On the Possibility of Pareto-improving Pension Reform.

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Abstract

The aim of this paper is two-fold. First, it provides a simple framework for the analyses of the transitions between two steady states with different fiscal policies. This allows us to clarify the existing results on the possibility of Pareto-improving transitions from pay-as-you-go to fully funded pension systems. We show that the reduction in the marginal tax rate is a sufficient condition for the possibility of such pension reforms. Second, the paper investigates the features and the duration of the shortest Pareto-improving pension reform in an open economy.

Keywords: *Pension reform, Pareto-improving transition, The shortest transition.*

JEL-codes: *H21, H55, E62.*

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1 Introduction

The reform of pension systems is one of the key issues in contemporary policy. Over the last thirty years there have been dramatic demographic changes that have resulted in significant increases in the age-dependency ratio across all OECD economies. Coupled with reduction in the growth of the payroll, these changes have placed pay-as-you-go pension systems under considerable stress. Both economists and policy makers world-wide predict that this tendency will continue for at least the next fifty years (Word Bank 1994, 2003). If pay-as-you-go systems are to continue in their current form, governments will either have to increase taxes or else reduce the pension benefits for future generations. Such moves would cause a decrease in welfare levels, as compared to those guaranteed by the present pension schemes. Thus, Feldstein and Samwick (2000) predict that the U.S. "current pure pay-as-you-go system can only maintain the benefits specified in current law by raising the payroll tax rate from 12.4 percent today to more than 17 percent by 2037 and nearly 19 percent by the end of the actuaries' 75-year forecast period." The situation in Europe is no better for the World Bank (2003) has projected that the age-dependency ratio in EU15 will double by 2050. This will compel the governments to increase significantly the ratio of taxes to GDP. For example, according to Disney (2003), the UK will have to raise tax by 3.5% GDP. The corresponding figures for Italy, Germany and Austria are 11.4%, 9.7% and 15.4%.

Pay-as-you-go systems are inefficient even in the present demographic environment. The fully funded pension systems perform better if the rate of return on capital exceeds the growth rate of wages (Samuelson, 1975).

The inefficiency arises from the indirect taxation on pension contributions imposed by pay-as-you-go systems, where the tax rate equals the difference between the interest rate and the growth rate of the wage bill.

Even though fully funded systems may be preferable, the transition from pay-as-you-go systems can be costly. On the one hand, the older generation must be compensated for their contributions to a pay-as-you-go system, and therefore the younger generation has to pay twice: They are taxed to support the older generation but they also have to contribute to their own fully funded pension. On the other hand, a reduction in the payroll tax rate leads to higher net wages and may increase labour supply, thereby providing some financing for a transition. The key issue is whether those funds are sufficient for a Pareto-improving reform.

General equilibrium analyses have provided mixed messages on the possibility of a Pareto-improving transition from pay-as-you-go to a fully funded system. The early literature investigates pay-as-you-go systems with lump-sum forms of pension benefits. Thus, Breyer (1989), using a simple two-period overlapping generations model argued that an efficient debt-financed transition for both closed and open economies was impossible. However, that result was due to the fact that the source of transition finance–increased labour supply–was assumed away. Subsequently, Homburg (1990) did indeed show that a Pareto-improving transition is possible for an open economy with an endogenous labour supply. Breyer and Straub (1995) also proved the existence of a Pareto-efficient transition for a closed economy, where labour is endogenously chosen. The Pareto improvement became possible due to the elimination of the labour distortion caused by pay roll tax which can be significant, as demonstrated by Feldstein (1996).

Although Breyer and Straub (1995) documented the possibility of an efficient transition, they did not spell out a policy mechanism for its implementation. The open economy dimension also raises complicated issues of the externalities that may arise when countries do not coordinate a switch from one social security regime to another. These issues have been analysed by Pemberton, (1999, 2000) who argues that the welfare implications of non-coordination may be substantial.

