Empirical Analysis under Additive /Multiplicative Output Uncertainty

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Abstract

This paper extends the existing estimation methods to allow the empirical estimation of two common forms of output uncertainty: additive risk and multiplicative risk, as well as price risk.

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1. Introduction

Empirical studies dealing with price uncertainty are abundant; for example, Arshanapalli and Gupta (1996) derived estimating equations by applying uncertainty analogues of Hotelling's lemma and Roy's identity to the indirect expected utility function (see Pope, 1980, and, Dalal 1990). However, their method is not applicable to the models with price and output uncertainty. Few empirical studies included both price and output uncertainty and focused on hedging. For example, Rolfo (1980) computed the ratio of hedge to expected output for cocoa producers. Lapan and Moschini (1994) calculated the same ratio for soya bean farmers.

Assuming simultaneous price and output uncertainty, this paper empirically estimate the most two common forms of output risk: additive risk and multiplicative risk (see Honda,1983, and, Grant 1985). Then it empirically determines which form is more suitable. The theory does not provide a conclusive criteria for the choice between additive risk and multiplicative risk (see Honda,1983). Therefore, the choice should be empirical.

2. The model

A competitive firm faces an uncertain output price given by $\tilde{p} = \bar{p} + \sigma\tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is random with $E\tilde{\varepsilon} = 0$ and $Var(\tilde{\varepsilon}) = 1$, so that $E\tilde{p} = \bar{p}$ and $Var(\tilde{p}) = \sigma^2$. The level of output realized at the end of the production process is also not known ex ante. Output has both a random and a nonrandom component and is given by $\tilde{q}$ and defined as $\tilde{q} = y + \theta\tilde{\eta}$ under additive output uncertainty and as $\tilde{q} = (1 + \theta\tilde{\eta})\, y$ under multiplicative output uncertainty; where $\tilde{\eta}$ is random with $E\tilde{\eta} = 0$ and $Var(\tilde{\eta}) = 1$, so that $Var(\tilde{q}) = \theta^2$ and the expected value of output is $E\tilde{q} = y$. We assume that $Cov(\varepsilon, \eta) = 0$.[1] Costs are known with certainty and are given by the cost function $c(y, \mathbf{w})$. The firm may be thought of as having y as its target level of output and committing inputs that would generate this level in the absence of any random shocks. The cost function is then the minimum cost of producing any arbitrary output level y given the input price vector $\mathbf{w}$. Thus, profit is $\tilde{\pi} = \tilde{p}\tilde{q} - c(y, \mathbf{w})$. The firm is risk-averse and seeks to maximize the expected utility of profit. Thus its objective is

$$\underset{y}{Max} EU(\tilde{\pi}),$$

and the first-order condition is

$$EU'(\tilde{\pi})(\tilde{p}v - c_y(y, \mathbf{w})) = 0, \tag{1}$$

[1] We empirically verified this assumption in Section 4. Theoretically, this assumption is used by Viaene and Zilcha (1998), Honda (1983), Losq (1982), etc.

where $v = 1$ and $1 + \theta\tilde{\eta}$ with additive output uncertainty and multiplicative output uncertainty, respectively.

3. Estimating Equations

In order to derive estimating equations with additive output uncertainty, we rewrite (1) as

$$\bar{p} - c_y(y, \mathbf{w}) + \sigma \frac{EU'(\tilde{\pi})\tilde{\varepsilon}}{EU'(\tilde{\pi})} = 0. \tag{2}$$

We need to derive expression for $EU'(\tilde{\pi})\tilde{\varepsilon}$ since it is random. To do so, consider the following Taylor series expansion at the arbitrary point of approximation $\hat{\pi}$

$$U'(\tilde{\pi}) \approx U'(\hat{\pi}) + U''(\hat{\pi})(\tilde{\pi} - \hat{\pi}).$$

Taking expectations of both sides yields

$$EU'(\tilde{\pi}) \approx U'(\hat{\pi}) + U''(\hat{\pi})(E\tilde{\pi} - \hat{\pi}).$$

Similarly,

$$EU'(\tilde{\pi})\tilde{\varepsilon} \approx U''(\hat{\pi}) E\tilde{\pi}\tilde{\varepsilon} = U''(\hat{\pi})\sigma y,$$

letting $\beta \equiv \dfrac{U''(\hat{\pi})}{U'(\hat{\pi}) + U''(\hat{\pi})(E\tilde{\pi} - \hat{\pi})}$, we have $\dfrac{EU'(\tilde{\pi})\tilde{\varepsilon}}{EU'(\tilde{\pi})} \approx \beta\sigma y$; substituting this back into (2), we obtain

$$\bar{p} - c_y(y, \mathbf{w}) + \beta\sigma^2 y = 0. \tag{3}$$

Now, in order to derive an expression for $c_y(y, \mathbf{w})$, we first derive an expression for $c(y, \mathbf{w})$. To do so, we approximate $c(y, \mathbf{w})$ by a second-order Taylor series expansion around the arbitrary point of expansion $\psi = (\hat{y}, \hat{\mathbf{w}})$

$$\begin{aligned} c(y, \mathbf{w}) &\approx c(\psi) + c_y(\psi)(y - \hat{y}) + \sum_i c_i(\psi)(w_i - \hat{w}_i) + \frac{1}{2} c_{yy}(\psi)(y - \hat{y})^2 \\ &+ \frac{1}{2} \sum_i \sum_j c_{ij}(\psi)(w_i - \hat{w}_i)(w_j - \hat{w}_j) \\ &+ \sum_i c_{yi}(\psi)(y - \hat{y})(w_i - \hat{w}_i). \end{aligned} \tag{4}$$

Differentiating (4) with respect to y yields

$$c_y(y, \mathbf{w}) \approx c_y(\psi) + c_{yy}(\psi)(y - \hat{y}) + \sum_i c_{yi}(\psi)(w_i - \hat{w}_i) \tag{5}$$

Substituting (5) into (3) and solving for y, we obtain

$$y = \frac{-\bar{p} + c_y(\psi) - c_{yy}(\psi)\hat{y} + \sum_i c_{yi}(\psi)(w_i - \hat{w}_i)}{\beta\sigma^2 - c_{yy}(\psi)}. \tag{6}$$

All the partial derivatives of c, as well as β, are parameters that can be estimated.

