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Abstract

This paper extends the existing estimation methods to allow empirical estimation and hypothesis testing under simultaneous price and output uncertainty.

Running Head: price and output uncertainty
JEL Classification: D21, D81.
Key Words: estimating equations, hypotheses testing, output uncertainty, price uncertainty, utility.

0.1 Introduction

There is hardly any literature that provides an empirical analysis of the behavior of the competitive firm under uncertainty in the absence of hedging. Some studies conducted in the presence of hedging include Arshanapalli and Gupta (1996), Rolfo (1980), and Lapan and Moschini (1994). Arshanapalli and Gupta included price uncertainty but not output uncertainty. They derived estimating equations by applying uncertainty analogues of Hotelling's lemma and Roy's identity to the indirect expected utility function. However, as will be apparent later in this paper, their method is not directly applicable to the present models. The other studies included both price and output uncertainty in the presence of hedging. For example, Rolfo (1980) computed the ratio of hedge to expected output for cocoa producers. Lapan and Moschini (1994) calculated the same ratio for soya bean farmers.

The goal of this paper is to devise a method that enables us to empirically estimate a simultaneous price and output uncertainty model; Moreover, it makes develops methods to test important hypotheses regarding the functional form and the attitudes toward risk, such as risk neutrality, separability, quadratic utility, and constant absolute risk aversion CARA.

0.2 The model

A competitive firm selling a single output faces an uncertain output price given by $p = \bar{p} + \sigma\varepsilon$, where ε is random with $E[\varepsilon] = 0$ and $Var(\varepsilon) = 1$, so that $E[p] = \bar{p}$ and $Var(p) = \sigma^2$. The level of output realized at the end of the production process is also not known ex ante. Output has both a random and a nonrandom component and is given by q, where q is random and defined as $q = y + \theta\eta$,[1] where η is random with $E[\eta] = 0$ and $Var(\eta) = 1$, so that $Var(q) = \theta^2$ and the expected value of output is $E[q] = y$. We assume that $Cov(\varepsilon, \eta) = 0$.[2] Costs are known with certainty and are given by a cost function, $c(y, \mathbf{w})$, which displays positive and increasing marginal costs so that $c_y(y, \mathbf{w}) > 0$ and $c_{yy}(y, \mathbf{w}) > 0$. While y represents expected output, it may usefully be thought of as the level of output which would prevail in the absence of any random shocks to output. The firm may be thought of as having y as its target level of output and committing inputs that would generate this level in the absence of any random shocks. The cost function is then the minimum cost of producing any arbitrary output level y given the input price vector $\mathbf{w}$.

[1] The model's fit is empirically tested in Section 6.

[2] This assumption is empirically verified in Section 6.

Thus, profit is $\pi = pq - c(y, \mathbf{w})$. The firm is risk-averse and seeks to maximize the expected utility of profit. It therefore seeks to solve the problem

$$\underset{y}{Max} E[U(\pi)] = E[U(pq - c(y, \mathbf{w}))]$$

0.3 Estimating Equations

In order to derive estimating equations with additive output uncertainty, we modify the expression for profits to include a shift parameter B, which has an initial value of 0. Thus, profit is

$$\pi = (\bar{p} + \sigma\varepsilon)(y + \theta\eta) - c(y, \mathbf{w}) + B,$$

and the maximization problem is

$$\underset{y}{Max}\ E[U((\bar{p} + \sigma\varepsilon)(y + \theta\eta) - c(y, \mathbf{w}) + B)].$$

The shift parameter also is interpreted as an end-of-period lump sum tax (if negative) or subsidy (if positive). Such shift parameters have been used in theoretical work by Dalal (1990) and have been exploited for empirical estimation by Arshanapalli and Gupta (1996) and Satyanarayan (1999).

The maximization problem implies the existence of an indirect expected utility function V, such that

$$V(\bar{p}, \sigma, \theta, \mathbf{w}, B) = E[U(p(y^* + \theta\eta) - c(y^*, \mathbf{w}) + B)], \tag{1}$$

where y^* is the optimal value of y. Let π^* represent the value of π corresponding to y^*. The envelope theorem applied to (1) implies

$$\frac{\partial V}{\partial \bar{p}} \equiv V_{\bar{p}} = y^* E[U'(\pi^*)] + \theta E[U'(\pi^*)\eta], \tag{2}$$

and

$$\frac{\partial V}{\partial B} \equiv V_B = E[U'(\pi^*)], \tag{3}$$

so that (2) and (3) imply

$$\frac{V_{\bar{p}}}{V_B} = y^* + \frac{\theta E\left[U'\left(\pi^*\right)\eta\right]}{V_B}. \tag{4}$$

In a model with no output uncertainty, (4) would provide a basis for deriving an estimating equation for y^*, since we would have $\eta \equiv 0$, and then $y^* = \dfrac{V_{\bar{p}}}{V_B}$. However, since η is random, this doesn't work, and hence a different procedure is needed to circumvent this problem.