Naturally, different policies have also been investigated to find out whether their implementation would allow for intergenerationally-efficient pension reforms. Raffelhuschen (1993) and Kotlikoff (1995) provided simulations of such transitions from pay-as-you-go with lump-sum benefits when government uses both public debt issue and lump-sum taxes and transfers for compensating transitional generations. Demmel and Keuschnigg (1999) demonstrated the feasibility of a debt-financed Pareto-improving transition from a system with lump-sum forms of benefits, where the government gradually reduces payroll taxes.

However, in an important contribution, Fenge (1995) demonstrated the Pareto-efficiency of pay-as-you-go systems in an open economy with an elastic labour supply and where benefits are proportional to contributions. In particular, he showed that a Pareto-improving transition was infeasible, even if the government could simultaneously vary payroll tax rates, change the rate of implicit return on contributions, and draw on debt-financing (by virtue of being an open economy). So far, Fenge (1995) has provided the most cogent case against the possibility of a Pareto-improving transition from a pay-as-you go to a fully funded pension system in representative agent economy. However, that conclusion may rely on rather strong assumptions. For example, Hirte and Weber (1997), successfully simulated Pareto-efficient transitions from a pay-as-you-go with positive but not full benefit-contribution linkage with compensation financed by proportional income and consumption taxes.

As I show below, this whole debate, at least as analysed using simple overlapping generations models, turns on whether there exists a proportional tax code and full linkage of benefits to contributions. In this paper I shall develop a simple framework that can demonstrate analytically the disparate results currently discussed in the literature. I will provide a theoretical rationale for the Hirte and Weber result which holds for any quasiconcave utility function. In practice, most fiscal systems can be regarded as comprising a proportional tax code along with a positive lump sum component to the pension benefit. Consequently, I characterise a wide class of feasible transitions, and I uncover a sub-class of Pareto-efficient transitions. From that class, I characterise the shortest Pareto-improving pension reform and investigate its features.

I will focus on the open economy case. Whilst this simplifies the formal analysis, it is also widely believed that reform in the open economy context is more complex. In a closed economy the distortion of capital accumulation caused by a pay-as-you-go pension system could be removed by implementing a saving subsidy, as shown in Belan, Michel and Pestieau (1998)[1]. Moreover, Pemberton (1999, 2000) shows that while Pareto-improving transition is feasible for closed economies, it is impossible for open economies, when we consider an environment with constant labour supply and positive production

[1] See also Wigger (1998).

externalities generated by the accumulation of capital.

The rest of this paper has the following structure. Section 2 describes the model. Section 3 constructs the shortest transition between two different fiscal policies and calculates its duration. Section 4 deals with Pareto-efficiency. Section 5 provides the shortest Pareto-improving termination of a pay-as-you-go system with defined benefit. It also comments on findings in the literature that have argued against the feasibility of efficient transitions, and discusses the intuition behind these results, and the results in this paper. Section 6 concludes the paper.

2 The model

I will consider a two period overlapping generations model. This framework affords much analytical convenience and is in widespread use in the literature. As part of my aim in this paper is to offer a unifying structure to assess competing arguments, it is an obvious modelling choice, which I adopt with no further justification.

2.1 Individuals

The representative agent consumes in both periods of his life. He works in the first period of his life and receives a pension in the second. He formulates consumption and labour plans, subject to a lifetime budget constraint, in order to maximizes lifetime utility. His optimisation problem is formally stated as follows:

$$
\begin{aligned}
\max_{c_y, c_o, l} U_t &= U(c_{yt}, c_{ot+1}, 1 - h_t); && (1) \\
st. \quad &: \quad c_{yt} + \frac{c_{ot+1}}{(1+r)} = \\
&= wh_t(1-\tau_t) + Tr_t + \frac{\Omega_{t+1}\tau_t wh_t + P_{t+1}}{1+r} = \\
&= wh_t\left(1 - \tau_t + \frac{\Omega_{t+1}}{1+r}\tau_t\right) + \frac{P_{t+1}}{1+r} + Tr_t. && (2)
\end{aligned}
$$

