South Sea Company Subscription Shares and Warrant Values in 1720

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ABSTRACT

The values of the famous Subscription Shares issued by the South Sea Company in 1720 have to be split into two components before they can be understood. One component was a fractional claim upon one original share in the firm. The other component, however, was a bundle of share warrants. The information contained in share warrant values is potentially helpful in understanding the South Sea Bubble. Warrant values might also be especially sensitive to "events" and "news" and could provide new ways of marking the turning points in the South Sea Bubble and testing for efficiency of markets. The level and volatility of subscription share prices are both consistent with the hypothesis that the subscription shares were essentially share warrants.

Introduction

The South Sea Bubble continues to be cited as the prime historical example of mania or panic in financial markets. In popular secondary historical works and in technical pieces for the specialist, it is routinely trotted out as Exhibit No.1. In the last 40 years there has been a revolution in the theory of corporate finance,[1] but important parts of that revised theory have never been used by economic historians to explain the South Sea Bubble. The classic descriptions of the changing capital structure of the South Sea Company by Scott and Dickson, for example, were both penned before this revolution took place. Almost contemporaneous with Dickson's historical work were the famous capital structure irrelevance theorems by Modigliani and Miller,[2] but in the more than 40 years since these works were published there has never been a revision nor an extension of Dickson's work in light of Modigliani and Miller's. What prevents this? One impediment is that the South Sea data themselves have to be transformed into units that make sense in light of modern financial theory. The definitions and labels of South Sea Company capital structure are inherited from the 18th century terms and they do little to help and do much to hinder our understanding.

To illustrate the problem consider the following simple question. What was the South Sea Company worth in 1720? The modern financial analyst usually first looks at such a problem as one of adding up the observed and estimated market values of the firm's liabilities. Valuing and summing up the liabilities are usually easier than valuing and adding up the assets simply because firms' liabilities are often marketable and traded, whereas most firms' assets are unique, nontradeable or else have markets in which values are hard to observe. If the analyst then believes that there exist certain

[1] Merton's ‘Capital Market Theory and the Pricing of Financial Securities’ is a most useful overview of this revolution.
[2] The seminal article in 1958 was 'The Cost of Capital, Corporation Finance and the Theory of Investment'. Merton cites Modigliani and Miller's work as one of the cornerstones of the revolution in modern

financial and asset market efficiencies, the total value of liabilities should be a reasonable estimate of the total value of assets. Even when such efficiencies do not exist, the analyst would still need estimated liability values in order to measure the market inefficiencies. So, can we yet attempt to answer the question of what the South Sea Company was worth? We cannot until we have a simple accounting of the liability structure of the firm in modern units. It is a premise in this paper and in our research programme that a reworking of the corporate financial structure of the South Sea Company into modern units is necessary or else there will be little understanding of the South Sea Bubble.

This paper is a step in redressing this problem and we start with a little understood component of the Company's equity capital, the subscription shares. The subscription shares have been referred to also as subscription contracts. They were share purchase agreements that required a down-payment and subsequent payments to follow in scheduled instalments. Subsequent events in 1720 and in later years suggest, however, that the general investing public did not see these contracts as simple instalment purchase contracts for shares. It appears that the Company had a large problem in enforcing the subscription contracts later in 1720. Although by that time it was well in the interest of the Company's management and the Government that the subscription contracts be enforced to the letter, there is good evidence to suggest that the investing public viewed these contracts as a complex package of call options on shares. We show that the investing public accepted (and priced) the subscription contracts as if they were call options on shares.

In this paper we demonstrate that the South Sea subscription shares were not simply South Sea shares in another guise. In terms of their value and the volatility of their returns they were very different from normal South Sea shares. Subscription

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corporate finance and the Nobel Prize committee agreed when it cited this work in awarding the prize to Miller in 1990.

share values were far too high if the shares are seen as nothing more than fractional claims on full South Sea shares. It is also shown that subscription share values obey a simple bound with respect to South Sea share values and that the bound is consistent with efficient financial arbitrage. Later in the paper we decompose the value of the subscription shares into two components: a) a fractional South Sea share value and b) a residual that must be viewed as the value of a bundle of South Sea share options. In the paper's final section we show that these residual values have returns whose volatilities have some of the defining features of volatilities that come from share option returns. Along the way we show that the modelled time-series of warrant volatilities mark out some of the major events of the South Sea Bubble in ways that are not captured by time series of share values themselves. We conclude with a discussion of the implications of this research for how future histories of the South Sea Bubble should be written. In the next section of the paper is described the subscription shares and their place in the capital structure of the South Sea Company.

The Subscription Contracts and South Sea Company Equity

In this section is described the subscription contracts and their place within the Company's equity capital structure. We present graphical evidence that the subscription share values had a distinctly different nature than had the values of ordinary South Sea shares. Our hypothesis is that most of the value of subscription shares can be thought of as share option value.

The inflation and collapse in South Sea equity values was experienced by all classes of shares. The main division was between i) straight share equity and ii) subscription shares. The structure of South Sea share equity in 1720 is illustrated in Table 1.

Table 1 – South Sea Company Original Shares and Subscription Shares in 1720

Original Shares Issued Pre-1720	Shares to be Issued via Exchanges announced 19 May 1720	1st Subscription Shares Issued 14 April 1720 (nominal)	2nd Subscription Shares Issued 29 April 1720 (nominal)	3rd Subscription Shares Issued 17 June 1720 (nominal)	4th Subscription Shares Issued 12 August 1720 (nominal)
109,428	32,427	22,250	15,000	5,000	1,250

At the height of equity values in late June 1720, subscription shares accounted for at least 10 percent of all South Sea Company equity value.[3] They were issued to meet the immediate cash needs of the firm. Some of the cash would have had to be used in the exchange packages the Company was offering to government annuitants, but by far the largest use of the cash was in the form of loans to shareholders who were pledging their shares as collateral for loans.[4]

The subscription shares were shares issued to the public and they could be purchased in instalments.[5] The 1st Subscription series started 14th April 1720 and the 2nd series started soon thereafter. The 1st Subscription, for example, was for shares priced at £300 per share (p.s.). A £60 deposit was required and thereafter every two months a 10 percent or £30 instalment would be called. The 1st Subscription shares had a £30 call upon them due to be paid on 14th June. On that date the possession of a subscription share would represent 30 p.c. of an original share and the obligation or

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[3] In another manuscript we have presented a first trial accounting of original share and subscription share equity in May and June of 1720. See the description in the Basic Data Table in "Rational Pricing of Options during the South Sea Bubble: Valuing the 22 August 1720 Options", mimeo, June 2004.

[4] In Add. Ms. 25,499, *South Sea Company. Court Minutes No. 6*, page 104, for 21 April 1720 we have one of the more remarkable statements from the directors of the Company concerning their intended financial management of the firm. The disposition of cash from the sale of subscription shares figures prominently in their statements. The standard descriptions of the annuity exchanges and the loans-on-shares programmes are found in Scott and Dickson. To discuss these programmes further would offer too great a distraction from the objects of this paper, but there is some technical discussion of the loans-on-shares programme in Appendix 3. In particular refer to fn. 32.

right to make 7 more bi-monthly instalments of £30 each until ownership in one full original share resulted. The 2nd Subscription required a 10 p.c. deposit on a share priced at £400. After the first £40 deposit, a further 9 £40 quarterly instalments would follow. The 3rd Subscription did not commence until 17 June and required a £100 deposit on a share that was to be priced at £1000. Nine further semi-annual instalments of £100 each would be required before a full original share was credited to the owner. In Appendix 1 we describe the full instalment schedules and terms for all four subscriptions issues and the limitations on useable data that pertain to them.

What were these subscription shares and what determined their value? There are two quite opposite answers to these questions. The subscriptions were sometimes referred to as subscription contracts and as such may be viewed as ironclad agreements between the issuer and the subscriber. That is, the subscriber was under strict contract to pay the instalments on time and in full otherwise the issuer had full legal right to recover the due instalments at the subscriber's cost. The subscriber on the other hand had a similar legal right to obtain further fractional shares in the company at the fixed date of each instalment. After the collapse of the South Sea Bubble and the near collapse of the Company itself, this was an interpretation of the subscription contracts that particularly suited the purposes of the company's management and there is substantial documentary evidence of the Company's pursuit of defaulting subscribers.[6]

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[5] The 18th and 19th century usage of the term "call" often refers to a request for the payment of an instalment. That is how we shall use the term in this paper and not as reference to a call option.

[6] There are a number of Court Minutes that record the difficulty of collecting instalments and several instances of postponements of instalments. The pursuit practically ended in the spring of 1721. In Add. Ms. 25,544, *South Sea Company. Court Minutes Jan 31 1721 - Jan 19 1732, No. 2*, page 6, for 9 March 1721, it was resolved that subscribers to the 2nd, 3rd and 4th subscriptions would no longer be asked to pay instalments. It was proposed that instalments on the 1st and 2nd subscriptions "be made stock", that is, moved off the subscription books and be credited in the ledgers that kept track of original shares. These and other new arrangements between the Company

If this was the proper way to view the subscription contracts, how would their value be determined? In this case, a subscription share would be worth one full share minus the discounted present value of all the remaining instalments. On 20 June 1720, for example, a full share was worth about £806 and a subscription share from the first series was worth £625.4[7]. The difference between the two prices was £181.2. The present value of the remaining 7 instalments (calls) of £30 each would have to have been smaller than £210, but a realistic estimate would have been also greater than £200. Thus we see that the subscription share was a bit too valuable to be in accord with the strict interpretation of the subscription contract. The difference between the values of original shares and subscription shares became even more narrow later in 1720. On 3 September, for instance, a share was worth £687.9 whilst a 1st Subscription share was worth as much as £612.2. The present value of the remaining instalments to be made on these shares was greater than £176. The behaviour of 1st Subscription prices relative to share prices is illustrated in the graph below, in which

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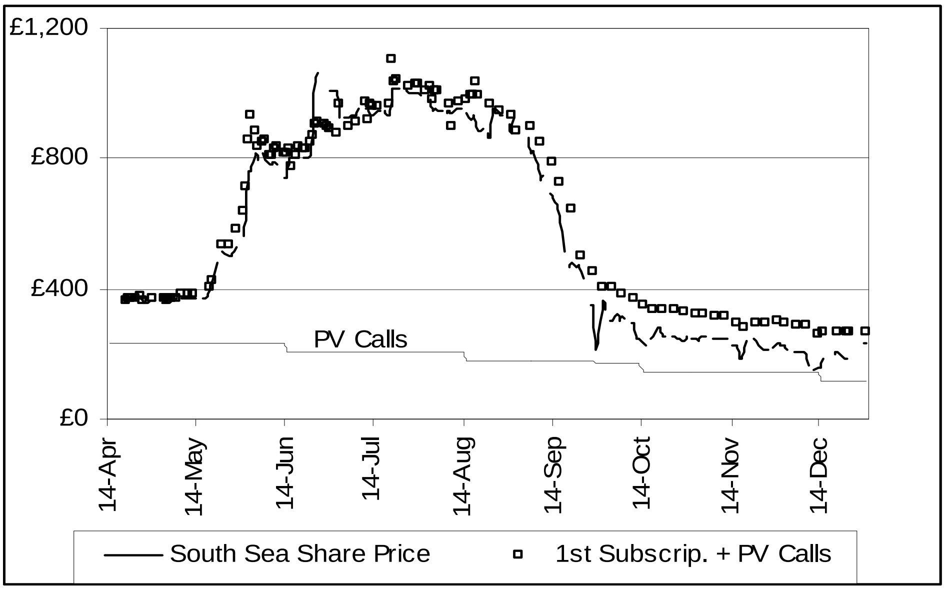
and its owners and creditors were made final and given legal sanction in 7Geo1,stat.2 in August 1721.

[7] It is useful here to describe how we have adjusted value data for equity liabilities of the Company. The quoted value for shares in *The Course of the Exchange* was £760 8s, but to convert that value into the value of a pre-1720 original share, which received a 6 p.c. stock dividend in 1716, we have to multiply the quoted value by 1.06 to obtain £806. The quoted 20 June value for a 1st Subscription share was a £500 premium over what was due to the Company by an owner of such a share. At that time £90 was due to the Company making the value of a subscription share £590. In terms of a claim on an original share, this value too has to be multiplied by 1.06 to obtain £625.4. It was declared in several meetings of the General Court that original shares would receive a 10 p.c. stock dividend. Latterly the stock dividend was extended to the first three series of subscription shares as well. This would dilute share values after July 20 (the official ex-dividend date) by 10 p.c. and make such values incomparable to share values prior to 20 July unless another adjustment factor (1.10) was applied to the ex-dividend prices. We have applied this adjustment factor as well to such prices. The transfer ledgers for trade in original shares were closed from 22 June to 22 August, so for most persons the effective ex-dividend date was 22 June. It was clear, however, that Directors could engage in sales and purchases of all Company liabilities throughout the summer and in using value data from this period we have to be especially careful (when the sources admit) to treat values on shares "with dividend" and values "without dividends" differently. Both appear in the abstracts of brokers' ledgers collected by the Committee of Secrecy. Appendix 1 contains more details on these matters.

the general point is well illustrated; subscription shares were too valuable to be in accord the strict enforceability interpretation of the subscription contracts.[8]

The first thing to note about the graph is that the subscription share value plus the present value of its remaining instalments appears to have the South Sea original share price as a lower bound. Regardless how subscription share contracts were valued, this lower bound should hold if arbitrage could be easily performed. If the present value of a subscription share and its future instalments were ever less than the value of an original share, an arbitrageur could purchase a subscription share, borrow the present value of the instalments and thereby obtain the equivalent to an original share for less than it would cost in the market for original shares. This kind of arbitrage would lead us to expect that the bound $P_{\text{subscription share}} + PV_{\text{calls}} \geq P_{\text{original share}}$ would hold.