Under multiplicative risk, the first-order condition becomes

$$\bar{p} - c_y(y, \mathbf{w}) + \sigma \frac{EU'(\tilde{\pi})\tilde{\varepsilon}}{EU'(\tilde{\pi})} + \bar{p}\theta \frac{EU'(\tilde{\pi})\tilde{\eta}}{EU'(\tilde{\pi})} + \sigma\theta \frac{EU'(\tilde{\pi})\tilde{\varepsilon}\tilde{\eta}}{EU'(\tilde{\pi})} = 0. \tag{7}$$

Following the procedures above, we obtain the following approximations

$$\frac{EU'(\tilde{\pi})\tilde{\varepsilon}}{EU'(\tilde{\pi})} \approx \beta\sigma y; \frac{EU'(\tilde{\pi})\tilde{\eta}}{EU'(\tilde{\pi})} \approx \beta\bar{p}\theta y; \frac{EU'(\tilde{\pi})\tilde{\varepsilon}\tilde{\eta}}{EU'(\tilde{\pi})} \approx \beta\sigma\theta y.$$

Thus (6) becomes

$$y = \frac{-\bar{p} + c_y(\psi) - c_{yy}(\psi)\,\hat{y} + \sum_i c_{yi}(\psi)\,(w_i - \hat{w}_i)}{\beta\left(\sigma^2 + \bar{p}^2\theta^2 + \sigma^2\theta^2\right) - c_{yy}(\psi)}. \tag{8}$$

Equation (8) is comparable to (6) and thus we can empirically decide which form better fits the data.

4. Empirical Results

We use a data set constructed by Berndt and Wood (1986) for aggregate U.S. manufacturing data for the period 1947-1981. For a more detailed data description, see Berndt and Wood, 1986. The aggregate manufacturing output (q) is produced using four inputs: materials (m), energy (e) capital (k), labor (l), with prices given, respectively, by w_m ,w_e, w_k, and w_l; and p is the gross output price. We generated data series for $\bar{p}$, σ^2, θ^2 (and thus for ε and η) using Chavas and Holt's method (see Chavas and Holt, 1996). We empirically tested the assumption $Cov(\varepsilon, \eta) = 0$ and we strongly accepted the null hypothesis that $Cov(\varepsilon, \eta) = 0$. Using (6) and (8) our estimating forms are

$$y = \frac{-\bar{p} + c_y(\psi) - 633c_{yy}(\psi) + c_{ym}(\psi)\,\hat{m} + c_{ye}(\psi)\,\hat{e} + c_{yk}(\psi)\,\hat{k} + c_{yl}(\psi)\,\hat{l}}{\beta\sigma^2 - c_{yy}(\psi)} \tag{9}$$

$$y = \frac{-\bar{p} + c_y(\psi) - 633c_{yy}(\psi) + c_{ym}(\psi)\,\hat{m} + c_{ye}(\psi)\,\hat{e} + c_{yk}(\psi)\,\hat{k} + c_{yl}(\psi)\,\hat{l}}{\beta\left(\sigma^2 + \bar{p}^2\theta^2 + \sigma^2\theta^2\right) - c_{yy}(\psi)}, \tag{10}$$

where ^denotes the deviations from the point of approximation.

We estimated (9) and (10) using non-linear least squares regressions. The results appear in Table 1. While the additive uncertainty model has an excellent fit (based on both F-test and likelihood ratio test; the statistics are reported in Table 1), this is not the case for the multiplicative uncertainty model. The latter has a very poor fit and thus we reject the model altogether and conclude the data is more consistent with the additive output uncertainty model.

To conclude, in at least this instance it appears that additive output uncertainty might be the more appropriate method of modeling output uncertainty. But this does not imply that, in general, multiplicative risk should be ruled out. Since some microeconomic theorists prefer the multiplicative form, the results are illustrative that additive risk shouldn't be ruled out without empirical evidence.

Table1. Empirical Results

	Additive	Multiplicative		Additive	Multiplicative
$c_y\,(\psi)$	.8368	.2711	$c_{yk}\,(\psi)$	-.0241	-.0024
	(.0090)	(20702751)		(.0639)	(5243921)
$c_{yy}\,(\psi)$	-.0011	-137.74	$c_{yl}\,(\psi)$	1.091	.0012
	(.00015)	(.43E-10)		(.1338)	(17636966)
$c_{ym}\,(\psi)$	.1669	-.0010	β	-.0030	1.190
	(.1089)	(23874727)		(.0029)	(37841618)
$c_{ye}\,(\psi)$	-.0375	.0104			
	(.0160)	(4363864)			
Log L	-140	-284.97			
Rest.Log L	-200.82	-200.82			
F (prob value)	197.96 (%0)				
Adj R^2	.98	-219.65			

Standard errors are in parentheses

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