Here, c_{yt} and c_{ot} denote the consumption of young and old individuals in period t, respectively, and h_t is labour. Terms r and w represent the interest rate and the wage rate, respectively. The pension benefit consists of two parts. First, there is a lump-sum pension benefit, P_{t+1}. Second, there is a proportional term, $\Omega_{t+1}\tau_t wh_t$, where Ω_{t+1} is the coefficient of return on contributions. τ_t represents the payroll tax rate, which the government can use to finance its expenditures, and Tr_t is the value of lump-sum transfers to each currently young individual.

2.2 The Government

The government has five policy choices. Each period it may (i) issue or retire debt, D_t; (ii) define pension benefits in terms of the linkage between benefits and contribution, Ω_{t+1}; (iii) determine lump-sum pension benefits, P_{t+1}; (iv) set the pay-roll tax rate, τ_t and (v) determine per capita lump-sum transfers to the young generation, Tr_t. Hereafter, when I refer to a fiscal policy or regime, I mean the collection of sequences $(D_t, \Omega_{t+1}, P_{t+1}, \tau_t, Tr_t)$.

Let n be the (constant) population growth rate. The government faces the following budget constraint normalized by the size of the working population:

$$D_t + Tax_t = \frac{Pen_t + (1+r)\, debt_{t-1}}{1+n}, \tag{3}$$

where $Tax_t := \tau_t w h_t - Tr_t$ equals the net per capita receipt from younger generation and $Pen_t := \Omega_{t+1}\tau_t w h_t + P_{t+1}$ represents per capita transfers to the older generation. The left hand side of equation (3) represents inflows to the budget, which are equal to new borrowing plus tax revenue. The right hand side represents the outflows equal to pensions plus debt service.

2.3 The Policy Criterion

We shall ultimately be interested in the class of policies that meet the Pareto criterion. The Pareto-improving condition requires that for any individual

$$U_t \geqslant U_{ss}, \tag{4}$$

where U_t is the lifetime utility of the individual born at period t, while U_{ss} is the utility level of the individual living in an initial steady state. In other words, a policy shift at time t needs to leave an agent at least as well as off as under the previous fiscal regime cum steady-state.

3 The Shortest Transition

This section characterises the available, or feasible, transitional fiscal policies and demonstrates how to choose the shortest reform from this feasible set. I will show that financing of the reform by borrowing with further debt repayment is possible only if the government increases the net present value of fiscal receipts. To minimize the debt repayment period, the government should set policy to maximize the net present value of tax and transfers paid by each individual. This statement is formalized and proved in proposition 1.

Proposition 1 *The shortest Pareto-improving transition has the following properties:*

i) Fiscal policy is the same for all generations living during a transition;

ii) Fiscal policy in transition maximizes the net present value of financial inflows from one individual subject to the Pareto improving constraint.

Proof. Let us assume that the transition begins in period 1. At the beginning of the transition the government's existing debt, normalized by the currently young generation, equals pension obligations net of tax and transfers

$$D_1 + Tax_1 = \frac{Pen_{ss}}{1+n},$$

where Pen_{ss} is the value of per capita pensions paid in the initial steady state.

Future period budget constraints evolve in the following way

$$\begin{aligned} D_{i+1} + Tax_{i+1} &= \frac{(1+r)\,Di + Pen_{i+1}}{1+n} \\ &= \frac{(1+r)}{(1+n)}\,(Di + Tax_i) - \frac{(1+r)}{(1+n)}\left(Tax_i - \frac{Pen_{i+1}}{1+r}\right). \end{aligned}$$

By induction it follows that in period k

$$D_{k+1} + Tax_{k+1} = \left(\frac{1+r}{1+n}\right)^K \frac{Pen_{ss}}{1+n} - \sum_{i=1}^{K} \left(\frac{1+r}{1+n}\right)^{K-i} \left[Tax_i - \frac{Pen_{i+1}}{1+r}\right], \tag{5}$$

where term $\frac{Pen_{ss}}{1+n}$ equals initial debt and $Tax_i - \frac{Pen_{i+1}}{1+r}$ equals the net present value of fiscal inflow from an individual born in period i.