South Sea Share Prices vs. 1st Subscription Share Prices



In what circumstances could the bound hold exactly? One circumstance in which the bound could hold precisely is when the subscription contracts were perfectly and costlessly enforceable. That is, if the Company could costlessly pursue a

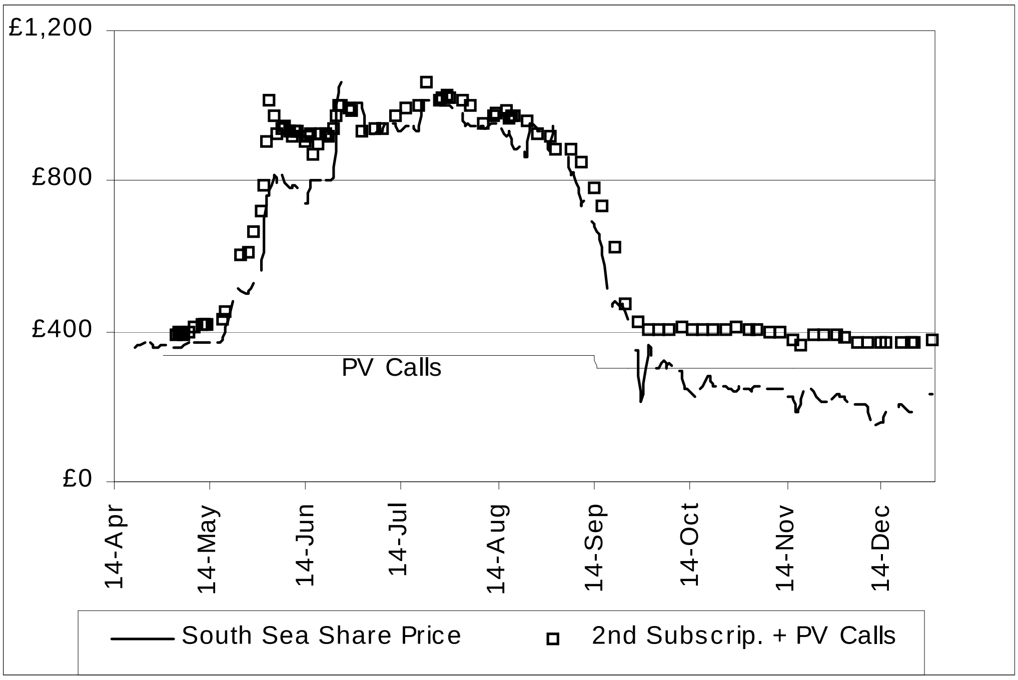
[8] Similar graphical depictions first appeared in Helen Paul's, *The Macroeconomic Basis of the South Sea Bubble,* 2004.

defaulting subscriber for missed instalment payments and interest, then all persons would know that the owner of a subscription share could never escape the liability of paying the calls on the subscription share. The other circumstance in which the bound could hold precisely is when the payment of calls is optional, but when that option is itself worthless. This would occur when it is practically certain that the owner of a subscription share would wish to make the instalment payments regardless of whether the payments were optional or not. These circumstances would most likely arise when original share prices were far higher (and likely to remain so) than the present value of the calls.[9] In the usual language of option analysis we would say that the option to transform a subscription share into an original share by paying the calls is "deep in the money". We see some of these features in the graph on the previous page. Certainly in the latter half of 1720 as the original shares approach "near the money" levels, an additional value to the subscription shares becomes more apparent. It is not going too far to observe that as the equity values were collapsing and original share values start to come close to PV_{calls}, subscription shares appear to have a special added value. We hypothesise that this added value was associated with the option to default on future calls if they did not appear worth paying.

These features are observable in similar depictions for the other subscription shares, such as the 2nd Subscription series illustrated on the next page.

[9] This is precisely how subscription share values for the Royal African Company behaved and is demonstrated in Appendix 2.

South Sea Share Prices vs. 2nd Subscription Share Prices



In the case of the 2nd Subscription series, the lower bound represented by $P_{original}$ is clearly being obeyed and, in fact, it is clear that the 2nd series (plus PV_{calls}) is further above $P_{original}$ than are the values of the 1st series (plus PV_{calls}). Is this anomalous? Because the subscription price of the 2nd series was £400, whereas the subscription price of the 1st series was £300, any option value the 2nd series possesses should *ceteris paribus* be less valuable than the same option embedded in the 1st Subscription series. We might think then, at first glance, that the 2nd Subscription series shares are too valuable relative to 1st series shares.[10]

We are here forming the hypothesis that special values are possibly attachable to the subscription shares because such contracts could never perfectly impose an instalment-payment liability upon the subscriber. An extreme version of this

[10] The same observation might be made for the premia for 1st and 2nd Subscription series shares as they were reported in *The Course*. Quite often the premia for 2nd series shares were equal to or in excess of the premia for 1st Subscription shares. This is probably not anomalous when we take into account the different instalment schedules for the two series. This is discussed in more detail near the end of Appendix 3.

hypothesis would state that the payments of calls were purely optional from the perspective of the subscriber. Viewed this way the 1st Subscription share value of £625.4 on 20 June 1720 is easier to understand. There was no certainty that over the next 14 months (covering the period of 7 bi-monthly instalments) that South Sea share prices would remain above £300 per share. The subscription shares could therefore be viewed as a package of small call options on shares in the firm and should be valued accordingly. To be more precise, since such call options were being issued by the Company itself instead of being sold by third-party option writers, we should say that these contracts represented packages of share warrants.

Before we turn to further analysis of such warrant values, we might look at the further evidence presented by the 3rd and 4th Subscription series. There is only a little data pertaining to these shares and we believe the data are of dubious quality. (See the discussion of data quality in Appendix 1.) They were issued both with subscription prices of £1000 p.s. and as original share values fell far and firmly below £1000, reliable value quotations for these series quickly disappeared from the record. In terms of the kinds of graphs[11] we have looked at so far we can at least conclude that the lower bound formed by $P_{original}$ was certainly being obeyed.

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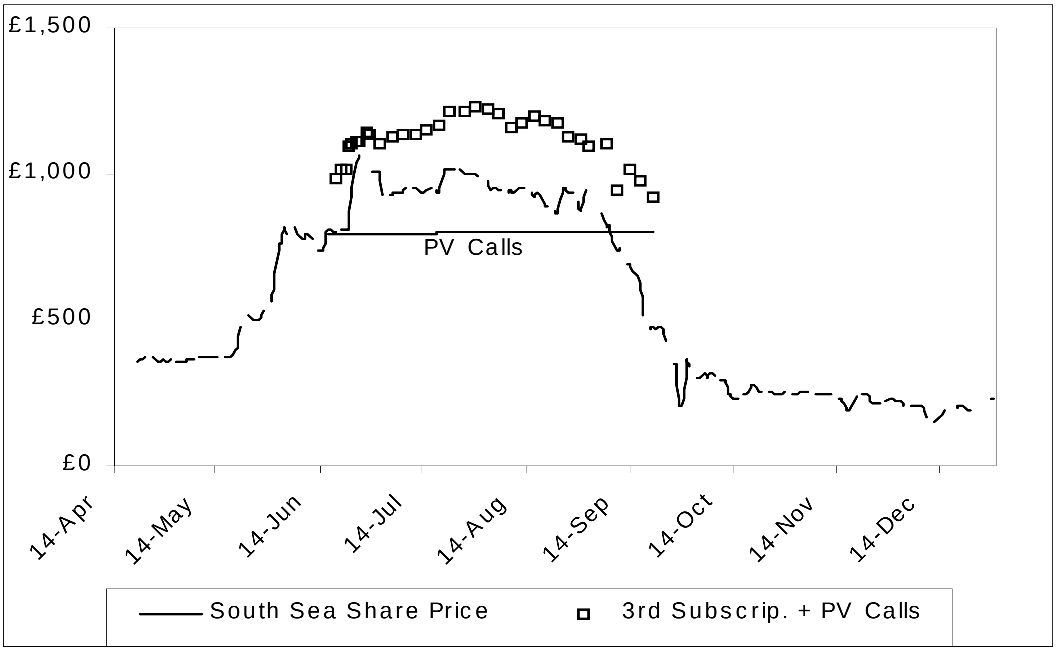
[11] The graph for the 4th Subscription series is reproduced in the Appendix on Subscription Shares.

South Sea Share Prices vs. 3rd Subscription Share Prices



There are some other interesting features of these graphs that require analysis. Note that through the rise and fall of the 'bubble' that the subscription share and original share values tend to have similar volatility. That is, even though their level is quite different, the way in which they vary looks very much the same. That picture, however, changes once original share values start to approach the PV of subscription instalments. What we see in the 1st and 2nd subscription share values is that their volatility relative to the original share value volatility clearly decreases.[12] Does this make sense in light of our hypothesis? It might, but there are several effects on volatility that need to be separated one from the other before we can say for sure. Standard option pricing theory (see Appendix 5) suggests that as share prices decline, share options become less valuable (this is clear in the graphs) and their returns become more variable relative to returns on shares. Do our graphs deny the appearance of this latter effect? Not necessarily. As share values decline and become

[12] The data series for the 3rd and 4th series are not long enough to make such phenomena visually clear.

less volatile, so does the value of the fractional share component of subscription shares. The proportion of subscription share value attributable to the fractional share component is also rising. It is hard to detect what is happening to the size and volatility of any component warrant values and their returns until we can look at them directly. This we do in the next section.

So far we have presented an informal case for our hypothesis that South Sea subscription share values contain significant warrant value components. We have also shown that subscription share values obey a simple arbitrage bound. If the share markets were grossly inefficient because of limits on arbitrage, we might have expected these bounds to be violated. The extent to which they were obeyed demonstrates some degree of financial market efficiency in 1720.

The Subscription Contract Residual and Warrant Values

So far the evidence indicates that the subscription shares were valued like packages of warrants. In order to further our enquiries we need to look directly at the warrant values themselves, but how do we measure such values? The approach that we propose here is straightforward. From the values of a subscription series we subtract the value of a claim to a fraction of an original share that is embodied in the subscription share. To take the example of the 1st Subscription share on 20 June 1720 again, we have already seen that its value was £625.4. This share, however, represented a claim on a fraction of an original share. By 20 June £90 was supposed to have been paid on instalments on these shares. At a subscription price of £300 p.s. this means that a subscription share represented a claim on at least £90/£300 of an original share. An original share on this date was worth £806, so clearly (£90/£300) × £806 = £241.8 is the value of a fractional claim on an original share that is embodied in the £625.4 subscription share value. The residual value is £625.4 - £241.8 = £383.6 and it is this residual we wish to associate with a warrant value or, to be precise, a package of warrant values.

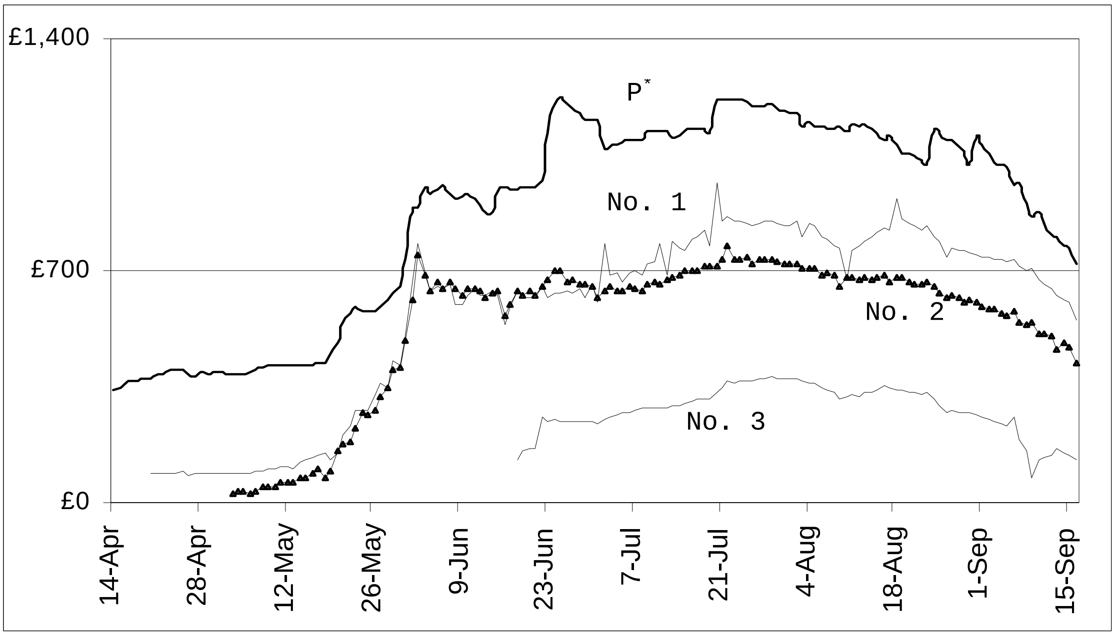
What was the structure of such a package? A share warrant is a call option on a share issued by the Company.[13] Every time a subscriber paid a call on his subscription share, he obtained a fraction of a share. On 20 June the 1st Subscription shares still had 7 instalment payments to go and each instalment paid gave the owner 1/10 of an original share. If these payments were optional, this is tantamount to the owner of the subscription share holding a bundle of warrants. In the case of the 20 June 1st Subscription share, this would imply there were seven warrants remaining with seven different values, but altogether they were worth £383.6. The warrants were worth nearly £55 each on average. This is not an unreasonable value. Each warrant, if exercised at an exercise price of £30, would give the owner 1/10 of an original share currently worth about £81.

We shall refer to the warrant packages henceforth simply as warrants; from each of the subscription series we have calculated a residual value series and have named them respectively No. 1 Warrant, No. 2 Warrant, No.3 Warrant and No. 4 Warrant. There are a number of further corrections and related issues regarding data that become rather technical and are dealt with in appendices. The corrections that are necessary to make the warrant values and the values for South Sea shares strictly comparable are discussed in Appendix 3. Thereafter, missing values for data were filled in by fixed interval smoothing. This is also a relative technical matter and is discussed in Appendix 4. From Appendix 4 we therefore obtain the corrected and

[13] It can be valued just like a call option on shares written by third-party option writers, but special attention has to be paid to some special features. The exercise of warrants changes the number of shares issued and outstanding since the Company has to issue new shares whenever a warrant is exercised. This creates some dilution in the values of shares. On the other hand, when a warrant is exercised more cash is injected into the firm and this tends to raise the value of shares. The details of the necessary adjustments are discussed in Appendix 3.

smoothed value data for original shares and warrants that can be used in our subsequent analysis. The graph below depicts these warrant values against the values of original shares.