Approximating $E\left[U'\left(\pi^*\right)\eta\right]$ by a second-order Taylor series approximation around the arbitrary point of expansion $\hat{\pi}$, we obtain

$$U'\left(\pi^*\right) \simeq U'\left(\hat{\pi}\right) + U''\left(\hat{\pi}\right)\left(\pi^* - \hat{\pi}\right).$$

Multiplying through by η and taking expectations of both sides,

$$\begin{aligned} E\left[U'\left(\pi^*\right)\eta\right] &\simeq U'\left(\hat{\pi}\right)E\left[\eta\right] + U''\left(\hat{\pi}\right)E\left[\pi^*\eta\right] \\ &= U''\left(\hat{\pi}\right)E\left[\left(\bar{p}y^* + \sigma y^*\varepsilon + \bar{p}\theta\eta + \theta\sigma\eta\varepsilon - c + B\right)\eta\right] \\ &= U''\left(\hat{\pi}\right)\theta\bar{p}. \end{aligned} \tag{5}$$

Now, since $\hat{\pi}$ is a constant, $U''\left(\hat{\pi}\right)$ is a parameter which can be estimated. Letting $\beta \equiv U''\left(\hat{\pi}\right)$, we can approximate $E\left[U'\left(\pi^*\right)\eta\right]$ by $\beta\theta\bar{p}$, and substituting this into (4) yields

$$y^* = \frac{V_{\bar{p}} - \beta\theta^2\bar{p}}{V_B}. \tag{6}$$

In order to get expressions for $V_{\bar{p}}$ and V_B, we need to have an expression for V. Since the form of the indirect expected utility function is not known, we approximate it by a second-order Taylor's series expansion about the arbitrary point $A = \left(\hat{p}, \hat{\sigma}, \hat{\theta}, \hat{\mathbf{w}}, \hat{B}\right)$, where $\hat{B}$ is set equal to its initial value of 0. Letting subscripts denote partial derivatives, and taking the partial derivatives of $V\left(\bar{p}, \sigma, \theta, \mathbf{w}, B\right)$ with respect to $\bar{p}$ and B, respectively, we obtain

$$\begin{aligned} V_{\bar{p}}\left(\bar{p},\sigma,\theta,\mathbf{w},B\right) &\simeq V_{\bar{p}}\left(A\right)+\sum_{i} V_{\bar{p}i}\left(A\right)\left(w_{i}-\hat{w}_{i}\right)+V_{\bar{p}\bar{p}}\left(A\right)\left(\bar{p}-\hat{p}\right) \\ &+V_{\bar{p}\sigma}\left(A\right)\left(\sigma-\hat{\sigma}\right)+V_{\bar{p}B}\left(A\right)B+V_{\bar{p}\theta}\left(A\right)\left(\theta-\hat{\theta}\right), \quad (7) \end{aligned}$$

$$\begin{aligned} V_{B}\left(\bar{p},\sigma,\theta,\mathbf{w},B\right) &\simeq V_{B}\left(A\right)+V_{BB}\left(A\right)B+V_{\bar{p}B}\left(A\right)\left(\bar{p}-\hat{p}\right) \\ &+\sum_{i} V_{Bi}\left(A\right)\left(w_{i}-\hat{w}_{i}\right)+V_{\sigma B}\left(A\right)\left(\sigma-\hat{\sigma}\right) \\ &+V_{\theta B}\left(A\right)\left(\theta-\hat{\theta}\right). \quad (8) \end{aligned}$$

Using (7) and (8), we can rewrite (6) as

$$y^{*}=\frac{V_{\bar{p}}+\sum_{i} V_{\bar{p}i}\tilde{w}_{i}+V_{\bar{p}\bar{p}}\tilde{p}+V_{\bar{p}\sigma}\tilde{\sigma}+V_{\bar{p}B}B+V_{\bar{p}\theta}\tilde{\theta}-\beta\theta^{2}\bar{p}}{V_{B}+V_{BB}B+V_{\bar{p}B}\tilde{p}+\sum_{i} V_{Bi}\tilde{w}_{i}+V_{\sigma B}\tilde{\sigma}+V_{\theta B}\tilde{\theta}}, \quad (9)$$