If taxes and debt are zero, then the government has no obligations and the transition is complete; the transition is over when $D_k + Tax_k = 0$. To implement the shortest transition we need to minimize the right hand side of (5). This is equivalent to maximizing $NPV_t := \left[Tax_i - \frac{Pen_{i+1}}{1+r}\right]$, the net present value of a fiscal regime affecting the individual born in period i. ■

Proposition 2 estimates the duration of the shortest reform.

Proposition 2 *The duration of the shortest transition is denoted by k, where*

$$k = \text{integer}\left[\frac{\ln NPV_t - \ln e}{\ln(1+r) - \ln(1+n)}\right].$$

and NPV_t is the net present value of fiscal policy during the transition, $e := NPV_t - NPV_{ss}$- is the difference between the net present values of fiscal policies during transition and in the initial steady state.

Proof. *See Appendix.* ■

We can conclude from proposition 2 that, given the same size of pension system in an initial steady state, the transition period will be smaller for the economy with a higher rate of return on capital and smaller population growth rate.

Propositions 2 also shows that finite time debt-financing reform is possible if and only if the government can implement a policy which increases the net present value of fiscal inflows from representative individual to the budget.

Finally, proposition 2 demonstrates that the shortest pension reform has another very desirable property; the government could implement the same fiscal policy during the entire transition period. The stability of fiscal policy is a very encouraging feature since time-varying tax codes, in particular, may be costly.

3.1 The Classes of Equivalent Transition Policies

In this subsection I will show that there exists a continuum of fiscal policies which are equally good for individuals and for the government. To show this, recall the individual consumption-investment problem:

$$\max U(c_{y,} c_o, l);$$
$$s.t. \quad : \quad c_{y+}\frac{c_o}{1+r} = I.$$

Here $I = (1-l)\,\eta w + T$, $\eta := \left(1 - \tau_t + \frac{\Omega_{t+1}}{1+r}\tau_t\right)$ and $T := \frac{P_{t+1}}{1+r} + Tr_t$ is the net present value of government lump-sum transfers to a particular individual. The next proposition shows that two fiscal regimes with the same η and T have similar outcomes for both government and individuals.

Proposition 3 *Any two fiscal policies with the same η and T are equivalent since (i) the agent's optimal consumption-saving program is invariant across policies and (ii) they generate the same net present value of fiscal inflow.*

Proof. *The first statement is true because parameters η and T solely define the budget constraint. This is a Ricardian Equivalence-type result. To prove the second statement we need to calculate the net present value of fiscal policy with respect to one individual $NPV_t = h_t(1-\eta)w - T_t$. This depends only on η and T and h.* ■

Proposition 3 shows us that the shortest transition is not unique. It also allows us to consider only the one representative from each class of equivalent transitional policies. Therefore we can chose the policies which affect the young generations only: $(P_{t+1} = 0,\ \Omega_{t+1} = 0)$. Implementing this policy the government could establish a large fully funded system at the beginning of the transition. This could be encouraging news if one considers the privatization of a fully funded system, since the larger size of pension accounts implies smaller average administrative costs[2].

4 Pareto efficiency

In this section I will select the shortest pension reform from those which are Pareto-efficient. Proposition (3) shows us that all available fiscal policy could be represented as a budget line in life time consumption-leisure space. Now I would like to analyze the Pareto constraint in the same space. For this purpose I will construct the indirect utility function $W(I, l)$, which depends on lifetime consumption, I, and leisure, $l = 1 - h$, and is defined as the solution to

$$\begin{aligned} W(I, l) &= \max U(c_y, c_o, l); \qquad (6) \\ S.T. &: \quad c_{y+} \frac{c_o}{1+r} = I. \end{aligned}$$

For further analysis I need to investigate the convexity of this function.