Corrected South Sea Share Prices (P^*) vs. Warrant Package Values



Warrant Volatility

If the residuals that we have defined and measured are truly associated with warrant values, there are other ways in which we can demonstrate the fact. Share warrants are but a type of call option on shares and call option volatilities have several well-established features. Standard option pricing theory tells us how large relative volatility between share and call option returns should be and how they are functions of the level of share prices. Both of these volatility implications of option theory can be checked in our data and each bolsters the case that the South Sea Subscription shares were indeed little more than packages of warrants. There are two basic relationships that we can demonstrate with the data we have at hand: 1) warrant return volatility is greater than is share return volatility and 2) relative volatility is sensitive to how "near the money" our warrants are. We describe the theory behind these propositions in Appendix 5.[14]

The first proposition refers to the fact that a call option, such as a warrant, can always be viewed as a levered investment in the share on which the call is written. In option pricing theory, as in all other financial asset pricing theory, an investment that is financed with borrowed money (a levered investment) has to have a higher expected return (and volatility of return) than has the asset itself. The volatility of a call option relative to the volatility of a share is the call option's elasticity (Ω) with respect to the share. The basic relation is:

$$\nu_{\text{warrant}} = \Omega \nu_{\text{original}},$$

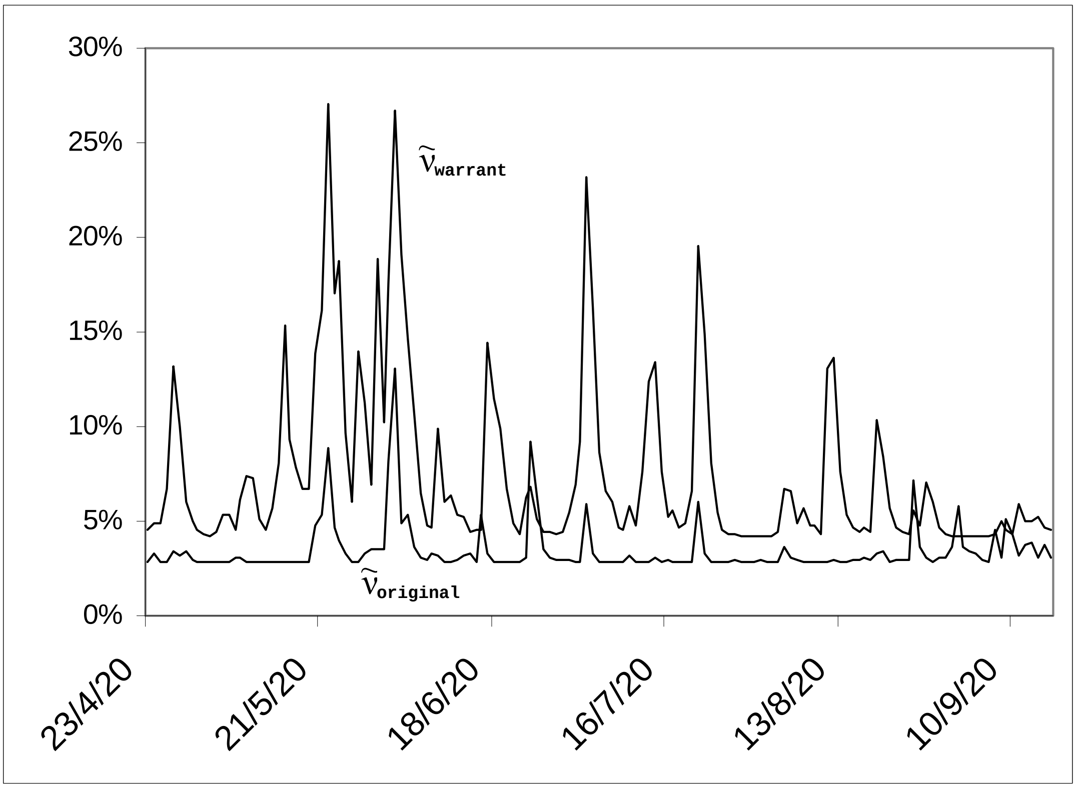
where ν refers to the time-varying standard deviation of return, with $\Omega \geq 1$. Our first test will depend upon a joint estimate of the time-varying volatility of both warrants and shares. We refer the reader again to Appendix 5 that describes the procedures that we employ to further prepare our data for analysis. The econometric techniques employed are described in Appendix 6.

The first test we propose is one in which we estimate jointly the volatility of original shares and our warrants. The volatilities of warrant returns can be nearly the same as the volatility on original shares ($\Omega \cong 1$) as long as the warrants are deep in the money, but as long as the warrants are near the money or, even more so when they are out of the money, the volatility of warrant returns should exceed the volatility of share returns. In no case should there be any evidence that share returns are more volatile than warrant returns. The model we use is the BEKK bi-variate GARCH model described in Appendix 6. The estimated model is capable of generating estimated time series of ν_{warrant} and ν_{original}. We label these estimates $\tilde{\nu}_{\text{warrant}}$ and

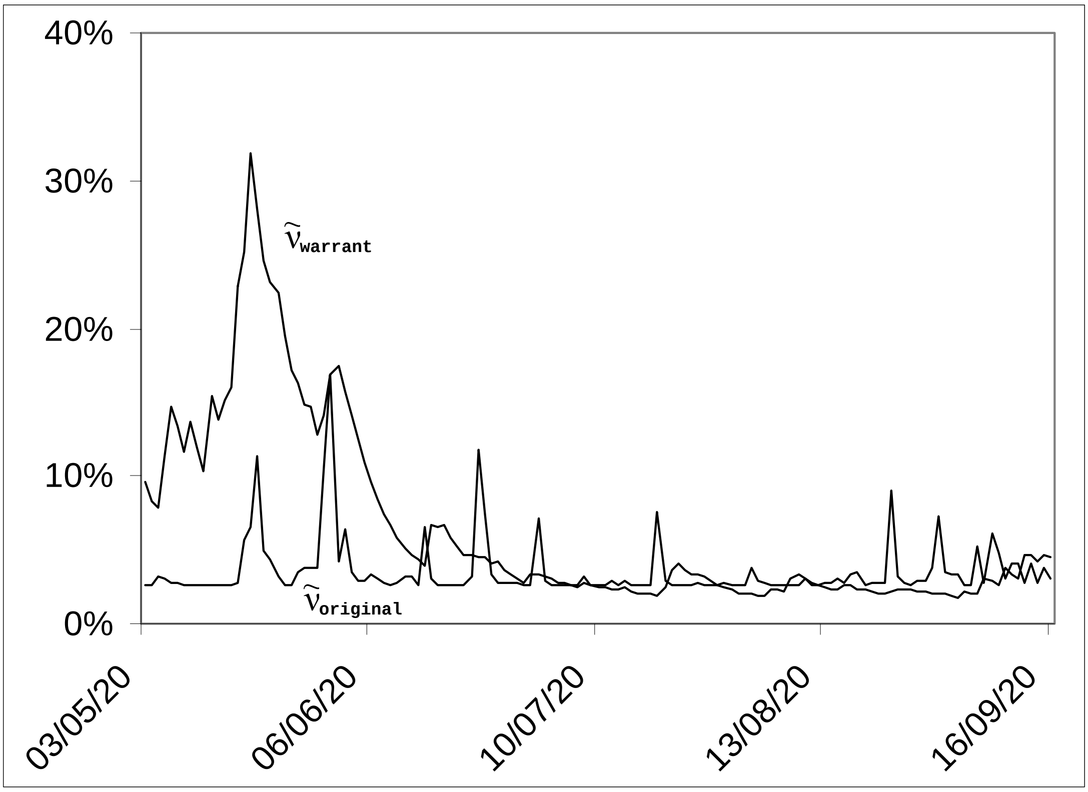
[14] Almost any basic textbook in option pricing will discuss these relations. In this and other papers we make reference to Cox and Rubinstein (1985) for basic theory of this type.

$\tilde{v}_{original}$ and make an informal inspection of them in the graphs that follow. (In Appendix 6 we more formally test the hypothesis $\tilde{v}_{warrant} - \tilde{v}_{original} \geq 0$.) As usual in this paper, we leave the mathematics and the other more technical matters to an appendix and here present our results in a graphical form. What these graphs show is that $\tilde{v}_{warrant}$ is almost always at least as great as is $\tilde{v}_{original}$ and there are times when it is greatly in excess of $\tilde{v}_{original}$. The timings of the greatest peaks in relative volatility are also of interest because they point possibly to the arrival of significant news to the financial markets in 1720.

Returns' Volatility - Original Shares vs. Warrant No. 1



Returns' Volatility - Original Shares vs. Warrant No. 2



The two graphs show a volatility surge in warrant returns immediately before, during and after the announced results of the May exchanges of government annuities. One of the major results in the ARCH modelling literature is that daily share returns tend to be highly persistent, but we do not see such persistence in the short sample of South Sea returns that we analyse.[15] The relative persistence, however, in warrant volatilities (especially for the No. 2 Warrants) is of interest. Simple option volatility theory indicates that if there appears to be some added persistence in $\nu_{warrant}$ than there appears to be in $\nu_{original}$, the added persistence must be coming through the returns' elasticity, Ω. Any exogenous factor that has more than a transitory influence on Ω is a

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[15] French, Schwert and Stambaugh's 1987 article is the seminal paper in this literature. Volatility persistence means that a disturbance to a share return results in a movement in return volatility that tends to persist longer than does the disturbance to the return. Given the shortness of our time-series sample, lack of such persistence in South Sea share returns is itself not a cause for concern, but we might wish to later investigate if it was a general feature of shares' returns over longer periods of the 18th century.

potential factor in explaining the behaviour of $\nu_{warrant}$ relative to the behaviour of $\nu_{original}$.

From the BEKK model estimates it appears that share return volatility is quite transitory relative to warrant return volatility. Indeed, persistence in warrant volatility seems to be typical of the period surrounding the 19th May announcement of the government annuity exchanges. The bi-variate BEKK model is a little crude in its depiction of the joint variation in volatilities for warrants and shares. It does have the advantage of allowing for the two sets of time-varying volatilities to be jointly estimated, but the method does not let us model permanent and transitory influences on volatilities. One particularly appealing class of alternative models is the class of Component ARCH Models of time-varying volatility.[16] These models specify that there are both permanent and transitory components to the volatility of returns. In the case of a warrant return, its volatility ($\nu_{warrant}$) has a long-run, but time-varying, component (ν_{perm}). The transitory component is the residual $\nu_{warrant}$ - ν_{perm}. There is a representative equation for both components and both equations contain so-called ARCH and GARCH effects that are explained in Appendix 6. Since observable and BEKK-modelled volatility in original share returns appears to be highly transitory, we might treat such share return volatility itself as an exogenous variable in the transitory volatility component of $\nu_{warrant}$.

Among the possible influences upon the relatively persistent level of volatility must be the degree to which the warrant is near the money. For example, for warrants that are deep in the money volatility elasticity (Ω) is near 1, and so short-run and long-run $\nu_{warrant}$ should be practically the same as $\nu_{original}$. As warrants get near the

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[16] The model used is the standard components ARCH model devised by Engle and Lee in "A Permanent and Transitory Component Model of Stock Return Volatility."

money, however, the long-run level of $\nu_{warrant}$ should become different from that of $\nu_{original}$. (More explanation of these effects are given in Appendix 5.) Warrant volatility may have substantial transitory volatility components, but the permanent component in volatility should be associated clearly with "near-the-moniedness".

In the models whose estimated parameters appear in Table 2 (Appendix 6) we find indeed that a measure of "near-the-moniedness" appears with correct sign and with statistical significance as a part of persistent warrant volatility, just as standard option theory would suggest. The model indicates as well, at least for No. 1 Warrant, that original share return volatility has a measurable transitory influence on $\nu_{warrant}$, again, just as standard option theory would suggest. These regression results are consistent as well with the BEKK-volatility graphs that we have presented. It is clear in the graphs that No. 1 Warrant volatility has a larger transitory component than has No. 2 Warrants. The picture also suggests that the transitory peaks and troughs in No. 1 $\nu_{warrant}$ are highly correlated with the peaks and troughs in $\nu_{original}$.

The volatility of returns for our hypothesised warrants is measurably larger than the volatility of returns on original shares. The volatility of South Sea share returns happened to be quite transitory, but there appeared to be some persistence in the volatility of warrant returns. The persistence appears to have been a function of how deep in the money or out of the money a warrant was. All these features of returns volatility are consistent with standard option pricing theory.

Conclusions for Further Research

The South Sea subscription shares were packages of share warrants. The values of subscription shares and the volatilities of their returns indicate that they priced like warrants in 1720. There is evidence that markets were sufficiently efficient to guarantee that a simple arbitrage bound between South Sea original share and subscription share values was obeyed. This bound was precisely obeyed by Royal African Company shares as well. But how far did such efficiencies extend in the financial markets of 1720? The historical literature on the South Sea Bubble emphasises that irrationality was endemic in persons' behaviour in financial markets, but in the literature there is no special role played by limits to arbitrage in curtailing market efficiency.[17] If there was an irrational mania driving the fundamental value of South Sea liabilities across time, why is not the mania evident in the pricing of derivatives (warrants) relative to shares?

The theoretical relationships between values of derivatives and the financial liabilities on which they are written are based upon arbitrage. There may be no more productive place to search for such efficiency limits in 1720 than in the relative pricing of securities and their derivatives. The South Sea literature is not only not empty of results from research in these areas, but the literature has yet even to acknowledge that the derivatives existed and, in the case of warrants, were a substantial part of the South Sea Company's equity capital.

There is much work to be done therefore in even simple accounting of the South Sea Company's liabilities. Straight shares and warrants embedded in the subscription shares we can account for, but this is not where the accounting of South Sea share warrants will end. The Company was also engaged in a substantial programme of loans to shareholders that it hoped would somehow support the market value of shares. In running this programme the Company was quite busy in handing out share conversion privileges (warrants) to privileged recipients of these loans. These warrants were not traded and so direct observations of their values will not be easy to obtain. But these warrants were not invisible either. We can account for them, study

[17] Financial market efficiency can be lessened in a number of ways. Human irrationality and limits to arbitrage, together or separately, can make markets less than efficient. See Shleifer’s *Inefficient Markets*, Introduction and Chapter 2.

their terms, measure their size and, perhaps later, we can attach proper values to them so that they can take their place alongside straight share and the subscription shares as part of the South Sea Company's equity capital. The estimated values of warrant packages presented in this paper will be step towards measuring the last missing component of South Sea equity. The simple question that was posed in the Introduction might then be answerable. What was the South Sea Company worth in 1720?

APPENDIX 1 – SOUTH SEA SUBSCRIPTION SHARES AND INATLMENT PAYMENTS

In this Appendix we describe the mechanics of the South Sea Company's Subscription Issues. There were four issues. The first two issues in April 1720 provide us with most of our useable data. The third and fourth subscriptions priced shares initially at £1000 each and were quickly "out-of the-money" as share prices declined. The fourth subscription indeed offers very little in the way of data. Each issue is described below along with its instalment schedules. At the end of the Appendix we explain how the present values of instalments are calculated.

FIRST and SECOND SUBSCRIPTION ISSUES

The first subscription issue was planned soon after the Company's enabling Act came into force on 7 April 1720. Formal issues began in the week of the 14th of April and in the Court Minutes of the 19th and 20th of April the management of the first issue and the newly planned second issue were discussed. The subscription price was set to £300 to be paid in 9 instalments. The first issue's instalment schedule was thus:

1st Subscription Shares Instalment Schedule	
14 April 1720	£60
14 June 1720	£30
15 August 1720	£30
14 October 1720	£30
14 December 1720	£30
14 February 1721	£30
14 April 1721	£30
14 June 1721	£30
14 August 1721	£30

In the 29th April meetings of the Company's General Court a schedule for the 2nd Subscription was determined. The subscription price was £400 to be paid in 10 equal instalments.

2nd Subscription Shares Instalment Schedule	
29 April 1720	£40
14 September 1720	£40
14 January 1721	£40
14 May 1721	£40
14 September 1721	£40
14 December 1721	£40
14 March 1722	£40
14 June 1722	£40
14 September 1722	£40
14 December 1722	£40

These schedules were printed from time to time, and with more or less accuracy, in *The Course of the Exchange* from mid-July 1720, but it was not until the 9th of August that *The Course* began to print accurate instalment schedules. The values of 1st Subscription shares did not appear in *The Course* until 20th April and 2nd Subscription share values appeared on 29th April. Subscription share values were reported almost as frequently as were the values of original South Sea shares, but by the reopening of Company share transfer ledgers in late August, *The Course* was reporting the values of subscription issues only once or at most twice in a week. Traded subscription share values are found also in the abstracts of brokers' ledgers collected by the Committee of Secrecy set up to investigate the affairs of the Company.[18] Dates of trades found in these ledgers are precisely the same as the dates for which values are reported in *The Course*. At this time we can only speculate until further evidence is found, but it appears that transfers in subscription shares were more restricted than was the case for original shares. Perhaps the Company's transfer ledgers for subscription shares were opened less frequently than were the ledgers for original shares.

