **five auctions**. This is done as follows: **Five times a random valuation is determined** that you have for the object. Similarly for the other bidder five random valuations are determined. You see the following screen:²¹

²¹This figure does not show the bidding function in the graph and the specific bids, gains and losses that would be shown during the experiment. Figures are slightly treatment dependent.

Round:	1 of 12	Remaining time [sec]: 113
You receive 0 ECU if you make the smallest bid in an auction The other bidder receives 0 ECU if he makes the smallest bid in the auction Your valuation will be a number between 50 and 100 The valuation of the other bidder will be a number between 50 and 100.		
	<p>Auction 1: Your randomly determined valuation is ... ECU. According to your entered bidding function you make a bid of ... ECU. You entered the larger bid. Your income from this auction is ... ECU.</p> <hr/> <p>Auction 2: Your randomly determined valuation is ... ECU. According to your entered bidding function you make a bid of ... ECU. You entered the larger bid. Your income from this auction is ... ECU.</p> <hr/> <p>Auction 3: Your randomly determined valuation is ... ECU. According to your entered bidding function you make a bid of ... ECU. You entered the smaller bid. The other bidder has made a bid of ... ECU. Your income from this auction is ... ECU.</p> <hr/> <p>Auction 4: Your randomly determined valuation is ... ECU. According to your entered bidding function you make a bid of ... ECU. You entered the smaller bid. The other bidder has made a bid of ... ECU. Your income from this auction is ... ECU.</p> <hr/> <p>Auction 5: Your randomly determined valuation is ... ECU. According to your entered bidding function you make a bid of ... ECU. You entered the larger bid. Your income from this auction is ... ECU.</p>	
Your income from all auctions in this round is ... ECU		Continue with the next round

For each of your five valuations the computer determines your $\{A1, B1, C1: \text{bid}\}$ $\{A2, C2: \text{maximum bid}\}$ according to the graph from stage 1. If a valuation is precisely at 50, 60, 70, 80, 90, or 100 the computer takes the $\{A1, B1, C1: \text{bid}\}$ $\{A2, C2: \text{maximum bid}\}$ that you gave for this valuation. If a valuation is between these points your $\{A1, B1, C1: \text{bid}\}$ $\{A2, C2: \text{maximum bid}\}$ is determined according to the joining line. In the same way the $\{A1, B1, C1: \text{bids}\}$ $\{A2, C2: \text{maximum bids}\}$ of the other bidder are determined for his five valuations. $\{A1, B1, C1: \text{Your bid is compared with the one of the other bidder. The bidder with the higher bid has obtained the object.}\}$ $\{A2, C2: \text{In each of the five auctions the price at which one bidder stops bidding will be determined from your maximum bid and the maximum bid of your co-bidder. The price is equal to the smaller one of both maximum bids. The bidder who remains alone in the auction obtains the object.}\}$

Your income from the auction:

For each of the five auctions the following holds:

- $\{A1, B1, C1: \text{The bidder with the higher bid obtains the valuation he had for the object in this auction added to his account minus his bid for the object.}\}$ $\{A2, C2: \text{The bidder who remains alone in the auction obtains the valuation he had for the object in this auction added to his account minus the price of the object. The price is given by the smaller one of both maximum bids, ie. the price at which one of the bidders stops bidding.}\}$
- $\{A1: \text{The bidder with the smaller bid obtains no income from this auction.}\}$ $\{B1 \text{ and } C1: \text{The bidder with the smaller bid obtains the randomly determined payment that [B1: is used in this round.] [C1: he is informed about.]} \}$ $\{A2, C2: \text{That bidder that first stops bidding in the auction obtains}\}$ $\{A2: \text{no income from this auction.}\}$ $\{C2: \text{the randomly determined payment that he is informed about.}\}$