Proposition 4 *If $U(c_y, c_o, l)$ is quasiconcave, then indirect utility $W(I, l)$ defined in (6) is quasiconcave.*

Proof. See the appendix. ■

[2] For the discussion about the administrative costs in private retirement systems see Mitchell (1998).

Now I will use propositions 1 and 4 to describe the shortest Pareto-improving pension reform. For graphical illustration let us consider Figure 1.

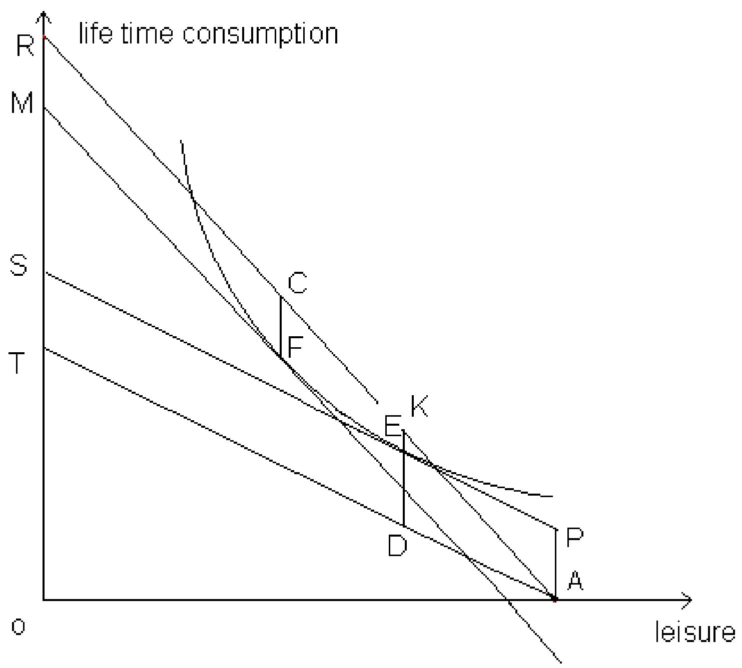

Figure 1: The shortest pension reform

Line AR is a budget constraint for a fully funded pension system. Line AT represents the budget constraint for an individual paying payroll tax and receiving pension benefit which is proportional to contribution. Line PS corresponds to a policy with positive net present value of government transfers ($|AP| = T = \frac{P_{t+1}}{1+r} + Tr_t$). Point E is an individual's choice in the initial steady state. The distance $|KD|$ equals the tax revenue per young worker minus the net present value of the share of pension benefit which is proportional to contribution. The distance $|KE|$ equals the net present value of government policy with respect to one individual, $|KE| = NPV_{ss}$. Curve EF represents the indifference curve for indirect utility function $W(I, l)$ in an initial steady state. Proposition (4) proves that EF is convex.

Propositions (1) and (2) describe the properties of the shortest transitions between different fiscal policies. They prove that a necessary and sufficient condition for the reform is the existence of a fiscal policy generating a higher net present value: $NPV_t > NPV_{ss}$. To implement the shortest Pareto-improving reform, the government ought to chose a fiscal policy which results in individual choice F which lies on or above the line EF (for Pareto-efficiency) and maximizes the net present value of net financial inflows to the government (for the shortest duration). Therefore the government should conduct policy to obtain the individual's choice corresponding to the largest vertical distance between curve EF and line AR. If the government did not face any further restriction, the lump-sum tax policy, MF, enforcing individual's choice, F, would be the solution.

5 The Termination of Defined Benefit Pension Systems in a Pareto-improving Way

If lump-sum taxes are not feasible, then the government needs to implement a fiscal policy which has the highest net present value among the available policies. Since the indirect utility function, $W(I, l)$, is quasiconcave, the policy associated with the highest net present value would correspond to the budget line with the highest slope which is equal to $1-\tau_t$. Therefore, if only a proportional tax code is available, the best fiscal policy during the transition would be zero pension benefits with a tax rate ensuring Pareto-efficiency. An example of such a policy is demonstrated in figure 2.