THIRD and FOURTH SUBSCRIPTION ISSUES

The last two series of subscription shares were small issues of expensive shares. South Sea share values were at their peak and the subscription price was set at £1000 for both issues. Evidence suggests, however, that these issues were not popular. In the Committee of Secrecy's broker ledgers Directors' trade in the 4th Subscription never appears and trade in the 3rd Subscription appears only rarely. The Company's accounting to the Committee of Secrecy of its loans on shares programmes rarely shows the 3rd subscription series being accepted as security for a loan and the 4th series never.[19] We know that more than £500,000 was still outstanding on the first £100 deposit for the 3rd Subscription as late as February 1721. Nor have we yet found any archival material that relates to private ownership in the 4th Subscription. We have no way of knowing whether these issues were fully subscribed or not. They could well have been, but subsequent trade in the shares quickly died away so that little evidence of these shares' value remains.

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[18] The Committee of Secrecy made its reports to the House of Commons in 1721 and left behind a valuable, but little used collection of papers at the House of Lords Record Office. The Committee was keenly interested in Directors' trade in South Sea liabilities and thus we have a number of abstracts of ledgers belonging to brokers who had dealings with South Sea directors. (See Box 158 of the Parchment Collection, HLRO).

[19] *Abstracts of the Ledger of the Loan*, Box 157, Parchment Collection, HLRO.

The first accurate reporting of the instalment schedule for the 3rd Subscription does not appear in *The Course* until the 9th August issue, but the schedule printed there was adopted on 15th June.[20] That schedule was:

3rd Subscription Shares Instalment Schedule, 15 June 1720	
16 June 1720	£100
2 January 1721	£100
2 July 1721	£100
2 January 1722	£100
2 July 1722	£100
2 January 1723	£100
2 July 1723	£100
2 January 1724	£100
2 July 1724	£100
2 January 1725	£100

The 3rd Subscription was also given rights to the 10 p.c. stock dividend that was promised to other shares in April. The only value data for these shares comes from the period when the transfer ledgers on original shares were closed. There was also some earlier data reported in Freke. We have noted already that we have yet no reliable information about how the books for the transfer ledgers were being managed. It could well be the case that values found in *The Course* are values for spot transactions in shares or, as for original shares, were values that contained a forward delivery premium of some sort.[21] Given that the 3rd Subscription was new just when the books on original shares were closed in late June, we believe there is a good chance that the values quoted in *The Course* were being established in a spot market for such shares. We have also noted that Directors' transactions in such shares are dated to the very same dates for which value quotations appear in *The Course*. We speculate that these were the dates that the Company actively managed the transfer ledgers in subscription shares and so were also the dates that the Directors dealt personally in such shares and placed such shares in private hands on behalf of the Company.

By mid-August 1720 it was clear that the firm was starved for cash. There was a noticeable panic in the measures the Directors proposed for raising more cash. The Court Minutes record a proposed unworkable scheme for getting subscribers to pre-pay their instalments and then at the same meeting the 4th and final cash subscription

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[20] Add. Ms. 25,499, *South Sea Company. Court Minutes No. 6*, (page 136) 15 June 1720.

[21] Trying to ascertain which value quotations are for forward delivery and which are for spot delivery is practically quite difficult. *The Course* was not consistent in printing informative qualifiers to quotations. Useful qualifiers such as "for the opening", "with deposit", "premium" or "discount" appear and disappear for reasons that are hard to fathom. Careful comparison with Directors' trades in subscription shares confirms that subscription share values found in *The Course* were quoted in terms of premia or discounts from the instalments paid. The same examination, however, suggests that *The Course*'s subscription share values were for spot transactions and not forward transactions, even in the period when original shares' values were generally expressed for forward delivery.

of shares was proposed, again at a subscription price of £1000 and with the following instalment schedule.[22]

4th Subscription Shares Instalment Schedule, 12 August 1720	
25 September 1720	£200
25 August 1721	£200
25 August 1722	£100
25 August 1723	£100
25 August 1724	£100
25 February 1725	£100
25 August 1725	£100
25 February 1726	£100

At the same meeting, however, they also decided to amend the instalment schedule for the 3rd Subscription (£100 supposedly already paid). One indication that the 3rd subscription issue was failing was that the Company allowed a 25 September deposit date for the first £100 to those individuals who had not yet subscribed. This would have been inequitable to those subscribers who paid their deposit on 16 June as originally required and then were required to come onto the following revised instalment schedule:

3rd Subscription Shares Instalment Schedule, 12 August 1720	
25 September 1720	£100
15 August 1721	£100
15 August 1722	£100
25 August 1723	£100
25 August 1724	£100
25 February 1725	£100
25 August 1725	£100
25 February 1726	£100
25 August 1726	£100
25 February 1727	£100

The Company soon changed its mind yet again with respect to the 4th Subscription and put forward the following revised shortened instalment schedule, which would again be quickly relengthened:

4th Subscription Shares Instalment Schedule, 23 August 1720	
at subscription	£200
25 March 1721	£200
25 September 1721	£200
25 March 1722	£200
25 September 1722	£200

[22] Add. Ms. 25,499, *South Sea Company. Court Minutes No. 6*, (pages 158-164) 12 August 1720.

The Court Minutes recorded (25th August) that something had to be done to make the payment schedules on the 3rd and 4th Subscriptions more equitable, but we do not see a resolution of this issue until after share values have largely collapsed. On 4 October and 7 October in *The Course* there were various reports of changed resolutions in the General Court. The 3rd and 4th Subscriptions' subscription prices were announced to both be revised downwards to £400 p.s.. The 3rd Subscription's revised instalment schedule became:

3rd Subscription Shares Instalment Schedule, 4 October 1720	
16 June 1720	£100
2 July 1721	£40
2 January 1722	£40
2 July 1722	£40
2 January 1723	£30
2 July 1723	£30
2 January 1724	£30
2 July 1724	£30
2 January 1725	£30
2 July 1725	£30

and the revised instalment schedule for the 4th Subscription became:

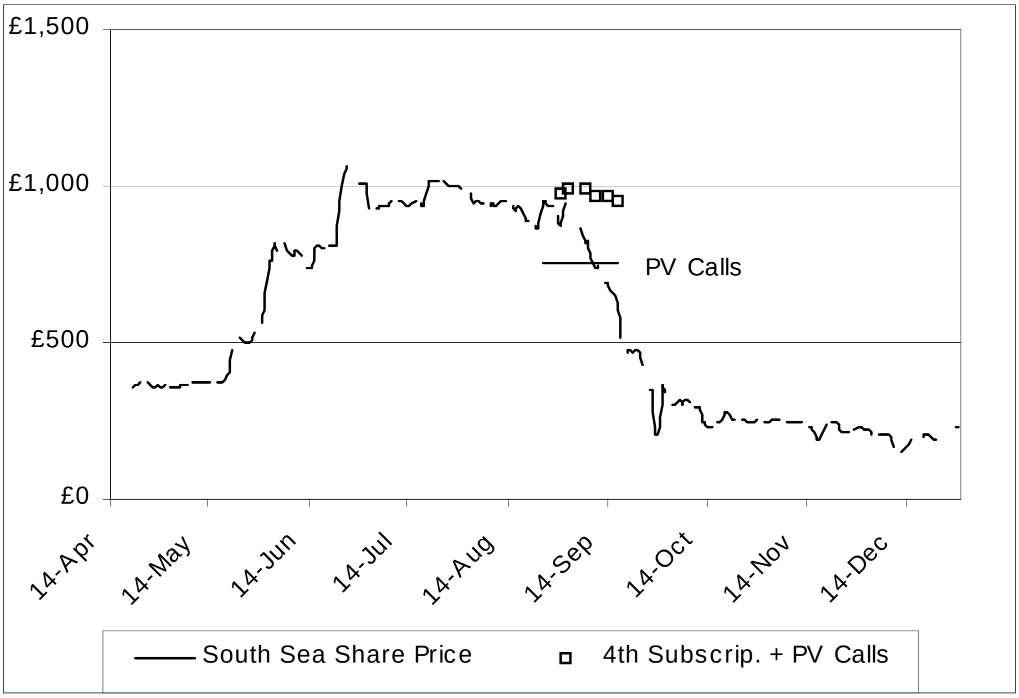
4th Subscription Shares Instalment Schedule, 7 October 1720	
6 August 1720	£200
26 September 1721	£25
26 March 1722	£25
26 September 1722	£25
26 March 1723	£25
26 September 1723	£25
26 March 1724	£25
26 September 1724	£25
26 March 1725	£25

The final gyrations in the arrangements for the 3rd and 4th Subscriptions were probably moot as far as making the issues attractive to potential subscribers. We are not inclined to use any of the values that appear in *The Course* after mid-September because it is difficult to ascertain to which instalment schedules they pertain. The quoted premia on these shares in late September and October also do not make sense. They would appear to make the 4th Subscription more valuable than the 3rd Subscription even though the 4th Subscription had a shorter instalment schedule and required larger up front payments on instalments. When we look again at the abstracts of brokers' ledgers collected by the Committee of Secrecy we find that there is no recorded Directors' trade at all in subscription shares beyond August 1720.[23] The value quotations disappear from *The Course* in any case in October and from thenceforward value data for the 3rd and 4th Subscriptions disappear from the historical record. The few useable prices for 4th Subscription shares and the only

[23] There were four mid-September instances in which Robert Knight placed some 3rd Subscription shares into circulation. Robert Knight was the subsequently disgraced and fugitive cashier for the Company.

reliable present values for their instalments that we can calculate are presented in the following graph.

South Sea Share Prices vs. 4th Subscription Share Prices



The present values of instalment payments for all subscription shares were straightforwardly calculated as the sum of the discounted present values of each of the calls. The discount rates used in these calculations are the yields on short-term East India Company bonds. Their quoted values are also found in regular issues of *The Course*. We use these yields as the approximate yields on readily-available risk-free loans. East India yields were steady and markedly lower than yields on either South Sea Bonds or on government annuities in 1720.

APPENDIX 2 - ROYAL AFRICAN COMPANY SUBSCRIPTION SHARES

The Royal African Company also issued subscription shares to the public in 1720. These shares and their values figured in the recent study by Carlos, Moyen and Hill (2002). The authors' (hereafter, CMH) immediate purpose was to use the value of the subscription shares along with the values of the firm's original shares to model a "fundamental", but did not ask if the two types of shares were in any important way different, which is the issue that we address here. CMH mention (page 68) there was a £10 dividend promised for April 1721 on the original shares and assert that this "led to different market prices for these shares." They confirm the reasonableness of this assertion by noting that the correlation between the values of the original and subscribed shares was almost perfect. We concur with these conclusions, but do not think that they are obvious and we believe they require careful demonstration.

First, how can we assume that subscribers to instalment shares did not value the possible right to defaulting upon promised subscription instalments? It is apparent that such "rights" had substantial value in the case of South Sea subscription shares. CMH even noted (pages 69-72) that in November the Royal African Company altered the terms of the instalment schedule on its subscription shares in order to forestall defaults. It thus would have been reasonable to wonder if there was some default option value embedded in the subscription share values.

The second reason we might wonder how these subscription shares were valued is that the relationship between original shareholders' rights and the rights held by new subscribers was a little bit more complex than implied by the story of the subscription told by CMH (page 68). The story also omits some significant features of the capital restructuring (the so-called "engraftment" of new capital) and is somewhat at odds with the account given by DuBois.[24] CMH state that Joseph Taylor purchased the entire issue for £75,696 and the reader is left with the impression that Mr. Taylor then had the issue in hand to sell on to the public, as if he was the underwriter of the issue. The authors indeed state that the £75,696 would be received by the Company "as a new cash infusion". If that was the only cash infusion coming to the Company, it is doubtful that original proprietors would have approved the subscription issue.[25]

The new shares were purposed to be sold for £25 each, but in April 1720 original shares stood at about £64 each. There could be as many as 15,696 new shares to go along with the 4,404 original shares taking the eventual number of shares issued and outstanding to 20,000. Once the new shares stood equal with the old shares, the

[24] DuBois (1938, n.68, page 394)gives a brief analysis of this "engraftment" of new capital and his analysis is based upon a documentary source (T70/115/199, *Report of the Trustees of the Proprietors of the Old Stock, Submitted to the Committee of Accounts of the Royal African Company, April 30, 1724*) not cited by CMH.

[25] Up to £392,400 (=15,696×£25) could have been collected in instalments on the new issue and by 1726 £387,967 had indeed been collected (see T70/1186, *Rough Drafts and Copy Book A* [contra 142], *"London April 1726, An account of the several sums of money paid in originally and since the Company's establishment by subscriptions, calls or otherwise"*). This was the actual cash infusion into the Company that resulted from the sale of subscription shares.

diluted value of shares would be about £33.4 per share.[26] The loss of about £40 per share in original shareholder wealth was at stake if some extra financial arrangements could not be made to accompany the issue of new shares. Otherwise, the new shareholders would end up owning more than 3/4's of the firm, but would have contributed not quite a 2/3's share to the total value of the firm's assets. The resolution of this problem was at the heart of the arrangements between the original proprietors, Mr. Taylor and other trustees. From the £392,400 that could be raised by subscription, £132,400 of it would be set aside for the sole use of the original proprietors in two ways. Mr. Taylor stood as a trustee for £80,000, of which he would keep £4,404 (his only fee as the devisor of the "engraftment"), and would eventually insure that the remaining £75,696 would be passed on to the firm's creditors. Original shareholders would thus be directly relieved of a debt obligation of almost £18 p.s.. If they could receive at least another £12 p.s. in compensation, their original wealth of about £64 p.s. would be preserved. This is where the promised cash dividend of £10 p.s. on original shares came in. Trustees would hold cash from the sale of new subscription shares sufficient to pay this cash over to the original shareholders and to relieve them of any further debts that might be undertaken on their behalf or subsequently discovered. This trust, however, would be paid no more than £52,400, which amounted to just over £12 p.s.. In the end this trust had to make payments to more creditors than was at first anticipated. DuBois reckoned that by 1724 original owners saw only a little more than £8 p.s. in the form of cash disbursements from the trustees.[27] In the end there was more than another £250,000 raised on the subscription shares that was for the Company's general use.[28]

The fact that the new shares were being issued at below "book" value (mentioned several times by CMH) was neither here nor there to original shareholders as long as their original wealth was at least preserved. This was done by the arrangements we have described. But this still leaves open the question as to how the value of the subscription shares should subsequently have been related to the value of original shares.