where tildes symbolize deviations from the point of approximation and all the first and second partial derivatives of V are evaluated at the point of expansion A. The derivatives of V and β in (9) are parameters and can be estimated. However, for estimation purposes, some normalization is required since (9) is homogeneous of degree zero in all the parameters. A convenient normalization is $V_B(A) = 1$. Also, we will set B equal to its initial value of 0. Thus (9) becomes

$$y^{*}=\frac{V_{\bar{p}}+V_{\bar{p}\bar{p}}\tilde{p}+\sum_{i} V_{\bar{p}i}\tilde{w}_{i}+V_{\bar{p}\sigma}\tilde{\sigma}+V_{\bar{p}\theta}\tilde{\theta}-\beta\theta^{2}\bar{p}}{1+V_{\bar{p}B}\tilde{p}+\sum_{i} V_{Bi}\tilde{w}_{i}+V_{\sigma B}\tilde{\sigma}+V_{\theta B}\tilde{\theta}} \quad (10)$$

0.4 Hypothesis Testing

We will use (10) to develop hypothesis tests for Pope's (1980) separable utility function, risk neutrality, CARA, and a quadratic utility function.

Separable Utility Function. This has the form $U(\pi) = a\pi - b(\pi - \bar{\pi})^2$, so that $U'(\pi) = a - 2b(\pi - \bar{\pi})$ and $E[U'(\pi)] = a$, a constant. This implies that V_B is constant since $E[U'(\pi)] = V_B$ and thus

$$V_{B\bar{p}} = V_{Bi} = V_{B\sigma} = V_{B\theta} = 0. \tag{11}$$

Then (10) is reduced to

$$y^* = V_{\bar{p}}(A) + V_{\bar{p}\bar{p}}\tilde{p} + \sum_i V_{\bar{p}i}\tilde{w}_i + V_{\bar{p}\sigma}\tilde{\sigma} + V_{\bar{p}\theta}\tilde{\theta} - \beta\bar{p}\theta^2, \tag{12}$$

and (11) implies that the number of independent restrictions imposed in (10) is $n+3$, where n is the number of inputs.

Risk Neutrality. In this case $U'' = 0$. This implies that U' is constant and hence so is $E[U'(\pi)]$. Thus V_B is constant, and all its partial derivatives are equal to zero. Thus (11) applies. In addition, $E[U(\pi)] = a + bE[\pi]$, implying that $EU(\pi)$ is independent of both price risk and output risk. This implies $V_\sigma = V_\theta = 0$, and hence all the second partial derivatives of V_σ and V_θ are also 0. Therefore, $V_{\bar{p}\sigma} = V_{\bar{p}\theta} = 0$. Further, since β is equal to $U''(\hat{\pi})$, we must also have $\beta = 0$. Thus, the parameter restrictions implied by risk neutrality are

$$V_{B\bar{p}} = V_{Bi} = V_{B\sigma} = V_{B\theta} = V_{\bar{p}\sigma} = V_{\bar{p}\theta} = \beta = 0, \tag{13}$$

which constitute $n+6$ independent restrictions. When (13) is substituted into (10), we get

$$y^* = V_{\bar{p}} + V_{\bar{p}\bar{p}}\tilde{p} + \sum_i V_{\bar{p}i}\tilde{w}_i. \tag{14}$$

Constant Absolute Risk Aversion. Recall that $V_B = E[U'(\pi^*)]$. Differentiating with respect to σ yields

$$\begin{aligned} V_{B\sigma} &= E\left[U''(\pi)\left(q\varepsilon + p\frac{\partial y^*}{\partial\sigma} - c_y\frac{\partial y^*}{\partial\sigma}\right)\right] \\ &= -kE\left[U'(\pi)\left((p - c_y)\frac{\partial y^*}{\partial\sigma} + q\varepsilon\right)\right], \end{aligned}$$

since with CARA $U''(\pi) = -kU'(\pi)$.

From the first-order condition, $E\left[U'(\pi)(p - c_y)\right] = 0$, and thus

$$V_{B\sigma} = -kE\left[U'(\pi)\,q\varepsilon\right] = -kV_{\sigma}. \tag{15}$$

Now $E\left[U'(\pi)\,q\varepsilon\right] = \dfrac{1}{\sigma}E\left[U'(\pi)\,q\,(p-\bar{p})\right] = \dfrac{1}{\sigma}Cov\left(U'(\pi)\,q, p\right) < 0.$ When evaluated at the point of expansion, (15) becomes

$$V_{B\sigma}(A) = -kV_{\sigma}(A), \tag{16}$$

and since $V_{\sigma}(A)$ is negative, it is clear that (16) implies $V_{B\sigma}(A)$ is positive. Thus constant absolute risk aversion implies $V_{B\sigma}(A) > 0$. We can test for $V_{B\sigma}(A) = 0$; if we cannot reject this hypothesis then we will reject CARA.