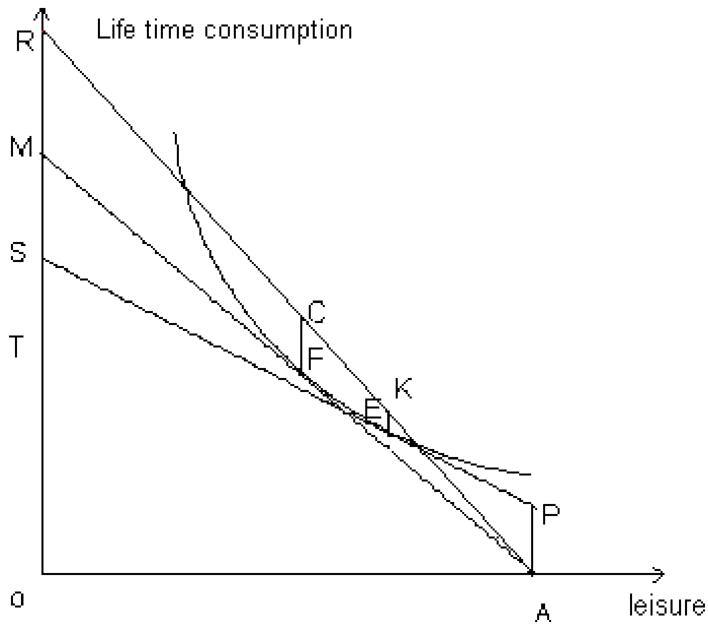

Figure 2: The shortest reform with proportional tax

The individual's budget constraint under a pay-as-you-go pension system ($Pen > 0$, $t > 0$, $\Omega = 0$, $Tr = 0$) is represented by line PS. Point E is an individual's choice of leisure and consumption in the initial steady state. The budget constraint of individuals living during the transition is represented by line AM, which is tangent to the indifference curve, EF, at F. Paying out zero pension benefits the government will collect tax $|FC|$, which is larger than $|EK|$, the net present value of cash inflow to the budget from one representative individual in the initial steady state. The first transitional generation should be compensated by borrowing. The same policy AM could be implemented for all periods until the entire debt is paid off.

Figure 2 helps to understand Homburg's (1990), and Demmel and Keuschnigg's (1998) results. They argue that Pareto-efficient transitions are possible partly as a result of endogenous labour supply responses (Homburg, 1990), and partly by virtue of debt financing (Demmel and Keuschnigg, 1998). It also illustrates that the assumption of the normality of leisure is not necessary for the existence of a Pareto improving pension reform, as it is indicated in Breyer and Straub (1993), since a higher marginal tax rate implies a larger distortion of labour supply for any quasiconcave utility function.

Fenge (1995) shows that pension systems with benefits proportional to contributions cannot be reformed in a Pareto-improving way in open economies, even if labour is chosen endogenously. However, he considered a set-up in which only three policy instruments were available; a proportional payroll tax at rate τ, debt and the degree of linkage between benefits and contributions, ($\Omega > 0$, $\tau > 0$, $P = 0$, $Tr = 0$). Using the described instruments the government may not be able to make the fiscal system less progressive and, at the same time, maintain individuals' utility. However, most fiscal codes have a progressive character even when benefits are proportional to contributions, for the marginal tax rate increases with income. This means that rich individuals still face the progressive budget constraint similar to that described in the previous section, (on figure 2 $|AP| > 0$, since $Tr > 0$). Therefore a Pareto-improving transition may still be possible.

6 Conclusion

In this paper I have provided a simple framework for analysing the debate on pension reform which helps isolate the key factors behind conflicting views as to the possibility of Pareto-improving reforms. I have shown that a reduction in the marginal tax rate is a sufficient condition for the implementation of a Pareto-improving pension reform.

