Production and Hedging Decisions in the Presence of Basis Risk: Note

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JEL Classification: D8.

Key Words: basis risk, output price uncertainty, futures markets, hedging

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Paroush and Wolf (1989) modeled output hedging in the presence of basis risk. They showed that (in the absence of scale shift) the optimal hedging and output fall in response to basis risk. However, they used a second-order Taylor's approximation of the utility function. It is worth noting that using such approximations is equivalent to assuming normality and constant absolute risk aversion or a quadratic utility; in their case, it is equivalent to assuming normality and constant absolute risk aversion since they explicitly assumed constant absolute risk aversion. Their results may not be obtainable if a higher-order Taylor's expansion is used. Thus, there is a loss of generality. Also, they did not show the impact of basis risk on the ratio of hedging to output (hedging as a fraction of output), which is a more relevant variable than the absolute change in either of the decision variables. The absence of such results constitutes a major gap in the hedging literature.

Consequently, this note provides two extensions. First, it generalizes Paroush and Wolf's results (Propositions 1 and 2) by using a general utility function and general distributions. Second, it shows the impact of basis risk on the ratio of hedging to output. Below is a brief description of the model.

The risk averse firm maximizes the expected utility of the profit π

$$\underset{y,h}{Max} Eu\left(\pi\right)$$

$$\pi = py + \left(b - g\right)h - c\left(y\right)$$

$$p = \bar{p} + \sigma\varepsilon;\;\; E\varepsilon = 0$$

$$g = p + \delta\eta \text{ (in the absence of scale shift)}; E\eta = 0; Eg = \bar{p}$$

where y is the output, h is the hedging, p is the stochastic spot price in period two with mean $\bar{p}$ and variance σ^2, b is the deterministic futures price in period one, η is an error term and independent of ε, and $c(y)$ is the cost function $(c'(y) > 0, c''(y) > 0)$.

The first-order conditions are

$$Eu'[p - c'(y)] = [\bar{p} - c'(y)]\, Eu' + Cov(u', p) = 0 \tag{1}$$

$$Eu'[b - g] = [b - \bar{p}]\, Eu' - Cov(u', p) - \delta Cov(u', \eta) = 0 \tag{2}$$

Adding (1) and (2) ,we obtain $c'(y) = b - \delta Cov(u', \eta)$.

Proposition 1: $y^* > y^\circ$

where y^* and $y^\circ$are the optimal production in the absence and presence of basis risk, respectively.

Proof: In the absence of basis risk, $c'(y) = b$; and thus $c'(y^\circ) < c'(y^*)$ since $Cov(u', \eta) > 0$.

Proposition 2: $h^\circ < h^*$

where h^* and $h^\circ$are the optimal hedging in the absence and presence of basis risk, respectively.

Proof: Define the sets A and $\sim A$ such that

$$A = \{p | p - c'(y^\circ) \geq 0\}$$

$$\sim A = \{p / p - c'(y^\circ) \leq 0\}$$

When $h^\circ < y^\circ$, for any $p \in A$ and $p \in \sim A$, we must have

$$\pi^\circ(p) \geq \pi^\circ(p')\,; p \in A, p \in \sim A$$

where π° is the profit with both risks. Since $u'' < 0$, the inequality above implies

$$u'(\pi^\circ(p)) \leq u'(\pi^\circ(p'))\,; p \in A, p \in \sim A$$

Therefore,

$$S \equiv \underset{p \in A}{Sup} u'(\pi^\circ) \leq I \equiv \underset{p \in \sim A}{Inf}\, u'(\pi^\circ)\,.$$

Since S and I are both positive, there must exist a positive constant t such that

$$\frac{E_\eta S}{u'(E_\eta \pi^\circ)} \leq t \leq \frac{E_\eta I}{u'(E_\eta \pi^\circ)},$$

so that

$$tu'(E_\eta \pi^\circ) \geq E_\eta S \geq E_\eta u'(\pi^\circ)\,,\; p \in A \tag{3}$$

where the last inequality in (3) holds since S is a maximum. Now (3) implies

$$[p - c'(y^\circ)]\, tu'(E_\eta \pi^\circ) \geq [p - c'(y^\circ)]\, E_\eta u'(\pi^\circ)\,,\; p \in A \tag{4}$$

Similarly,

$$[p - c'(y^\circ)]\, tu'(E_\eta \pi^\circ) \geq [p - c'(y^\circ)]\, E_\eta u'(\pi^\circ)\,,\ p \in\sim A \tag{5}$$

Thus,

$$[p - c'(y^\circ)]\, tu'(E_\eta \pi^\circ) \geq [p - c'(y^\circ)]\, E_\eta u'(\pi^\circ)\,, \forall p \tag{6}$$

Taking expectations with respect to ε, we obtain

$$E\varepsilon\, [p - c'(y^\circ)]\, u'(E_\eta \pi^\circ) = 0 \tag{7}$$

Now, let $\alpha \equiv E_\varepsilon [p - c'(y)]\, u'(E_\eta \pi)$, then (7) implies that $d\alpha = 0$ when basis risk is added. Totally differentiating α (and holding the parameters constant), we obtain

$$d\alpha = Eu''\, [p - b]^2\, [dy - dh] - c''(y)\, Eu' dy = 0 \tag{8}$$

and thus $dh < 0$ since $dy < 0$.This result holds when $h^\circ \geq y^\circ$ (the proof is similar). Another important implication of (8) is $d(y - h) > 0$.

Proposition 3: $h^\circ / y^\circ < h^* / y^*$ if $h \leq y$

Proof: From Proposition 2, $dy - dh > 0$ and thus $|dh| > |dy|$; therefore

$$d(h/y) = \frac{ydh - hdy}{y^2} < 0$$

in response to basis risk.

References

[1] Paroush, J. and Wolf, A. (1989): "Production and Hedging Decisions in the Presence of Basis Risk," *Journal of Futures Markets*, 9: 547-563.