Quadratic Utility. If the utility function is quadratic $\partial y^*/\partial\theta = 0$ and then we can use this to test for the existence of a quadratic utility function. If we reject this hypothesis, then quadratic preferences will be ruled out. To obtain the relevant parameter restrictions, differentiate (10) with respect to θ

$$\frac{\partial y^*}{\partial\theta} = \frac{1}{D^2}\left[D\left(V_{\bar{p}\theta} - 2\beta\theta\bar{p}\right) - NV_{B\theta}\right] = 0,$$

where D and N are the denominator and the numerator of (10), respectively. At the point of approximation, $D = 1$, $N = V_{\bar{p}}(A) - \beta\hat{\theta}^2\hat{p}$ and thus

$$V_{\bar{p}\theta} = 2\beta\hat{\theta}\hat{p} + \left\{V_{\bar{p}}(A) - \beta\hat{\theta}^2\hat{p}\right\}V_{B\theta} = 0,$$

yielding 1 independent restriction

0.5 Methods of Generating Moments

The data required for estimation of (10) include the mean and standard deviation of output and its price. Since these are not directly observable we have to generate these values from observable data. There is some arbitrariness in the method chosen to do so, since there is no unambiguously "best" approach. Some empirical studies have adopted an extremely simple approach such as Arshanapalli and Gupta (1996), who used a simple moving average process, while others use much more complex methods.

In order to generate a series of expected prices, we have chosen to use the method developed by Chavas and Holt (1996) where the price at time t is considered as a random walk with a drift. Thus,

$$p_t = \delta + \alpha p_{t-1} + \varepsilon_i,$$

where p_t is the price at time t, p_{t-1} is the previous year's market price, δ is a drift parameter, and ε_i is a random variable with $E[\varepsilon_i] = 0$. Hence

$$E_t [p_t] = \delta + \alpha p_{t-1},$$

Similarly, to generate a series for y^*, we use the method developed by and Lapan and Moschini (1994), and model output at time t by

$$q_t = \phi + \varphi q_{t-1} + u_i,$$

where q_t is the output at time t, q_{t-1} is the previous year's output, and u is an error term with $E[u] = 0$. Hence,

$$E[q_t] = y_t^* = \phi + \varphi q_{t-1}$$

To generate a series for σ we will also use Chavas and Holt's method:

$$\sigma_t^2 = \sum_{j=1}^{3} \omega_j \left(p_{t-j} - E_{t-j} p_{t-j}\right)^2,$$

where the weights ω_j are 0.5, 0.33 and 0.17. This is done to reflect the idea of declining weights. The price variance is thus measured as the weighted sum of squared deviations of the previous prices from their expected values. Similarly, the variance of output, is

$$\theta_t^2 = \sum_{j=1}^{3} \omega_j \left(q_{t-j} - y_{t-j}^*\right)^2.$$

0.6 The Data and Results

We used a data set constructed by Berndt and Wood (1986) for aggregate U.S. manufacturing data for the period 1947-1981. For a more detailed description, see Berndt and Wood (1986).

The aggregate manufacturing output (q) is produced using four inputs: materials (m), energy (e) capital (k), labor (l), with prices given, respectively, by w_m ,w_e, w_k, and w_l. And Gross output price is given by p.

Rewriting the estimating equations to explicitly introduce the 4 input prices we will be using, (10) becomes

$$y^* = \frac{V_{\bar{p}}(A) + V_{\bar{p}\bar{p}}\tilde{p} + V_{\bar{p}e}\tilde{w}_e + V_{\bar{p}l}\tilde{w}_l + V_{\bar{p}m}\tilde{w}_m + V_{\bar{p}k}\tilde{w}_k + V_{\bar{p}\sigma}\tilde{\sigma} + V_{\bar{p}\theta}\tilde{\theta} - \beta\theta^2\bar{p}}{1 + V_{\bar{p}B}\tilde{p} + +V_{Bm}\tilde{w}_m + V_{Be}\tilde{w}_e + V_{Bl}\tilde{w}_l + V_{Bk}\tilde{w}_k + V_{\sigma B}\tilde{\sigma} + V_{\theta B}\tilde{\theta}}; \tag{17}$$

we chose the mid-point in the data series as the point of expansion.