I have also described the policy for the implementation of the shortest Pareto-improving transition from a pay-as-you-go to a fully funded pension system. The older generation living in the first period of transition should be compensated by debt issue. To pay debt back in the shortest period the government should adopt the fiscal policy with the highest net present value. The same tax policy should be implemented until the entire debt is repaid. If government does not face any restrictions, a lump-sum tax policy would be optimal. If lump-sum tax is not available, the government should switch to a funded pension system and tax wages at the rate which maximizes the tax revenue subject to the Pareto-efficiency constraint.

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7 Appendix

7.1 Proof of proposition 2.

Proof. Let NPV_t denote the maximum net present value of fiscal inflow which government can receive from one individual. Then formula (5) can be rewritten as (7).

$$D_{k+1} + Tax_{k+1} = \frac{(1+r)^K}{(1+n)^K}\frac{Pen_{ss}}{1+n} - \sum_{j=1}^{K}\left(\frac{1+r}{1+n}\right)^j NPV_t < 0. \tag{7}$$

We can simplify it

$$\Lambda^K \frac{Pen_{ss}}{1+n} - NPV_t\left[\frac{\Lambda^{K+1}-1}{\Lambda-1}\right] < 0,$$

where $\Lambda := \frac{1+r}{1+n} > 1$

$$\Lambda^K \left[\frac{Pen_{ss}}{1+n} - \frac{\Lambda}{\Lambda - 1} NPV_t \right] < -\frac{NPV_t}{\Lambda - 1},$$

$$\Lambda^{K-1} > NPV_t / \left[NPV_t - \frac{Pen_{ss} (\Lambda - 1)}{(1+n) \Lambda} - \right]$$

$$\left[\frac{1+r}{1+n} \right]^{K-1} \geq NPV_t / \left\{ NPV_t - Pen_{ss} \frac{(r-n)}{(1+n)(1+r)} \right\}. \tag{8}$$

In the assumption that there is budget balance in the initial steady state, we see that

$$Tax_{ss} = \frac{1}{1+n} Pen_{ss}. \tag{9}$$

From (9), it follows that

$$NPV_{ss} = Tax_{ss} - \frac{1}{1+r} Pen_{ss} = \frac{r-n}{(1+r)(1+n)} Pen_{ss}. \tag{10}$$

From (8) and (10) we may now calculate the smallest K :

$$(K-1) \ln \left[\frac{1+n}{1+r} \right] \geq \ln NPV_t - \ln \{ NPV_t - NPV_{ss} \};$$

$$k > \text{integer} \left[\frac{\ln NPV_t - \ln (NPV_t - NPV_{ss})}{\ln (1+r) - \ln (1+n)} \right].$$

■

7.2 Proof of proposition 4

If $U(c_y, c_o, l)$ is quasiconcave, then indirect utility $W(I, l)$ defined in (6) is quasiconcave.

Proof. $U(c_y,c_o,l)$ is quasiconcave iff the Hessian D^2U is negative semidefinite on subspace $\{m, \nabla U * m = 0\}$ [3]. The gradient of indirect utility $W(I,l)$ can be represented in formula (11)

$$\nabla W = \nabla U * C_I, \tag{11}$$

where $C_I = \begin{pmatrix} \frac{\partial c_y}{\partial I} & \frac{\partial c_o}{\partial I} \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Moreover the Hessian of indirect utility W equals

$$D^2W = C_I\prime D^2UC_I. \tag{12}$$

Let consider any z such that $\nabla W * z = 0$. This means that $\nabla U * C_I * z = 0$. Consequently $C_I * z \in \{m, \nabla U * m = 0\}$ which implies that $z\prime C_I\prime D^2UC_Iz < 0$ because U is quasiconcave. From formula (12) we obtain $z\prime D^2Wz = z\prime C_I\prime D^2UC_Iz < 0$, which proves that W is quasiconcave. ■

[3] see Mas-Colell at all (1995).