As we did in the case of the South Sea shares, we first ask if the subscription shares were simply fractional claims upon ex-dividend original shares. To take an example, compare the value of subscription shares and original shares on 1 June. On that date a £5 instalment was due on the shares and there were still to be paid another £5 (due 1 September) and a final £7 (due 1 December). Thus on 2 June a subscription share

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[26] The diluted share value is simply $\frac{4{,}304 \times £64}{20{,}000} + \frac{15{,}696 \times £25}{20{,}000} = £33.4$

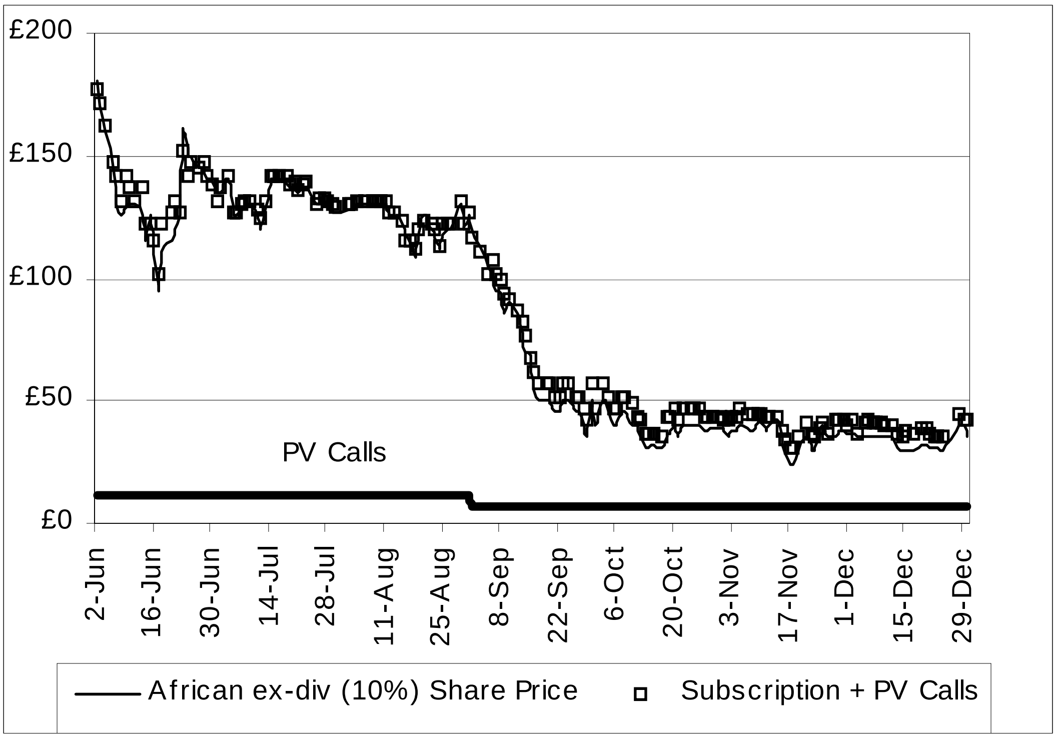
[27] He reported that from the £52,400 only about £35,395 was ever paid out to shareholders. The remainder of the fund went to creditors.

[28] It is clear in footnote 21 (page 69) that CMH have understated the scale of the "engraftment" and have not described key components of its operation correctly. In July 1720 the Company began to make loans to shareholders and CMH wonder where the cash for these loans came from. They note that the Company had money from Joseph Taylor, but state that this cash would not have been sufficient to run the loans on shares programme. This refers to the £75,696 that Mr. Taylor gathered and held in trust for the Company's creditors and so would not have been available for distribution to shareholders in any case. The implication is that CMH believe there was no other identifiable cash that was raised by the subscription shares, but there was not only cash for creditors, cash was available for dividend payouts and much was left over for running the firm's business and for making loans to shareholders.

would represent a fractional (£13/£25) claim on an original share that was shorn of its expected dividend; the value of that would be (£13/£25)×($P_{original}$ - PV[£10]).[29] We would expect that an arbitrage lower bound on subscription share values would be obeyed, thus

$$(£13/£25)\times(P_{original} - PV[£10]) + PV(instalments) \geq P_{original} .$$

If the bound was not obeyed, anyone who could buy a subscription share and borrow the present value of the remaining instalments would be able to obtain the equivalent of an original share at a cost smaller than that demanded in the market for original shares. If the bound appeared to be exactly binding, then we would have to conclude there was no possible value in the subscription shares attributable to a default option on instalments. In the figure below we depict the value quotations for African ex-dividend original shares and subscription shares plus the present value of the remaining instalments (calls).[30]



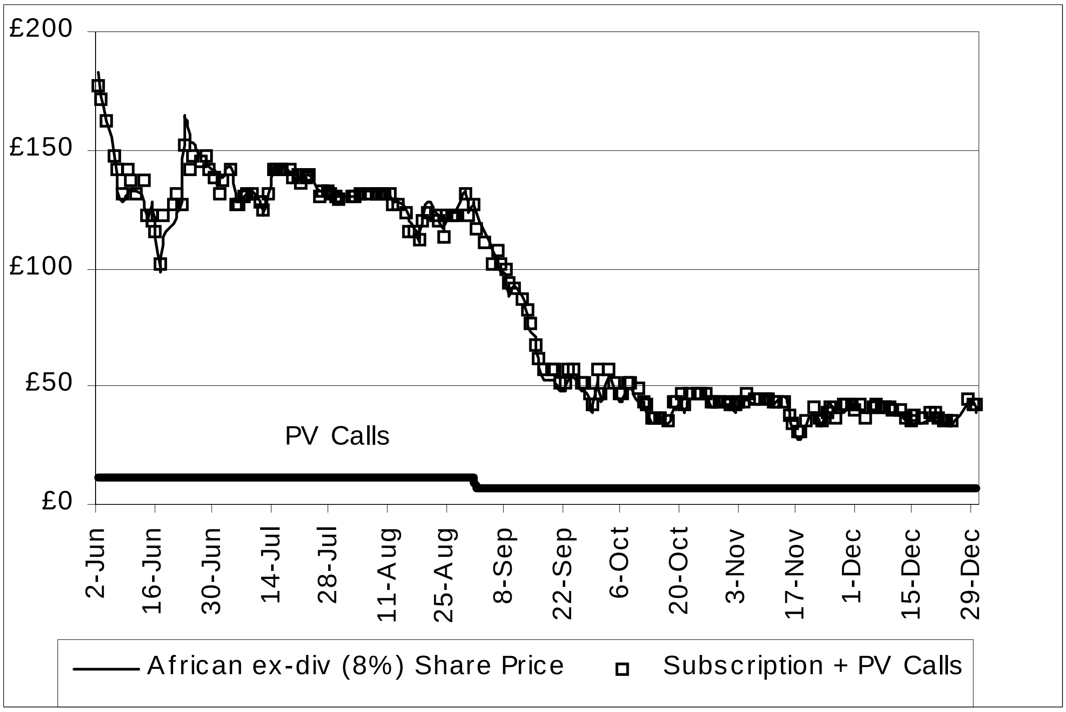
We can perhaps just persuade ourselves that the subscription shares have a distinctly greater value than they should have if they did not contain default option values. This added value appears clearly only after late September and can perhaps be discerned in the middle of June as well. If the added value is there, it is certainly very small. We must not forget, however, that in contemporary minds there was likely to be some confusion as to what an ex-dividend original African share would actually turn out to be. The expected dividend payout to original shareholders may well have been different from £10 and was likely to be lower. This could happen because the

29 We here assume momentarily that £10 was the expected dividend.

30 The present value of the instalments takes a slight step on 18 November (not perceptible in the graph) when the Company announced that only £4 would be due 1 December and not £7. The present value of the remaining £3 was calculated upon the assumption that it would not be due until 21 March 1721, which was the assumption made by CMH (page 79).

Company could discover or incur additional debt obligations (this did happen) or because the subscription issue itself would fail to collect sufficient instalment payments to sufficiently fill the £52,400-trust out of which cash dividends would be paid (this did not happen). In one way or another, the definition of an ex-dividend share was uncertain.

Incorporating a little of that uncertainty in our picture above removes all doubt that the Royal African subscription shares did not contain default option values. The actual ex-post dividend paid to shareholders was a longer delayed approximate £8 dividend, according to DuBois. If that could have been foreseen, then contemporaries could have persuaded themselves that ex-dividend original shares were a few pounds more valuable than we have depicted them above. The picture, modified for an expected 8% dividend, would be like the one below.



There is apparent not only the almost perfect correlation between original and subscription share values (noted by CMH), but there is also no difference in the level of the respective share values. There is no suggestion of a noticeable default option component to the subscription share values. Unlike the South Sea subscription shares, the Royal African subscription shares contained warrants that were very deep in the money. There was no likely future collapse in Royal African asset or share values that would make not exercising such warrants attractive. The warrant value itself was thus very close to the value of original (ex-dividend) shares minus the present value of the costs of obtaining the shares. There is, of course, also the possibility that the Company had other means of depriving subscribers of the option to default and this is why such option values do not appear in the subscription share values. We believe, however, that the former explanation is more plausible. Royal African Company subscription shares were packages of very-deep-in-the-money share warrants.

APPENDIX 3 - CALCULATING RESIDUAL WARRANT VALUES FROM SUBSCRIPTION SHARE PRICES

The residual warrant value is defined as the subscription share price minus the value of the fractional claim on an original share that is embodied in the subscription share. Let that fraction be denoted α. It is simply the sum of all instalments paid divided by the subscription price. On 20 June a 1st Subscription share was supposed to be £90-paid and the subscription price was £300. Thus on that date and for that subscription share α was £90/£300, or 30 p.c. For each subscription share we calculate α over time (foregoing time-subscripts in our notation) from the instalment schedules that appear in Appendix 1. Thereafter it is straightforward to calculate the residual warrant values as:

$$P_{\text{warrant}} = P_{\text{subscription}} - \alpha \times P_{\text{original}}$$

The warrant values so calculated represent values of claims upon fractions of original shares. These fractions change over time and are different for different warrant packages. In this paper we often make comparisons between the values of different warrant packages, but it would be pointless to do so unless we took steps to make sure the warrants and their values were as comparable as they could be. The first adjustment that we make is to restate warrant values so that the warrants represent a claim upon only <u>one whole</u> original share. To take the 20 June example again, a 1st Subscription share represented a future claim upon 70 p.c. of an original share (£210 in instalments yet to be paid, such that £210/£300 = 70 p.c.), but a 2nd Subscription share on the same date represented a future claim upon 90 p.c. of an original share. In other words, a No. 2 Warrant embodied in a 2nd Subscription share was a bigger warrant than a No. 1 Warrant. No.1 Warrant values should therefore be inflated somewhat relative to No. 2 Warrant values. Thus we reflate the warrant values by $1/(1-\alpha)$ to obtain the expression,

$$P^{*}_{\text{warrant}} = \frac{1}{1-\alpha} \times P_{\text{warrant}} = \frac{1}{1-\alpha} \times P_{\text{subscription}} - \frac{\alpha}{1-\alpha} \times P_{\text{original}} \ .$$

These adjustments certainly make No. 1 through No. 4 warrant values more validly comparable. There are, however, two more ways in which our warrants and their values are still not fairly comparable: 1) the warrant series and their values have different "dilution factors" associated with them and 2) our warrants are all of different maturities, or more properly, different durations. An approximate set of corrections for different dilution factors is not that difficult to achieve. This is because of the functional simplicity of the dilution factor itself. Making corrections in our warrant values that take into account different durations is not, however, so straightforward. This is because the maturity effect on an option's value is a function of a number of factors - a few of which are themselves hard to estimate, such as the volatility in South Sea share returns. In this Appendix we make an approximate correction for dilution factors. We do not, however, attempt to make corrections for different maturities. It is not necessary for the purposes of this paper and, in any case, is more properly delayed until we attempt to model and value the individual warrants embedded in our warrant packages.

Dilution Factors

Unlike call options on shares written by persons outside of the firm, a warrant is a call option written and issued by the firm itself. When a call option is exercised, the option writer receives the exercise price. In the case of a warrant being exercised, the recipient of the exercise price is the firm itself. This should raise the value of all the firm's shares. The firm, however, also has to issue new shares when the warrant is exercised and this should dilute the value of all the firm's shares. On a net basis, this latter dilution factor is the stronger. Standard textbook[31] methods for valuing warrants take this dilution factor into account. In these methods we first imagine a firm that has the same assets and in all other respects is the same as the South Sea Company, but this hypothetical firm has no warrants in its equity capital. The next step is to value a call option on a share in this firm when the call option has the same exercise price and maturity of the warrant that we ultimately wish to value. The value of our warrant is the value of this hypothetical call option multiplied by a dilution factor. The dilution factor is surprisingly simple in form. It is not a function of any of the warrant or call option characteristics; it is neither a function of the exercise price nor the maturity of the option or warrant. It is simply a function of the percentage increase in the numbers of shares that result from the exercise of the warrant. We call this percentage the "dilution q".

To illustrate these concepts imagine for the moment that the South Sea Company issued only one series of subscription shares. Let x stand for the number of original shares issued and outstanding and let y stand for the corresponding number of subscription shares. From our earlier discussion it is straightforward to write the value of total equity (V_E) as,

$$V_E = xP_{original} + yP_{subscription} = (x + \alpha y)P_{original} + yP_{warrant} =$$

$$(x + \alpha y)P_{original} + y(1 - \alpha)P^*_{warrant}$$

It is straightforward to see that percentage increase in the number of shares that would result if our warrants were exercised would be,

$$q = \frac{y(1 - \alpha)}{x + \alpha y}.$$

Now recall the hypothetical firm that has no warrants. We require that it has a value of equity equal to V_E and that V_E is spread over $x + \alpha y$ original shares. Of course our hypothetical firm does not have the same price of shares that our actual firm has; the value of each share in the hypothetical firm is $P^* = V_E/(x+\alpha y)$. The third-party option of interest to us is a call option written on a share in the hypothetical firm and this option has the same maturity and exercise price as the warrant we ultimately wish to value. Denote the call option value P_C. Then it is easy to show that

$$P_{warrant} = \frac{1}{1 + q} P_C.$$

[31] Brealey and Myers, *Principles of Corporate Finance* (2003), pages 646-647 is a good example.

The dilution q's for each of our warrants had to be quite different and we should take these differences into account if we are going to compare values across our four classes of warrants. First, from Table 1 it should be clear that subscription shares in the series Nos. 1 and 2 were by far more numerous than were shares in the series Nos. 3 and 4. The dilution q's for warrants contained in these series would have to have been quite different. For the latter two series the q's would have to be quite small, whereas for the former two series the q's could be quite large. When comparing warrant values and share values, it is more valid to make the comparisons between the series, P^*, $(1+q_1)P^*_{\text{No.1 Warrant}}$, $(1+q_2)P^*_{\text{No.2 Warrant}}$, $(1+q_3)P^*_{\text{No.3 Warrant}}$ and $(1+q_4)P^*_{\text{No.4 Warrant}}$, q_i standing for the dilution q pertaining to the i^{th} series warrant.