Before we proceeded with the estimation, we empirically tested the assumption $Cov(\varepsilon, \eta) = 0$ and we strongly accepted the null hypothesis that $Cov(\varepsilon, \eta) = 0$. We used non-linear least squares regressions to estimate (17). The results are presented in Table 1. The non-linear least squares regression yields maximum likelihood estimates and thus we can use the likelihood ratio test to test the hypotheses. The statistic has a chi-square distribution, with degrees of freedom equal to the number of independent restrictions.

We first tested for risk neutrality. The results of the estimation appear in Column 6 of Table 1. The test statistic is 37.5 while the critical value at .05 significance level and 10 degrees of freedom is 18.31. The hypothesis is rejected even at .005 significance level.

We rejected $\partial y/\partial\theta = 0$ at .05 significance level; where the test statistic is 12.8. This implies that a quadratic utility is ruled out. The results appear in Column 3 of Table 1.

Then, we tested for separability. The results of estimation appear in Column 5 of Table 1. The test statistic is 15.4 while the critical value at .05 significance level and 7 degrees of freedom is 14.07. Thus, the hypothesis is rejected.

To test for CARA, we need to implement an indirect test since we have an inequality restriction ($V_{B\sigma} > 0$). Thus we will first test for $V_{B\sigma} = 0$. If we accept this hypothesis, we will reject CARA. The test statistic is 2.64, while the critical value at .05 significance level and 1 degree of freedom is 3.84. We cannot reject the hypotheses and hence we reject CARA. The results are reported in Column 4 of Table 1. The model's fit is excellent; F-statistic=171.26; the significance level at which the alternative hypothesis, that each parameter equals 0, would be rejected (α) is .005. Thus we present this model as our final estimating form.

Table 1

Estimation Results [3]

Par.	Unrestricted	$\partial y/\partial\theta = 0$	$V_{B\sigma} = 0$	Separability	Risk N.
$V_{\bar{p}}\,(A)$	623.12	623.13	623.1	625.32	603.78
	(6.275)	(6.93)	(6.6)	(5.530)	(5.601)
$V_{\bar{p}\bar{p}}$	-331.18	-2497.69	-790.01	-684..78	-857.71
	(992.77)	(885.28)	(1072.36)	(138.36)	(121.67)
$V_{\bar{p}\sigma}$	-3646.17	-5532.11	-1106.92	-33.64	
	(1632.37)	(1571)	(631.36)	(215.21)	
$V_{\bar{p}\theta}$	-1.249		1.311	-1.507	
	(1.757)		(2.133)	(.4807)	
$V_{\bar{p}m}$	-510.56	-2042.7	-757.19	435.81	123.87
	(924.83)	(945.28)	(983.8)	(98.58)	(103.77)
$V_{\bar{p}e}$	-45.72	365.11	261.81	-66.71	-29.91
	(243.54)	(246.81)	(307.92)	(15.23)	(11.93)
$V_{\bar{p}k}$	300.68	189.66	293.41	-10.35	-33.88
	(190.18)	(202.74)	(243.33)	(41.66)	(57.06)
$V_{\bar{p}l}$	1042.79	3203.28	1423.41	672.97	978.46
	(991.54)	(935.06)	(10115)	(103.08)	(78.85)

[3] Standard errors in parentheses.

Table 1 (continued)

Par.	Unrestricted	$\partial y/\partial\theta = 0$	$V_{B\sigma} = 0$	Separability	Risk N.
β	-.2024E-01	.0149	-.247	.1584E-02	
	$(.1281E-01)$	(.01)	(.1472)	$(.7777E-02)$	
$V_{B\bar{p}}$	.8085	-2.63	-.2099		
	(1.484)	(1.32)	(1.654)		
$V_{B\sigma}$	-4.806	-6.53			
	(2.07)	(2)			
$V_{B\theta}$	.1912	-.0007	.5755		
	(.2773)	(.002)	(.3352)		
V_{Bm}	-1.365	-3.192	-1.863		
	(1.226)	(1.259)	(1.36)		
V_{Be}	.4508	.513	.3707		
	(.2995)	(.301)	(.3725)		
V_{Bk}	.4863	.2192	.4894		
	(.2676)	(.283)	(.2807)		
V_{Bl}	.4180	3.67	1.2783		
	(1.496)	(1.4)	(.028)		
Log-L	-121.747	-128.13	-123.069	-129.449	-140.514
# of rest.	0	1	1	7	10
χ^2		12.8	2.64	15.4	37.5
crit. χ^2		3.84	3.84	14.07	18.31

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