Because we are trying to measure the values of warrant packages rather than the values of individual warrants, measuring q's can only proceed via approximations. For example, in calculating q's we shall pretend that the warrant packages contain warrants that are all exercisable on the same maturity date. We know that was not the case. Each package consisted of a series of warrants that were exercisable at increasingly more distant maturity dates. In a sense the q's we shall calculate will be something like an average of the q's of the individual warrants contained in the package. Yet these approximations are potentially important and informative. Our data allow us to calculate α's precisely; they come from the instalment schedules on subscription shares. We have to be more approximate in estimating x and y through time. We have good information on actual numbers of share and subscription shares issued, but have to be a bit more careful in accounting for such shares actually outstanding. The South Sea Company purchased and held some numbers of original shares as treasury stock and we can account for these quite easily.[32] More important however were the effects of the Company's loans on shares programmes. It was the Company's stated purpose to use cash from the subscription series to make loans to shareholders. The Company would reclassify some of the shareholders' shares as treasury stock and use them as collateral for these loans. Thus the Company was quite busy in 1720 in taking back one form of liability (shares) and issuing in its stead other forms of liabilities.[33] These programmes operated on a very large scale and their net effect was to actually depress the numbers of shares outstanding. In fact we can quite clearly see that through most of 1720 the net numbers of shares outstanding actually tended to decline rather than increase. This of course is before we allow for the issues of shares that resulted from the exchanges for government annuities, but these did not happen until late 1720 and in 1721. Subscription shares were also taken as security for loans. Taking the issue numbers from Table 1 and thenceforward simply keeping track of the numbers and types of shares that were being booked as treasury stock is

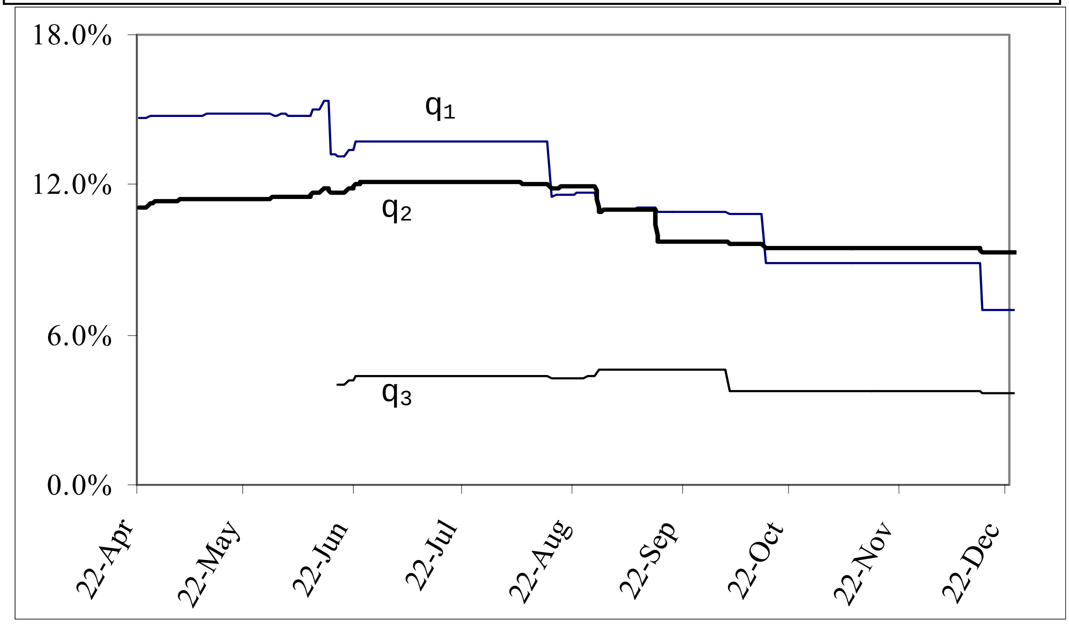
[32] See *Particulars of £332,250 Stock Bought for the Use of the Company*, Box 158, Parchment Collection, HLRO.

[33] The Company lent to shareholders cash and bonds, but because it was taking shares as collateral, the firm was handing conversion privileges (warrants) to shareholders. In this paper we keep track of shares that were taken back by the firm , but we do not add the conversion privileges back into equity. In other words, we are probably understating V_E. We allow the loans on shares programmes to reduce x, but we do not count the new warrants that the programme resulted in.

one way of approximating the numbers of shares and subscription shares outstanding.[34]

Before the first two subscription share series were issued, there were about 110,000 original shares issued and outstanding. Despite the fact that the subscription shares were subsequently issued and contained fractions of original shares within them and instalments on subscription shares were being paid and more original shares were thus being created, the numbers of original shares outstanding declined to about 95,000 by October. Of course there was to become a large issue of new shares (about 32,000) when the exchanges in May pledged by government annuitants were consummated. But this was not happen until late 1720 and the even larger issue of new shares that would result from the later exchanges arranged in August were to be delayed into 1721. On this basis we can calculate x's, y's and α's for our individual warrants series and thereby obtain q's. If we were to include in x the shares that would be eventually issued via the annuity exchanges, all q's would drop by nearly half, but their sizes relative to each other would not change much. In the graph below we plot these approximate q's. q_4 itself is only 1 p.c. or lower and we do not bother to plot it.

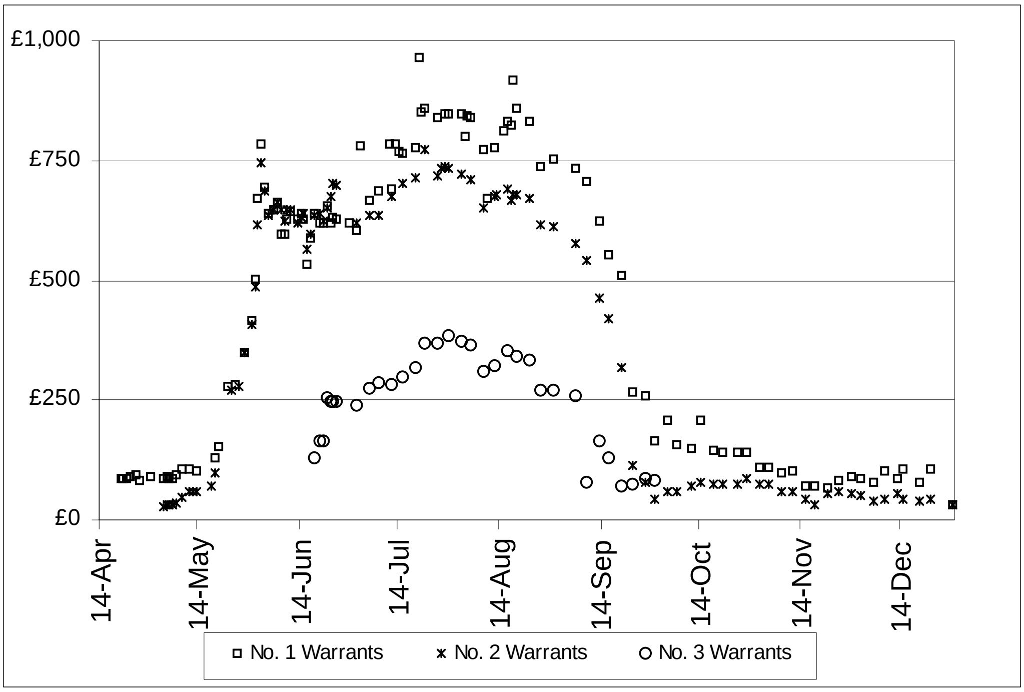
Approximate Dilution q's for South Sea Share Subscription Shares' Warrants in 1720



[34] Shares taken as collateral are accounted for in the *An Abstract of the Ledgers of the Loan*, Box 157, HLRO.

The series $(1+q_1)P^*_{\text{No.1 Warrant}}$, $(1+q_2)P^*_{\text{No.2 Warrant}}$, $(1+q_3)P^*_{\text{No.3 Warrant}}$ and $(1+q_4)P^*_{\text{No.4 Warrant}}$ can now be calculated. We shall refer to them as our 2nd approximation to the warrant values and we plot the first three below.

Approximation No. 2 for South Sea Share Warrant Values in 1720



A comparison of the levels of the warrant values in Approximation No. 2 requires explanation. For example, Warrants No.1 and No.2 values are at times not very far from each other. This is remarkable considering that the exercise price for a No. 1 Warrant was £300 and the exercise price for a No. 2 Warrant was £400. All other things the same, warrants or options with the higher exercise prices should be less valuable. If both warrants were very deep in the money we would normally expect that their values would be a near constant £100 apart. It is clear in the graph that their values are at times much less than £100 apart even when they are both deep in the money. What could account for this? The added value experienced by No. 2 Warrants could easily come from their longer duration.

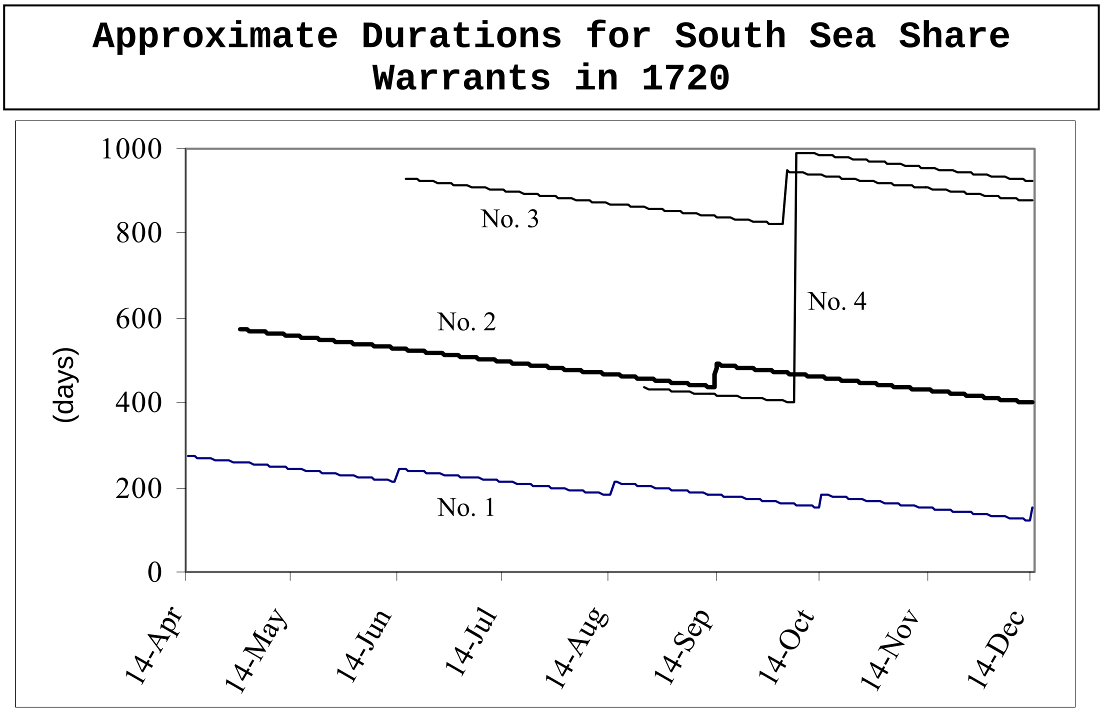
Maturity or Duration Factors

Warrant values are now more closely comparable in their levels, but there is still the issue of their different maturities or durations. In Appendix 1 we saw that the 1st and 2nd Subscription shares were designed to become fully paid by the end of 1721 and end of 1722 respectively, whereas the 3rd and 4th Subscription share series were of much longer duration. Their design went through many gyrations, but we can safely say that they were designed to become fully-paid ultimately by the middle or end of 1725. A call option's value is quite sensitive to the option's maturity, so we could expect that a large maturity adjustment would have to be made to make values of Warrant Nos. 1 and 2 comparable to values for Nos. 3 and 4. To proceed further in this analysis we have to have some way of defining the duration of our warrant packages.

Let us define duration of our warrant packages as the instalment-weighted average of the individual warrants' maturities that appear in the package. For example, on 20 June we have noted that a No. 1 Subscription share required 7 further £30 instalments at bi-monthly intervals, or in other words, Warrant No. 1 consisted of 7 warrants each with an exercise price of £30. The first warrant would mature in 2 months' time, the second warrant in 4 months and so forth. The duration of the Warrant No. 1 package on 20 June was therefore about

$$\frac{£30 \times 60 \text{ days} + £30 \times 120 \text{ days} + ... + £30 \times 420 \text{ days}}{£210}.$$

In this case the calculation is nothing more than the average maturity of all instalment payments since all the instalments are of the same size. But the 3rd Subscription's instalments were not all of equal size and in that case the instalment-weighted average of maturities would give a more accurate picture of the average time it would take for a subscriber to obtain an original South Sea share. Nevertheless the duration measures we present here, though crude, are certainly adequate for fixing the relative maturities of the warrant packages. A graph of the durations (in days) of each of our warrant packages is depicted below.



In the cases of Warrants Nos. 1 and 2 the stepwise jumps are simply a product of the way we have defined the duration approximation. Each time a warrant package "sheds" a warrant (every time a subscription instalment is paid) the average duration of the warrant jumps up by a little more than the instalment payment (in days). In the cases of Warrants Nos. 3 and 4 the jumps are a result of the redefined instalment schedules on Nos. 3 and 4 Subscription shares. What the chart shows quite accurately is the relative size of the warrant durations. The differences are actually quite enormous when considered in the light of the likely share returns' volatilities that were prevalent during the South Sea Bubble. For example, it is not puzzling that No.2 Warrants could be almost as valuable as No. 1 Warrants at times despite having an exercise price of £400 as compared to £300 for the No. 1 Warrant. No. 2 Warrant has more than twice the duration that No. 1 Warrant has. That raises the relative value of No. 2 to No. 1 Warrant and, in fact, it does not require very large volatilities to make No. 2 Warrant even more valuable than No. 1 Warrant. We estimate that a 100 p.c. annualised South Sea share return's volatility would be more than sufficient to make No.2 Warrant at least as valuable as No. 1 Warrant. This may seem a large number, but it is not. Over most of the summer of 1720, returns' volatility on South Sea shares was on the order of 1 p.c. to 2 p.c. per day.

Given that the durations of the warrant packages were so different, it will be difficult to apply any further adjustments that would make the warrant package values more comparable to each other in levels. What has been achieved in this Appendix, however, is a set of approximations that allows us to compare level and volatility of our warrant values individually to the level and volatility of shares' P*. In the next Appendix these values are smoothed to fill in the gaps in the data.

APPENDIX 4 - FIXED INTERVAL SMOOTHING OF DATA FOR MISSING VALUES VIA THE KALMAN FILTER

Our warrant and share value data are full of holes. Not only were there the usual non-trading days that were holidays and Sundays, but for reasons that are not at all clear, there were gaps - regular and irregular - in the reported data, especially in the data pertaining to subscription shares. We speculated about some of determinants of reporting frequencies for subscription share values in Appendix 1. In this Appendix we need to solve a very simple problem. Some of the econometric exercises that we perform are well-specified only for continuous data. To enable us to do these exercises, we purpose to fill in the missing data - to smooth it - using one of the popular econometric techniques for doing this type of thing - the Kalman filter.

The Kalman filter is an algorithm for forecasting data in the framework of a state-space model. The basic structure of state-space model is that it expresses the data (the signals) we wish to forecast or smooth as functions of observed and unobserved exogenous variables called states. When a dynamic multivariate model is written in this form, then maximum-likelihood estimates of the parameters of the forecast equations (the signal equations) and the equation parameters for the state-vectors (the parameters of the state processes) provide a setting in which the Kalman filter can be employed to forecast or smooth estimates of the state variables. From these estimates it is but another simple step to obtain forecasts or smoothed estimates of the signals. Harvey (1989) popularised the use of state-space models and the Kalman filter in a number of applications, including the smoothing over of missing values in times series data.[35]

Specifying an appropriate state-space model for smoothing of missing data is still very much of an art using intuition about the dynamics of unobserved state variables that will result in smoothed missing signal variables that "look right". The inspiration for the smoother used in this paper comes from some of the literature on smoothing missing temperature and other weather data.[36] Intra-seasonal and inter-seasonal variations in temperatures do not look all that different from the long-term and short-term variation in South Sea share values themselves and, in retrospect, it is not surprising that the smoothers that work well for missing weather data also work well for South Sea data.

One smoothing model takes the four times series, P^*, $(1+q_1)P^*_{\text{No.1 Warrant}}$, $(1+q_2)P^*_{\text{No.2 Warrant}}$ and $(1+q_3)P^*_{\text{No.3 Warrant}}$ (see Appendix 3) as the signals. The vector of state variables contains a unit root and the stationary components of the state variables are all ARMA(2,2). The signal equation for P^* also contains P_{original} and the intent of doing this was to make sure that P^* was not smoothed overly, but had some of the same variation that characterised P_{original}.

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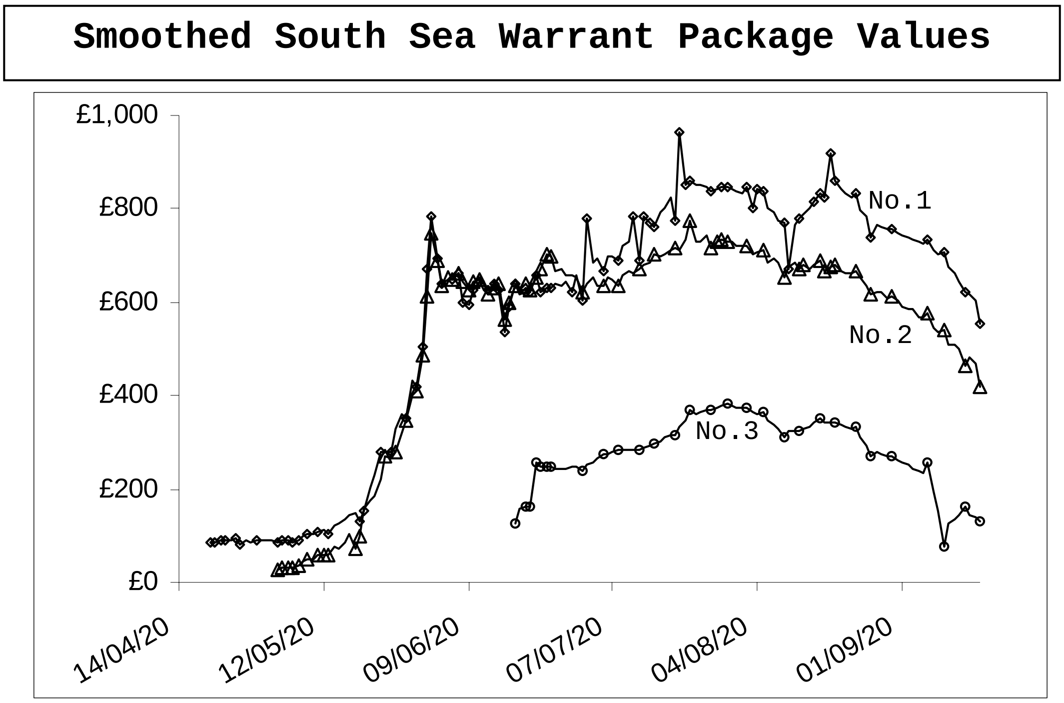
35 Harvey (1989, Section 8.7) discusses the application of methods that we employ here.

36 See Dunis and Karalis, "Filling Analysis for Missing Data: An Application to Weather Risk Management", in Dunis, C., Laws, J., and Naim, P. (eds.), *Applied Quantitative Methods for Trading and Investment*.

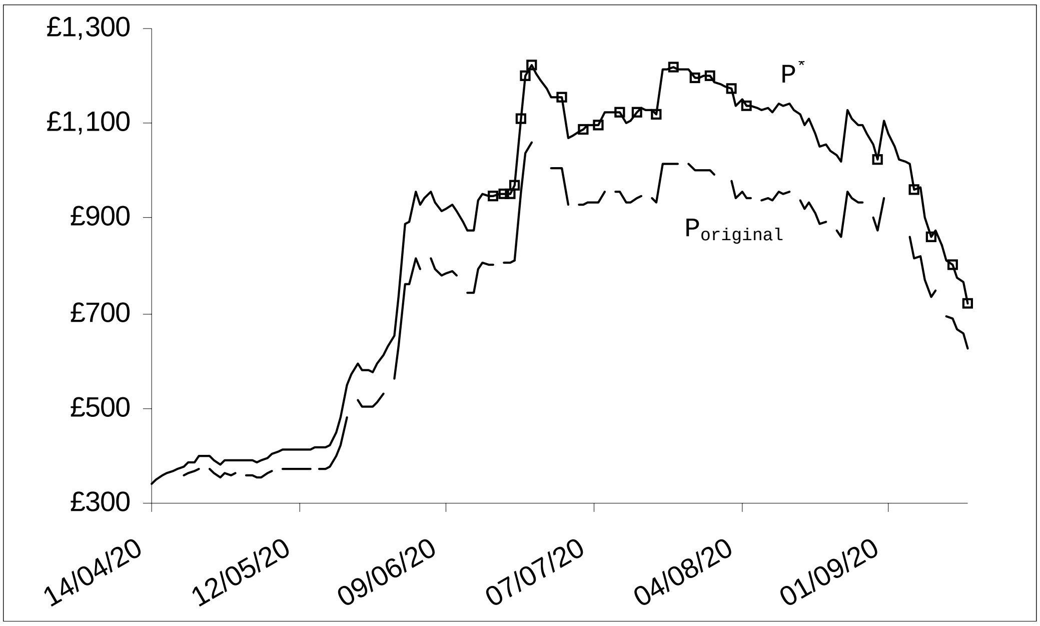
Because $P_{original}$, which also has gaps in its record, was used as an exogenous variable to smooth P^*, the smoothed record of P^* has gaps in its record as well. We have therefore smoothed this time series yet again to fill in the gaps. The gaps correspond to Sunday and holiday dates almost entirely. We used a simple univariate smoother in which we assumed that the smoothed series was driven by a state variable that was AR(1) with a unit root.

The graph below shows the smoothed values for the three time series $(1+q_1)P^*_{\text{No.1 Warrant}}$, $(1+q_2)P^*_{\text{No.2 Warrant}}$ and $(1+q_3)P^*_{\text{No.3 Warrant}}$. The lines represent the smoothed values and the symbols (◊, Δ, and °) represent the actual values that are being smoothed.



The next graph illustrates the relation between smoothed P^* and $P_{original}$. As explained in Appendix 3, P^* represents the per share equity value for the firm when it has no warrants. It therefore spreads the total equity value, which includes the value of warrants, over the number of original shares and thus must be a larger number than $P_{original}$ itself. It is the corrected version of original share value that is comparable in level and volatility to the smoothed warrant values illustrated above.

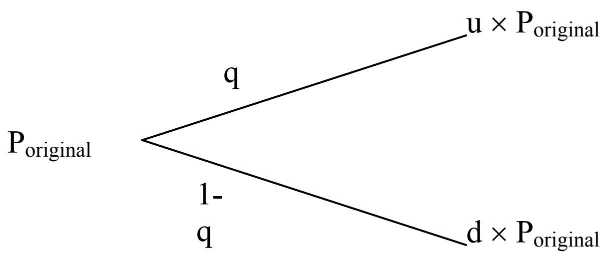
Smoothed P* and South Sea Original Share Prices



APPENDIX 5 - OPTION PRICING THEORY FOR VOLATILITIES

Our warrant packages should obey the same volatility relations that call options obey. In this and other papers we adopt the theory and terminology as presented in Cox and Rubinstein's *Options Markets*. One relevant set of relations that we test are described on pages 186-187 under the topics of *option elasticity* and *option risk*.

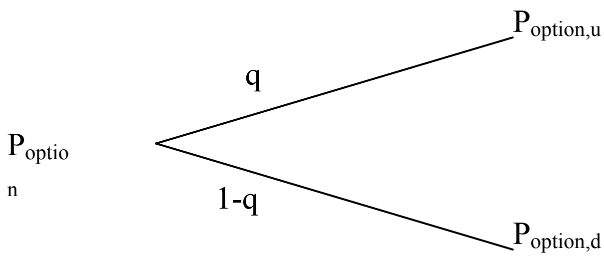
There are a number of frameworks in which to present the fundamentals of option pricing theory. The framework is determined largely by the way in which we choose to model the stochastic character of share prices. The range of choice in such stochastic models is immense, but in this paper we can make do with the simplest of them all - the binomial random walk. The binomial random walk is simple, but is also powerful. It leads to a number of exact pricing formulae for options that serve as accurate approximations to formulae that can be obtained from other more complex stochastic models of the share price fundamental. Its construction starts in the following simple manner. Over some appropriately short interval of time, the price of an original share is assumed to take a random walk either "up" or "down". The potential size of these steps is denoted u and d and the probability of an "up" step is q. This binomial random walk and its attendant probabilities is illustrated thus:



Note that u > 1 and 0 < d < 1. It is quite straightforward to show that the unconditional (constant) standard deviation of the share's return is:

$$\nu_{original} = [q(1-q)(u-d)^2]^{1/2}.$$

We can depict the option's random walk that corresponds to the random walk in the share price thus:



The standard deviation of the option's return is:

$$\nu_{option} = \left[q(1-q)\left(\frac{P_{option,u} - P_{option,d}}{P_{option}} \right)^2 \right]^{1/2}.$$

The pricing solution for P_{option} can be expressed in a number of ways, but the most convenient for our purposes is the expression that uses Δ, the first derivative of the P_{option} with respect to $P_{original}$. In the framework we have presented, it can be shown that

$$\Delta = \left(\partial P_{option} \middle/ \partial P_{original} \right) = \frac{P_{option,u} - P_{option,d}}{(u-d)P_{original}} .$$

The definition of an option's elasticity with respect to the price of an original share is:

$$\Omega = \left(P_{original} \middle/ P_{option} \right)\left(\partial P_{option} \middle/ \partial P_{original} \right) = \left(P_{original} \middle/ P_{option} \right)\Delta .$$

Cox and Rubinstein complete their demonstration on option risk by showing that:

$$\nu_{option} = \Omega\nu_{original}, \text{ with } \Omega \geq 1.$$

We do not clutter the expressions above with time subscripts, but it should be clear from our discussion that Ω, Δ and ν_{option} are all time-varying even though $\nu_{original}$ does not have to be time-varying. For options that are far out of the money ($P_{original}$ being far below X, the option exercise price) the option is nearly worthless, but Ω is quite large (because $P_{original}/P_{option}$ is quite large) and is quite sensitive to any further changes in $P_{original}$. For options that are deep in the money ($P_{original}$ being far in excess of X) the option is worth nearly $P_{original}$ minus PV(X), but Ω is very close to 1 and is not very sensitive to further changes in $P_{original}$.

Finally, there is the basic relationship between the value of the option and share return volatility. From the expressions above it is simple to write

$$\nu_{option} \times P_{option} = \nu_{original} \times P_{original} \times \Delta.$$

Since every item in this equation is positive, it follows that $\partial P_{option} / \partial \nu_{original} > 0$. For options that are in the money Δ will be nearly $P_{option} / P_{original}$ since Ω will be near 1. As $\nu_{original}$ gets very large, both Ω and Δ are approximately 1.

APPENDIX 6 - ECONOMETRIC MODELS AND RESULTS FOR VOLATILITIES

The BEKK GARCH Model

The model we use is the bi-variate version of the multi-variate BEKK GARCH (Generalised Autoregressive Conditional Heteroscedasticity) model presented by Engle and Kroner (1995). A more accessible presentation of the model and its advantages and disadvantages are found in Campbell, Lo and MacKinlay (1997, pages 490-494).

The model is of the form:

$$\Sigma_t = C'C + A'\vec{\varepsilon}_{t-1}\vec{\varepsilon}'_{t-1}A + B'\Sigma_{t-1}B,$$

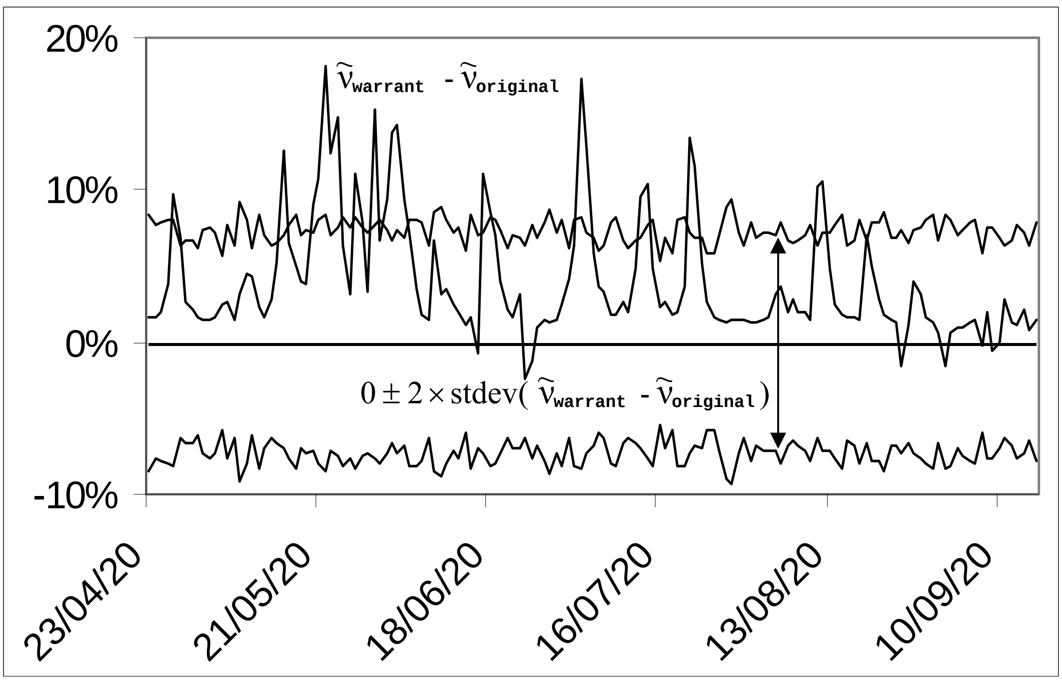
where A, B and C are all 2×2 matrices of estimable parameters and the 2×1 vector $\vec{\varepsilon}_t$ represents the time-t shocks to returns on warrants and shares. Σ_t is the 2×2 matrix containing $\nu^2_{warrant}$ and $\nu^2_{original}$ on the diagonal.

The model is estimated via maximum-likelihood and typically requires a large number of computational iterations to achieve convergence to a solution. It was no different in our three uses of the model in which several hundred iterations were required in each case. Instead of trying to construct asymptotic standard errors on estimated $\nu^2_{warrant}$ and $\nu^2_{original}$, we have resorted to bootstrap approximations to them. The theoretical principle behind the bootstrap is that the estimated model residuals should be useful for simulating of the sampling error of the vector $\vec{\varepsilon}$'s. The researcher samples (with replacement) from the estimated time series of $\vec{\varepsilon}$'s and then uses the sampled $\vec{\varepsilon}$'s to simulate the estimated model. In this way is generated what can be called a pseudo-sample, or what has come to be called a bootstrap sample, of estimated $\nu^2_{warrant}$ and $\nu^2_{original}$. In repeating this resampling procedure a large number of times, a bootstrap sample distribution of simulated data can be constructed and from this distribution we can construct a large number of measures of statistical uncertainty such as standard errors of estimation and forecast and confidence intervals. One of the great advantages of resampling plans such as the bootstrap is that statistics can be constructed for complex theoretical entities for which asymptotic statistics based upon the model's estimated parameters are difficult to derive and compute. A classic description of the bootstrap and other resampling plans is presented by Efron (1979).

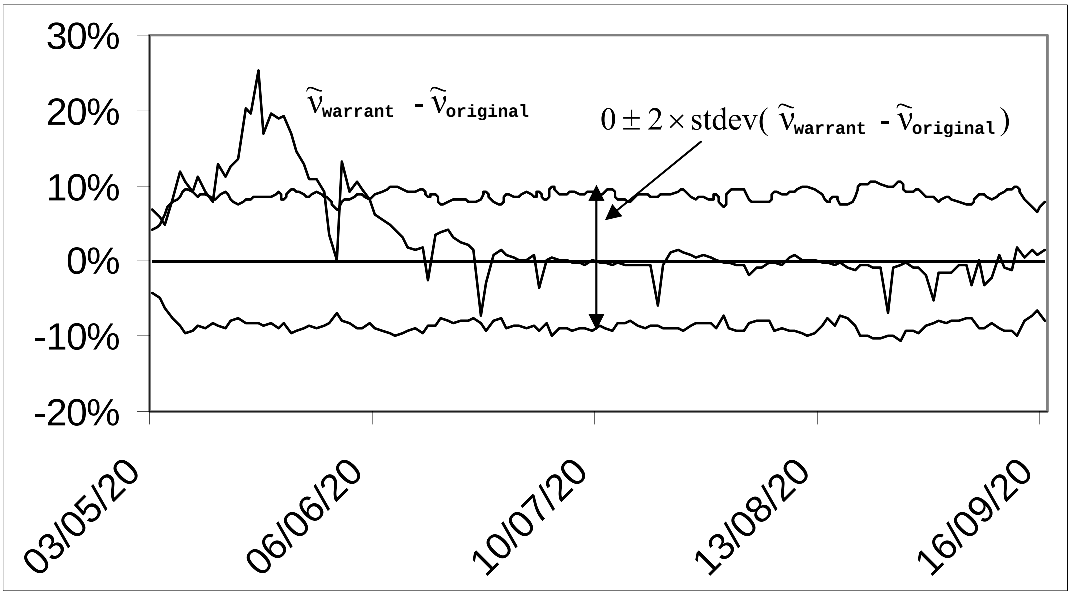
We have taken the square roots of estimated $\nu^2_{warrant}$ and $\nu^2_{original}$ and presented them as $\tilde{\nu}_{warrant}$ and $\tilde{\nu}_{original}$ in the main body of the text. The bootstrapped samples of the $\vec{\varepsilon}$'s were then used to simulate the estimated model many times to generate a bootstrap sample distribution for $\tilde{\nu}_{warrant}$ and $\tilde{\nu}_{original}$. We need to explain at this point that we have not re-estimated the model in each bootstrap sample. We would do this if we were trying to construct bootstrap distributions of estimated parameters. This is a common application of the bootstrap, but one that would probably be computationally too expensive in the context of a BEKK model. Therefore our bootstrapped distributions of $\tilde{v}_{warrant}$ and $\tilde{v}_{original}$ do not reflect uncertainty in BEKK parameter estimation in any way. If they did, then our bootstrapped estimates of $\tilde{v}$ dispersion might be a bit larger than we illustrate them to be. This is why we choose to show a rather wide band (two standard deviations) around zero for the "test" statistic $\tilde{v}_{warrant}$ - $\tilde{v}_{original}$, the excess volatility in warrant returns. It is a conservative way of illustrating our main point; $\tilde{v}_{warrant}$ - $\tilde{v}_{original}$ was probably at least as great as zero and at times was greatly in excess of zero.

For each of the three warrant packages, Nos. 1 through 3, we present estimated $\tilde{v}_{warrant}$ - $\tilde{v}_{original}$ and a band of two estimated standard deviations of $\tilde{v}_{warrant}$ - $\tilde{v}_{original}$ on either side of the zero line. The excess returns' volatility depicted in these graphs should be proportional to Ω. The volatility peaks correspond reassuringly to episodes in which the warrants are near the money or, as in the case of Warrant No. 3, are very near or just out of the money. Average $\tilde{v}_{warrant}$ - $\tilde{v}_{original}$ is positive for all three warrants, although the late summer volatility for Warrant No. 2 does not appear to be sufficiently positive. Certainly there are no anomalous negative spikes for $\tilde{v}_{warrant}$ - $\tilde{v}_{original}$ greater than two standard deviations below zero, but there are a small number of such spikes more than one standard deviation below zero. This is the worst that can be said for our results in terms of excess volatility in returns.

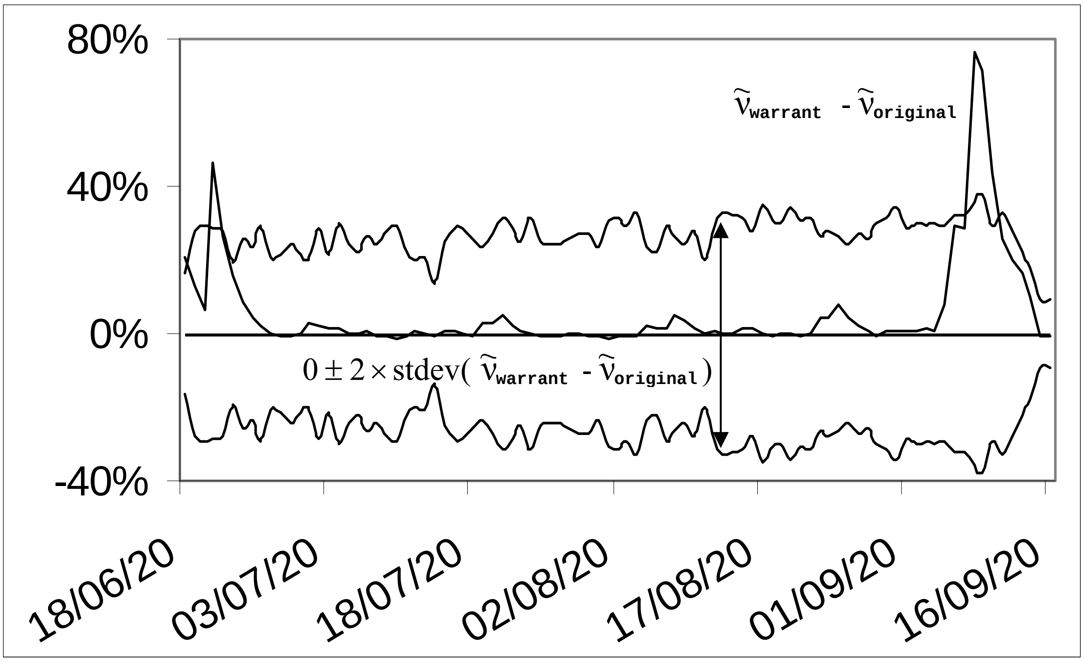
Warrant No. 1 Returns' Excess Volatility



Warrant No. 2 Returns' Excess Volatility



Warrant No. 3 Returns' Excess Volatility



The Component ARCH Model

In the main text we argued that a measure of "in-the-moniedness" influencing elasticity (Ω), should be associated with persistence in $\nu_{warrant}$. A very direct measure is to take the ratio of original share value to the present value of the calls on the subscription shares and call it "inout". We saw in the first section of the paper that as long as share values were way above the value of the instalments on subscription shares, any warrant component to subscription share values would have to be far in the money. Conversely, as share prices approached the present value of the subscription shares instalments, warrant components would have smaller value.

Log(inout) measures the percentage degree by which the warrant is "away" from the money. Right at the money (inout=1), log(inout) is zero. The percentage sensitivity of inout is small for large values of inout and the sensitivity is large for small values of inout. Log(inout) therefore has some features in common with theoretical volatility elasticity. Ω is large for warrants that are near the money or out of the money; log(inout) quickly becomes a large negative number as the warrant goes out of the money. Ω gradually goes to one for warrants that are going into the money; log(inout) increases, but at a decreasing rate as the warrant gets further into the money. A linear (negative) relationship therefore between Ω and log(inout) is reasonable. We include log(inout) in the component ARCH model therefore as an exogenous component of the persistent warrant volatility.

We also argued in the main text that original share volatility appeared to have a large transitory component. We are tempted therefore to use $\nu_{original}$ (actually the ARCH proxy for it - $r_{original}$ squared) as an exogenous component of the transitory part of warrant volatility.

Our component ARCH model is therefore of the following form:

$$r_{warrant} = c_r + \varepsilon_t \Leftrightarrow \text{a random walk in warrant values;}$$

$$\nu^2_{perm,t} = c_p + \lambda \log(inout_t) + \rho(\nu^2_{perm,t-1} - c_p) + \varphi(\varepsilon^2_{t-1} - \nu^2_{warrant,t-1});$$

$$\nu^2_{warrant,t} - \nu^2_{perm,t} = \delta r^2_{original,t} + \alpha(\varepsilon^2_{t-1} - \nu^2_{perm,t-1}) + \beta(\nu^2_{warrant,t-1} - \nu^2_{perm,t-1}).$$

In Table 2 we present estimated values and standard errors for the parameters of the component ARCH model for each of three warrant volatilities. The expected sign on λ is negative and the expected sign on δ is positive. In this table we find significantly negative estimates for λ for all three warrants and a significantly positive estimated value for δ for the No. 1 Warrant.

Further estimated features of the model are of interest. The first equation above is the mean equation, which provides the shocks (ε's). The next two equations are respectively the permanent and transitory component equations. Which equation plays the role of the permanent component and which equation is the transitory component depends on the relative size of the parameters. As long as $\rho > \alpha + \beta$ the second equation represents the permanent component. As long as $\rho < 1$, a shock (ε) to returns might have some long-term effect on volatility, but its persistence will not be perfect. The effect of the shock dies away at the rate of 1 - ρ per day (p.d.). In the absence of such shocks, the mean or average level of volatility coming from the second equation is c_p + Mean(log[inout])×λ/(1-ρ). The effect of a return's shock on the transitory component dies away faster than does the shock's effect on the permanent component. It removes itself at the rate of 1 - (α+ β) p.d. The average short-run volatility is Mean($r_{original}$-squared)×δ/(1 - (α+ β)) p.d.

In all three models the convergence to the mean transitory component of volatility is quite quick. Only Warrant No. 1 has a mean transitory component that is significantly different from zero, but it is quite small in size. It is calculated

Mean($r_{original}$-squared)×0.22/0.88 ≅ 0.001247×0.252 or about 1.8 p.c. p.d. when expressed in units of $\nu_{warrant}$. The average size of the permanent components for all three models are all the same order of magnitude. For example No. 1 Warrant has an estimated ρ of 0.98.This is quite large and means that the effects of a shock on the permanent component of volatility dies away at a rate of about only 2 p.c. p.d. In squared volatility units the average level that it converges to is c_p + Mean(log[inout])×λ/(1-ρ) ≅ 0.02 - 1.38×0.0002/0.02 ≅ 0.006 or about 7.7 p.c. p.d. in return volatility units. For Warrant No. 2. λ is larger and ρ is smaller. The effects of a shock die away much faster than they do for Warrant No. 1 volatility, but Warrant No. 2 volatility is a bit more sensitive (λ is larger) to shocks that take the warrant out of the money. The average level of permanent volatility is a little bit smaller than it is for Warrant No. 1; 0.02 - 0.98×0.0014/(1-0.88) ≅ 0.0039368 or 6.3 p.c. p.d. in returns volatility units. Warrant No. 3 starts its life in mid-June, but is quickly out of the money, yet its average level of permanent volatility is a little bit smaller than it is for Warrant No. 2; 0.01 - 0.29×0.0063/(1-0.79) ≅ 0.0032 or 5.7 p.c. p.d. in returns volatility units.

Table 2

Component ARCH Models for South Sea Warrants Returns

Dependent Variable: $\text{dlog}(1+q_1)P^*_{\text{No.1 Warrant}}$

Sample(adjusted): 4/21/1720 - 9/16/1720

	Estimated Coefficient	Standard Error	z-Statistic	Probability 0 not false
c_p	0.02	<0.01	61.44	<0.01
ρ	0.98	<0.01	205.76	<0.01
φ	-0.03	0.01	-2.62	0.01
λ	-0.0002	0.00005	-4.21	<0.01
α	0.26	0.12	2.24	0.03
β	-0.14	0.07	-2.15	0.03
δ	0.22	0.10	2.14	0.03

Mean dependent var	0.012
S.D. dependent var	0.076
Akaike info criterion	-2.756
Schwarz criterion	-2.595

Dependent Variable: $\text{dlog}(1+q_2)P^*_{\text{No.2 Warrant}}$

Sample(adjusted): 5/04/1720 - 9/16/1720

	Estimated Coefficient	Standard Error	z-Statistic	Probability 0 not false
c_p	0.02	0.01	1.33	0.18
ρ	0.88	0.11	7.96	<0.01
φ	0.44	0.15	2.90	<0.01
λ	-0.0014	0.0008	-1.77	0.08
α	-0.18	0.15	-1.19	0.23
β	0.72	0.35	2.05	0.04
δ	0.06	0.07	0.86	0.39

Mean dependent var	0.020
S.D. dependent var	0.088
Akaike info criterion	-3.281
Schwarz criterion	-3.109

Dependent Variable: $\text{dlog}(1+q_3)P^*_{\text{No.3 Warrant}}$

Sample(adjusted): 6/19/1720 - 9/16/1720

	Estimated Coefficient	Standard Error	z-Statistic	Probability 0 not false
c_p	0.01	<0.01	5.43	<0.00
ρ	0.79	<0.01	617.55	<0.00
φ	0.01	0.08	0.10	0.92
λ	-0.0063	0.0012	-5.38	<0.00
α	0.37	0.09	4.21	<0.00
β	-0.09	0.01	-8.61	<0.00
δ	0.10	0.34	0.29	0.77

Mean dependent var	<0.01
S.D. dependent var	0.117
Akaike info criterion	-3.017
Schwarz criterion	-2.795

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